Subdivision Surfaces

Homework 1:

• Questions/Comments?
Questions on Homework?

- What’s an illegal edge collapse?

- To be legal, the ring of vertex neighbors *must be unique* (have no duplicates)!

Last Time?

- Curves & Surfaces
- Continuity Definitions
  - $C^0, G^1, C^1, \ldots C^\infty$
- Interpolation vs. Approximation Splines
- Cubic Bezier & BSpline
Today

• **Spline Surfaces / Patches**
  – Tensor Product
  – Bezier Patches
  – Trimming Curves

• Subdivision Surface “Zoo”
• Misc. Mesh/Surface Vocabulary
• “Subdivision Surfaces in Character Animation”
• “Piecewise Smooth Surface Reconstruction”
• Interpolating Subdivision

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Tensor Product

• Of two vectors:

\[
\begin{bmatrix}
a_1 & a_2 & a_3 \\
b_1 & b_2 & b_3 & b_4
\end{bmatrix} \otimes
\begin{bmatrix}
a_1 & a_2 & a_3 & a_4 \\
b_1 & b_2 & b_3 & b_4
\end{bmatrix} =
\begin{bmatrix}
a_1 b_1 & a_2 b_1 & a_3 b_1 & a_4 b_1 \\
a_1 b_2 & a_2 b_2 & a_3 b_2 & a_4 b_2 \\
a_1 b_3 & a_2 b_3 & a_3 b_3 & a_4 b_3 \\
a_1 b_4 & a_2 b_4 & a_3 b_4 & a_4 b_4
\end{bmatrix}
\]

• Similarly, we can define a surface as the tensor product of two curves....

Farin, Curves and Surfaces for Computer Aided Geometric Design
Bilinear Patch

Bi-lerp a (typically non-planar) quadrilateral

\[
Q(s, t) = L(L(P_1, P_2, t), L(P_3, P_4, t), s)
\]

Notation: \( L(P_1, P_2, \alpha) \equiv (1 - \alpha)P_1 + \alpha P_2 \)

Bilinear Patch

- Smooth version of quadrilateral with non-planar vertices...

- But will this help us model smooth surfaces?
- Do we have control of the derivative at the edges?
Ruled Surfaces in Art & Architecture

http://www.bergenwood.no/wp-content/media/images/frozenmusic.jpg

Chiras Julia
Astri Isabella
Matiss Shteinerts

Antoni Gaudi
Children’s School
Barcelona

http://www.lonelyplanetimages.com/images/399954

Bicubic Bezier Patch

Notation: \( \text{CB}(P_1, P_2, P_3, P_4, \alpha) \) is Bézier curve with control points \( P_i \) evaluated at \( \alpha \)

Define “Tensor-product” Bézier surface

\[
Q(s, t) = \text{CB}(\text{CB}(P_{00}, P_{01}, P_{02}, P_{03}, s), \\
\text{CB}(P_{10}, P_{11}, P_{12}, P_{13}, t), \\
\text{CB}(P_{20}, P_{21}, P_{22}, P_{23}, t), \\
\text{CB}(P_{30}, P_{31}, P_{32}, P_{33}, t), \\
s, t))
\]

\(s\) and \(t\) range from 0 to 1.
Editing Bicubic Bezier Patches

Curve Basis Functions

Surface Basis Functions

Bicubic Bezier Patch Tessellation

- Given 16 control points and a tessellation resolution, we can create a triangle mesh

resolution: 5x5 vertices
resolution: 11x11 vertices
resolution: 41x41 vertices
Modeling with Bicubic Bezier Patches

- Original Teapot specified with Bezier Patches

- But it’s not "watertight": it has intersecting surfaces at spout & handle, no bottom, a hole at the spout tip, a gap between lid & base

Trimming Curves for Patches

Shirley, Fundamentals of Computer Graphics
Spline-Based Modeling Headaches

Questions?

