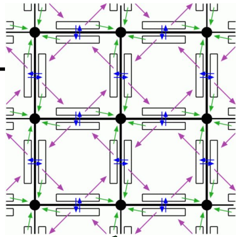
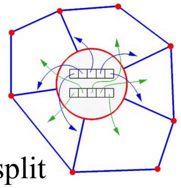
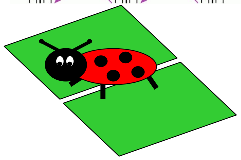
Curves & Surfaces

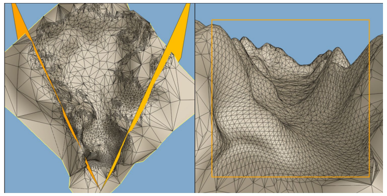
Last Time?



- Adjacency Data Structures
  - Geometric & topologic information
  - Dynamic allocation
  - Efficiency of access
- Mesh Simplification
  - edge collapse/vertex split
  - geomorphs
  - progressive transmission
  - view-dependent refinement







Progressive Meshes

- Mesh Simplification
  - vertex split / edge collapse (only one operation!)
  - geometry & discrete/scalar attributes
  - priority queue
- Level of Detail
  - geomorphs
- Progressive Transmission
- Mesh Compression
- Selective Refinement
  - view dependent

Selective Refinement

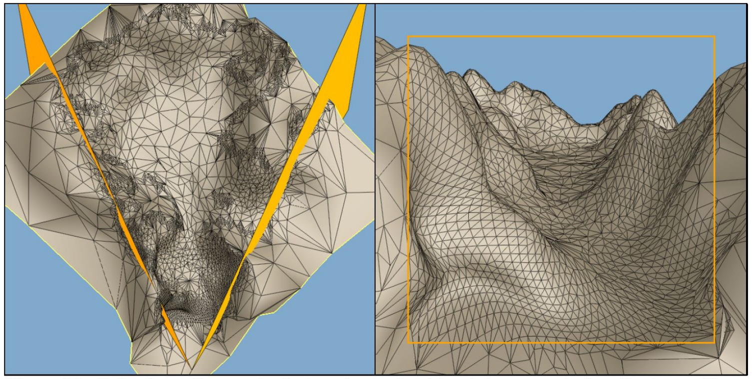
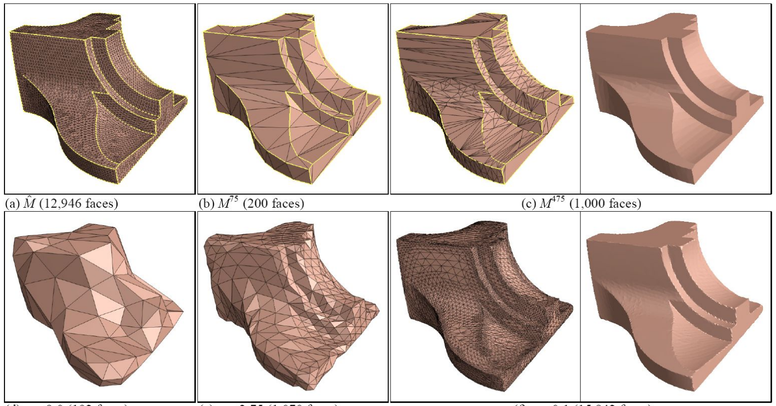


Figure 10: Selective refinement of a terrain mesh taking into account view frustum, silhouette regions, and projected screen size of faces (7,438 faces).

Preserving Discontinuity Curves



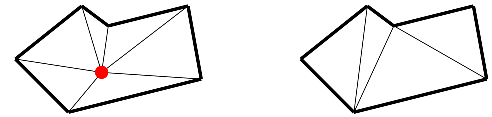
(a) $\hat{M}$ (12,946 faces) (b) M^{75} (200 faces) (c) M^{475} (1,000 faces)

(d) $\epsilon = 9.0$ (192 faces) (e) $\epsilon = 2.75$ (1,070 faces) (f) $\epsilon = 0.1$ (15,842 faces)

Figure 12: Approximations of a mesh $\hat{M}$ using (b–c) the PM representation, and (d–f) the MRA scheme of Eck et al. [7]. As demonstrated, MRA cannot recover $\hat{M}$ exactly, cannot deal effectively with surface creases, and produces approximating meshes of inferior quality.

