Rigid Body Dynamics, Collision Response, & Deformation

Last Time?
- Navier-Stokes Equations
- Conservation of Momentum & Mass
- Incompressible Flow

Today
- Rigid Body Dynamics
- Collision Response
- Non-Rigid Objects
- Finite Element Method
- Deformation
- Level-of-Detail

Rigid Body Dynamics
- How do we simulate this object’s motion over time?
- We could discretize the object into many particles…
  - But a rigid body does not deform
  - Only a few degrees of freedom
- Instead, we use only one particle at the center of mass
- Compute net force & net torque

Degree of Freedom (DOF)
- Rotations:
  - Translations count too… → 6 DOF

Collisions

- Detection
- Response
- Overshooting problem (when we enter the solid)

Collisions - Overshooting

- Usually, we detect collision when it’s too late: we’re already inside
- Solutions: back up
  - Compute intersection point
  - Compute response there
  - Advance for remaining fractional time step
- Other solution: Quick and dirty fix up
  - Just project back to object closest point

Collision Response

- tangential velocity \( v_t \) unchanged
- normal velocity \( v_n \) reflects:
  \[ v = v_t + v_n \]
  \[ v_n \rightarrow v_n - \epsilon v_n \]
- coefficient of restitution
  - \( 1 \) for elastic
  - \( 0 \) for plastic
- change of velocity = \(-(1+\epsilon)v\)
- change of momentum \( \text{Impulse} = -m(1+\epsilon)v \)

Energy & Rigid Body Collisions

- Total Energy stays constant if there is no damping and no friction
- Rotational Energy is constant between collisions

Collision Between Two Objects

- Suppose a vertex on body A is colliding into an edge of body B at point P. Define the following variables:

  \[ m_A, m_B = \text{mass of bodies A, B} \]
  \[ \mathbf{r}_{AP} = \text{distance vector from center of mass of body B to point P} \]
  \[ \mathbf{r}_{BP} = \text{distance vector from center of mass of body A to point P} \]
  \[ \vartheta_A, \vartheta_B = \text{initial pre-collision angular velocity of bodies A, B} \]
  \[ \vartheta_{AP}, \vartheta_{BP} = \text{final post-collision angular velocity of bodies A, B} \]
  \[ \psi_A, \psi_B = \text{initial pre-collision velocities of center of mass bodies A, B} \]
  \[ \psi_{AP}, \psi_{BP} = \text{final post-collision velocities of center of mass bodies A, B} \]
  \[ v_{1A}, v_{1B} = \text{initial pre-collision velocity of impact point on body A} \]
  \[ v_{2A}, v_{2B} = \text{initial pre-collision velocity of impact point on body B} \]
  \[ \mathbf{n} = \text{normal (perpendicular) vector to edge of body B} \]
  \[ \epsilon = \text{elasticity (0 = inelastic, 1 = perfectly elastic)} \]

Center of Mass & Moment of Inertia

- Center of Mass: mean location of all mass in the system
- Moment of Inertia: a measure of an object’s resistance to changes to its rotation
- If a solid cylinder & a hollow tube have the same radius & the same mass, which will reach the bottom of the ramp first?
Advanced Collisions

- What about Friction?
- What if the contact between two objects is not a single point?
- What if more than two objects collide simultaneously?

Rigid Body Dynamics

- Physics
  - Velocity
  - Acceleration
  - Angular Momentum
- Collisions
- Friction

http://www-cs-students.stanford.edu/~dalewis/cs448a/rigidbody.html

Collisions

- We know how to simulate bouncing really well
- But resting collisions are harder to manage

Victor J. Milerovic & Harald Schmidl
Optimization-Based Animation
SIGGRAPH 2001

Guentelmann, Bridson & Farkas
Nonconvex Rigid Bodies with Stacking
SIGGRAPH 2003

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Simulation of Non-Rigid Objects

- We modeled string & cloth using mass-spring systems. Can we do the same?
- Yes...
- But a more physically accurate model uses volumetric elements:

Strain & Stress

- Stress
  - the internal distribution of forces within a body that balance and react to the loads applied to it
  - normal stress & shear stress
- Strain
  - material deformation caused by stress.
  - measured by the change in length of a line or by the change in angle between two lines

\[ \varepsilon = \frac{\Delta l}{l_0} \]
Finite Element Method

- To solve the continuous problem (deformation of all points of the object)
  - Discretize the problem
  - Express the interrelationship
  - Solve a big linear system
- More principled than Mass-Spring

Diagram from Debunne et al. 2001

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Multi-Resolution Deformation

- Use Voronoi diagrams to match parent & child vertices.
- Interpolate values for inactive interface vertices from active parent/child vertices
- Need to avoid interference of vibrations between simulations at different resolutions


Level of Detail

- Interactive shape deformation
- Use high-resolution model only in areas of extreme deformation

Gilles Debunne, Mathieu Desbrun, Marie-Paule Cani, & Alan H. Barr
Dynamic Real-Time Deformations using Space & Time Adaptive Sampling
SIGGRAPH 2001

Pre-computation & Simulation

- FEM matrix pre-computed
- Level of detail coupling pre-computed for rest topology
- What to do if connectivity of elements changes?
  - Cloth is cut or torn
  - Surgery simulation

Multiple Materials

Mueller, Dorsey, McMillan, Jagnow, & Cutler
Stable Real-Time Deformations Symposium on Computer Animation 2002
Tree Stump

Images from Cutler et al. 2002

Readings for Today:

“Deformable Objects Alive!” Coros, Martin, Thomaszewski, Schumacher, & Sumner, SIGGRAPH 2012


How to read a research paper?

(eespecially an advanced paper in a new area)

• Multiple readings are often necessary
• Don't necessarily read from front to back
• Lookup important terms
• Target application & claimed contributions
• Experimental procedure
• How well results & examples support the claims
• Scalability of the technique (order notation)
• Limitations of the technique, places for future research
• Possibilities for hybrid systems with other work

Components of a well-written research paper?

• Motivation/context/related work
• Contributions of this work
• Clear description of algorithm
  – Sufficiently-detailed to allow work to be reproduced
  – Work is theoretically sound
    (hacks/arbitrary constants discouraged)
• Results
  – well chosen examples
  – clear tables/illustrations/visualizations
• Conclusions
  – limitations of the method are clearly stated
Reading for Friday:


- Fracture threshold
- Remeshing  
  - need connectivity info!
- Material properties
- Parameter tuning