- Bezier Patches?

  or

- Triangle Mesh?

Henrik Wann Jensen
Today

- Spline Surfaces / Patches
- **Subdivision Surface “Zoo”**
  - Doo Sabin (anything!)
  - Loop (triangles only)
  - Catmull Clark (turns everything into quads)
  - … many others!
- Misc. Mesh/Surface Vocabulary
- “Subdivision Surfaces in Character Animation”
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Chaikin's Algorithm
Doo-Sabin Subdivision

Idea: introduce a new vertex for each face
At the midpoint of old vertex, face centroid

Doo-Sabin Subdivision

Original Cube  The 1st subdivision  The 2nd subdivision
The 3rd subdivision  The 5th subdivision
http://www.ke.ics.saitama-u.ac.jp/xuz/pic/doo-sabin.gif
Loop Subdivision

Subdivision Rules. The masks for the Loop scheme are shown in Figure 4.3. For boundaries and edges tagged as crease edges, special rules are used. These rules produce a cubic spline curve along the boundary/crease. The curve only depends on control points on the boundary/crease.

Figure 4.3: Loop subdivision: in the picture above, $\beta$ can be chosen to be either $\frac{1}{2}(5/8 - (\frac{1}{2} + \frac{1}{2} \cos \frac{\pi}{8})^2)$ (original choice of Loop [16]), or, for $n > 3$, $\beta = \frac{1}{16}$ as proposed by Warren [33]. For $n = 3$, $\beta = 3/16$ can be used.

SIGGRAPH 2000 course notes
Subdivision for Modeling and Animation (page 70)
Catmull Clark Subdivision

\[ v^{i+1} = \frac{v' + v'' + \frac{v'''}{2} + \frac{v''''}{4}}{4} \]

where subscripts are taken modulo the valence of the central vertex \( v' \) (The valence of a vertex is the number of edges incident to it.) Finally, a vertex point \( v' \) is computed as

\[ v^{i+1} = \frac{n-2}{n} v' + \frac{1}{n} \sum_{j} \frac{1}{m_j} \sum_{j} v^{i+1} \]

Vertices of valence 4 are called ordinary; others are called extraordinary.

Figure 3: Recursive subdivision of a topologically complicated mesh: (a) the control mesh; (b) after one subdivision step; (c) after two subdivision steps; (d) the limit surface.

Adding creases to Loop Subdivision

- Vertex & edge masks
- Limit masks
  - Position
  - Tangent

"Subdivision Surfaces in Character Animation",
DeRose, Kass & Truong, SIGGRAPH 1998
Questions?

Justin Legakis

Today

- Spline Surfaces / Patches
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• **Genus:** The maximum number of disjoint simple closed curves which can be cut from an orientable surface of genus g without disconnecting it is g.


• **Homeomorphic/Topological equivalence:** a continuous stretching and bending of the object into a new shape

• **Dihedral Angle**:  
  - the angle between the planes of two triangular faces  
  - “looking down the edge” between two faces, the angle between the faces.

• **Valence (a.k.a. degree)**: the number of edges incident to the vertex.

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• **Warp & weft**: Yarns used in weaving. Because the weft does not have to be stretched in the way that the warp is, it can generally be less strong.

Today

- Spline Surfaces / Patches
- Subdivision Surface “Zoo”
- Seams In Subdivision
- Misc. Mesh/Surface Vocabulary
- “Subdivision Surfaces in Character Animation”
- “Piecewise Smooth Surface Reconstruction”
- Interpolating Subdivision

Reading for Today

- DeRose, Kass, & Truong, "Subdivision Surfaces in Character Animation", SIGGRAPH 1998

Figure 5: Geri’s hand as a piecewise smooth Catmull-Clark surface. Infinitely sharp creases are used between the skin and the finger nails.
Subdivision Surfaces in Character Animation

- Catmull Clark Subdivision Rules
- Semi-sharp vs. Infinitely-sharp creases
- Mass-Spring Cloth (*next week*)
- Hierarchical Mesh for Collision
- Texturing Subdivision Surfaces

Figure 11: (a) A texture mapped regular pentagon comprised of 5 triangles; (b) the pentagonal model with its vertices moved; (c) A subdivision surface whose control mesh is the same 5 triangles in (a), and where boundary edges are marked as creases; (d) the subdivision surface with its vertices positioned as in (b).