Other Simplification Strategies

- Remove a vertex & surrounding triangles, re-triangulate the hole



- Merge Nearby Vertices
  - will likely change the topology…

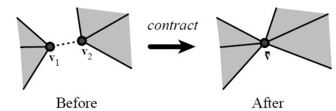


Figure 2: **Non-edge contraction.** When non-edge pairs are contracted, unconnected sections of the model are joined. The dashed line indicates the two vertices being contracted together.

from Garland & Heckbert, "Surface Simplification Using Quadric Error Metrics" SIGGRAPH 1997

When to Preserve Topology?

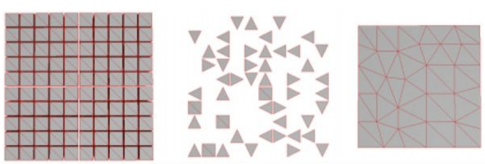


Figure 3: On the left is a regular grid of 100 closely spaced cubes. In the middle, an approximation built using only edge contractions demonstrates unacceptable fragmentation. On the right, the result of using more general pair contractions to achieve aggregation is an approximation much closer to the original.

from Garland & Heckbert, "Surface Simplification Using Quadric Error Metrics" SIGGRAPH 1997

Quadric Error Simplification

- Contract (merge) vertices v_i and v_j if:
  - (v_i, v_j) is an edge, or
  - $\| v_i - v_j \| < t$, where t is a threshold parameter
- Track cumulative error by summing 4x4 quadric error matrices after each operation:

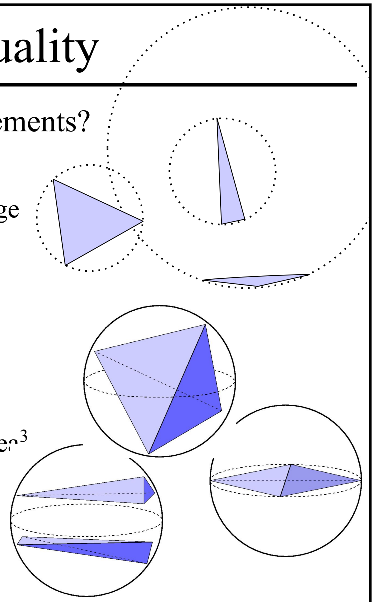
$$\begin{aligned}\Delta(\mathbf{v}) &= \sum_{\mathbf{p}\in \text{planes}(\mathbf{v})} (\mathbf{v}^\mathsf{T}\mathbf{p})(\mathbf{p}^\mathsf{T}\mathbf{v}) \\ &= \sum_{\mathbf{p}\in \text{planes}(\mathbf{v})} \mathbf{v}^\mathsf{T}(\mathbf{p}\mathbf{p}^\mathsf{T})\mathbf{v} \\ &= \mathbf{v}^\mathsf{T}\left(\sum_{\mathbf{p}\in \text{planes}(\mathbf{v})} \mathbf{K}_\mathbf{p}\right)\mathbf{v}\end{aligned}$$

$$\mathbf{K}_\mathbf{p} = \mathbf{p}\mathbf{p}^\mathsf{T} = \begin{bmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & c^2 & cd \\ ad & bd & cd & d^2 \end{bmatrix}$$

Garland & Heckbert, "Surface Simplification Using Quadric Error Metrics" SIGGRAPH 1997

Judging Element Quality

- How "equilateral" are the elements?
- For Triangles?
  - Ratio of shortest to longest edge
  - Ratio of area to perimeter^2
  - Smallest angle
  - Ratio of area to area of smallest circumscribed circle
- For Tetrahedra?
  - Ratio of volume^2 to surface area^3
  - Smallest *solid* angle
  - Ratio of volume to volume of smallest circumscribed sphere

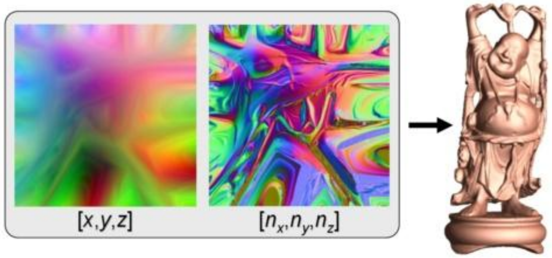


Reading for Today *(option A)*

- "Teddy: A Sketching Interface for 3D Freeform Design", Igarashi et al., SIGGRAPH 1999

- How do we represent objects that don't have flat polygonal faces & sharp corners? What are the right tools to design/construct digital models of blobby, round, or soft things? What makes a user interface intuitive, quick, and easy-to-use for beginners?

Reading for Today *(option B)*



- "Geometry Images", Gu, Gortler, & Hoppe, SIGGRAPH 2002

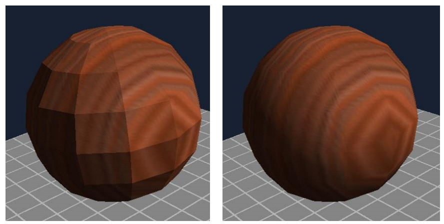
- Can we leverage existing image formats and image compression methods to store geometry? How do we take a complex 3D shape an unroll/flatten/stretch it to a square image? File size? Quality?

Today

- Limitations of Polygonal Models
  - Interpolating Color & Normals in OpenGL
  - Some Modeling Tools & Definitions
- What's a Spline?
  - Interpolation Curves vs. Approximation Curves
  - Linear Interpolation
- Bézier Spline
- BSpline (NURBS)
- Extending to Surfaces – Tensor Product

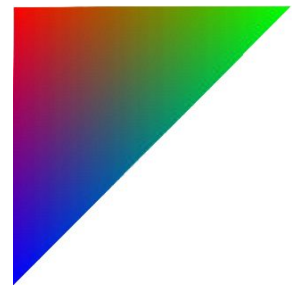
Limitations of Polygonal Meshes

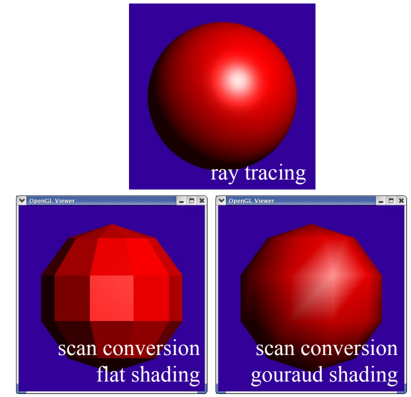
- Planar facets (& silhouettes)
- Fixed resolution
- Deformation is difficult
- No natural parameterization (for texture mapping)
- Incorrect collision detection
- Solid texturing problems



Color & Normal Interpolation

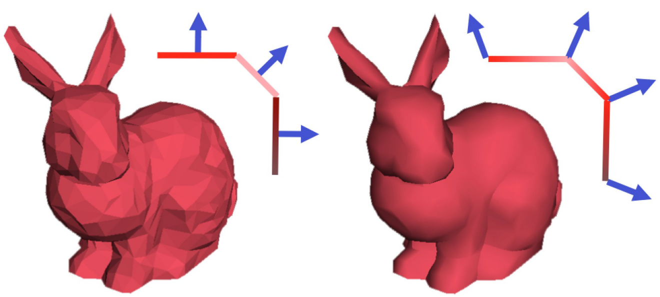
- It's easy in OpenGL to specify different colors and/or normals at the vertices of triangles:
- Why is this useful?





What is Gouraud Shading?

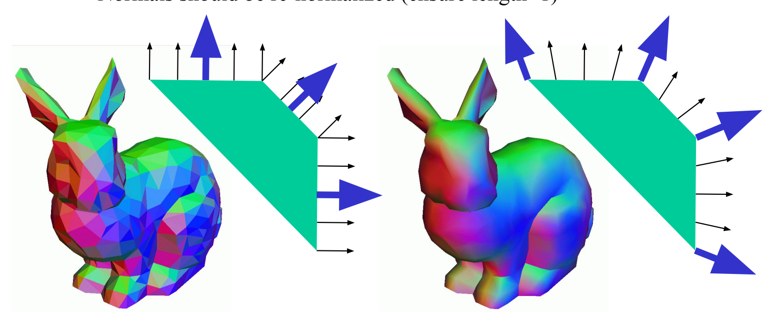
- Instead of shading with the normal of the triangle, we'll shade the vertices with the *average normal* and *interpolate the shaded color* across each face
  - This gives the *illusion of a smooth surface* with smoothly varying normals



- How do we compute Average Normals? Is it expensive??