Catmull-Clark in Pixar Production

- Based on quadrilaterals
  - Like NURBS, specifically cubic bsplines
  - Implicit adjacency in subdivided microgeometry
  - Better than triangles for symmetric objects
Today

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Reading for Today

• Hoppe et al., “Piecewise Smooth Surface Reconstruction” SIGGRAPH 1994
Piecewise Smooth Surface Reconstruction

- From input: scanned mesh points
  - Estimate topological type (genus)
  - Mesh optimization (a.k.a. simplification)
  - Smooth surface optimization

Adding creases to Loop Subdivision

- Vertex & edge masks
- Limit masks
  - Position
  - Tangent
Piecewise Smooth Surface Reconstruction

- Optimization
- Remeshing

- Crease subdivision masks *decouple* behavior of surface on either side of crease
- Crease rules cannot model a cone
- Optimization can be done locally
  - subdivision control points have only local influence

- Results
  - Noise?
  - Applicability?
  - Limitations?
  - Running Time
Today

- Spline Surfaces / Patches
- Subdivision Surface “Zoo”
- Seams In Subdivision
- Misc. Mesh/Surface Vocabulary
- “Subdivision Surfaces in Character Animation”
- “Piecewise Smooth Surface Reconstruction”
- *Interpolating Subdivision*

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**Interpolation vs. Approximation Curves**

- Interpolation Curve – over constrained → lots of (undesirable?) oscillations
- Approximation Curve – more reasonable?

![Interpolation vs. Approximation Curves Diagram]
Interpolating Subdivision

- Chaikin:

- Doo-Sabin:

of the centroids of each edge/face

Interpolating Subdivision

- *Interpolation* vs. *Approximation* of control points
- Handle arbitrary topological type
- Reduce the “extraneous bumps & wiggles”

"Efficient, fair interpolation using Catmull-Clark surfaces", Halstead, Kass & DeRose, SIGGRAPH 1993
Interpolation of Catmull-Clark Surfaces

• Solve for a new control mesh (generally “bigger”) such that when Catmull-Clark subdivision is applied it interpolates the original control mesh.

Vertex Position in Limit

• $V_n$ stores the center vertex & surrounding edge & face vertices as a big column vector.

\[ V_{n}^{i+1} = S_n V_{n}^{i} \]

• When $n = 4$: (n = valence)

\[ V_{n}^{\infty} := \lim_{i \to \infty} S_n^{i} V_{n}^{1} \]
Solve for New Positions

- Goal: Find the control mesh vertex positions, \( x \) (a column vector of 3D points), such that the position of the vertices in the limit match the input vertices, \( b \) (also a column vector of points)
- Use Least Squares to solve
  \[
  Ax = b
  \]
  where \( A \) is a square matrix with the interpolation rules and connectivity of the mesh

- See paper for extension to match limit normals

Fairing

- Fairing: an additional part or structure added to an aircraft, tractor-trailer, etc. to smooth the outline and thus reduce drag
- Subdivide initial resolution twice so that all constrained vertex positions are independent

Figure 5: Top row: Original mesh, Interpolating mesh, Faired interpolating mesh. Bottom row: Corresponding Catmull-Clark surfaces. Interpolation introduces wiggles which are removed by fairing.
Reading for Tuesday:  \textit{(pick one)}


- "Painting and Rendering Textures on Unparameterized Models", DeBry, Gibbs, Deleon, and Robins, SIGGRAPH 2002

Post a comment/question on the LMS discussion by 10am