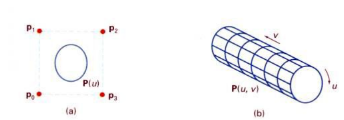
Phong Normal Interpolation (Not Phong *Shading*)

- *Interpolate the average vertex normals* across the face and compute *per-pixel shading*
  - Normals should be re-normalized (ensure length=1)

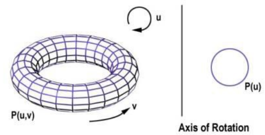


- Before shaders, per-pixel shading was not possible in hardware (Gouraud shading is actually a decent substitute!)

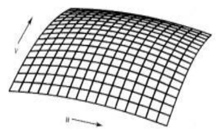
Some Non-Polygonal Modeling Tools



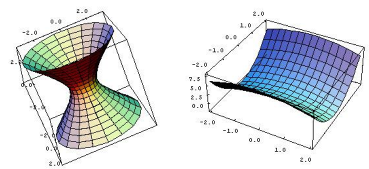
Extrusion



Surface of Revolution

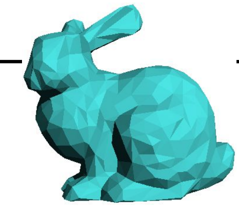


Spline Surfaces/Patches



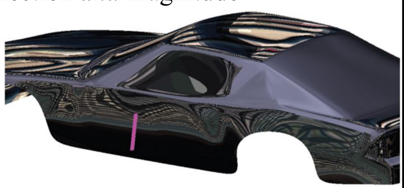
Quadrics and other implicit polynomials

Continuity definitions:



- C^0 continuous
  - curve/surface has no breaks/gaps/holes
- G^1 continuous
  - tangent at joint has same direction

- C^1 continuous
  - curve/surface derivative is continuous
  - tangent at joint has same direction *and* magnitude
- C^n continuous
  - curve/surface through n^{th} derivative is continuous
  - important for shading



"Shape Optimization Using Reflection Lines", Tosun et al., 2007

Questions?

Today

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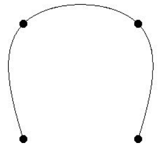
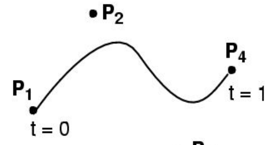
Definition: What's a Spline?

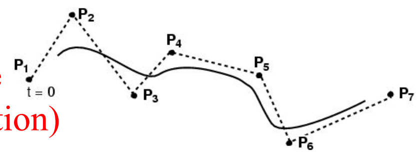
- Smooth curve defined by some control points
- Moving the control points changes the curve



Interpolation

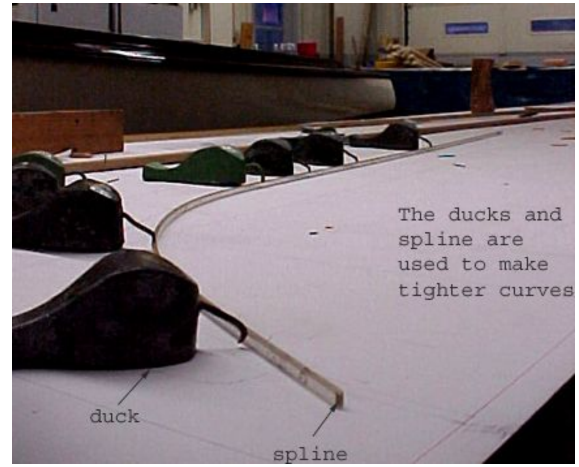


Bézier (approximation)



BSpline (approximation)

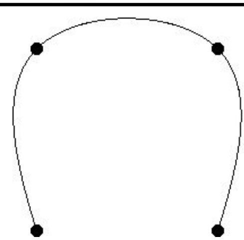
Interpolation Curves / Splines



www.abm.org

Interpolation Curves

- Curve is constrained to pass through all control points

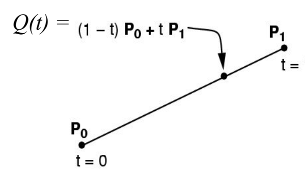


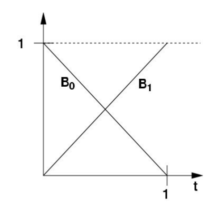
- Given points $P_0, P_1, \ldots P_n$, find lowest degree polynomial which passes through the points

$$x(t) = a_{n-1}t^{n-1} + \ldots + a_2t^2 + a_1t + a_0$$
$$y(t) = b_{n-1}t^{n-1} + \ldots + b_2t^2 + b_1t + b_0$$

Linear Interpolation

- Simplest "curve" between two points





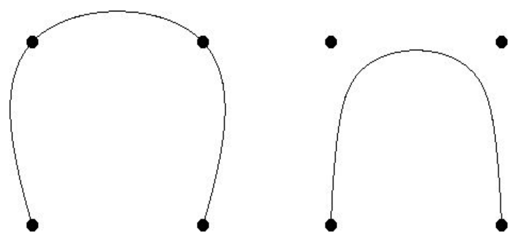
Spline Basis Functions

a.k.a. Blending Functions

$$Q(t) = \begin{pmatrix} Q_x(t) \\ Q_y(t) \\ Q_z(t) \end{pmatrix} = \begin{pmatrix} (P_0) & (P_1) \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} t \\ 1 \end{pmatrix}$$

$$Q(t) = \mathbf{GBT(t)} = \text{Geometry } \mathbf{G} \cdot \text{Spline Basis } \mathbf{B} \cdot \text{Power Basis } \mathbf{T(t)}$$

Interpolation vs. Approximation Curves



Interpolation

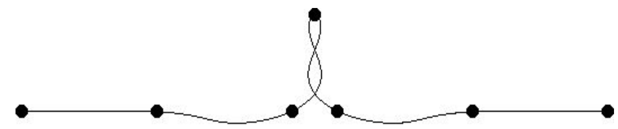
curve must pass through control points

Approximation

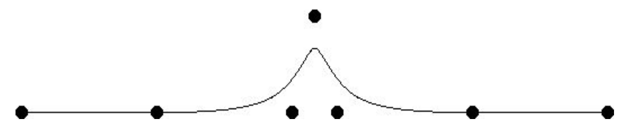
curve is influenced by control points

Interpolation vs. Approximation Curves

- Interpolation Curve – over constrained → lots of (undesirable?) oscillations



- Approximation Curve – more reasonable?



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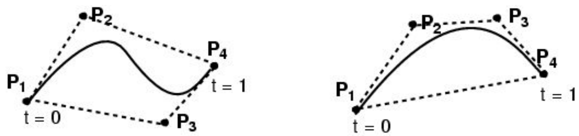
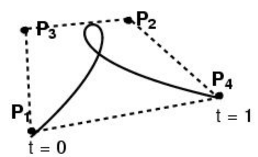
Cubic Bézier Curve

- 4 control points
- Curve passes through first & last control point
- Curve is tangent at $\mathbf{P}_1$ to $(\mathbf{P}_2\text{-}\mathbf{P}_1)$ and at $\mathbf{P}_4$ to $(\mathbf{P}_4\text{-}\mathbf{P}_3)$

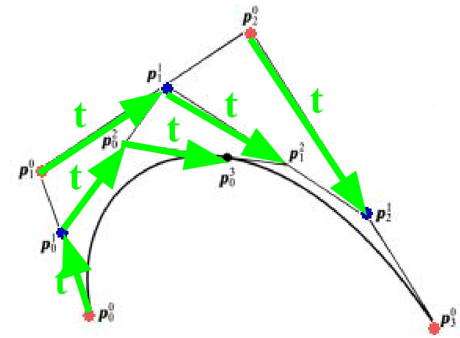




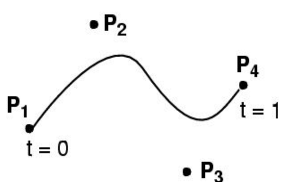
A Bézier curve is bounded by the convex hull of its control points.

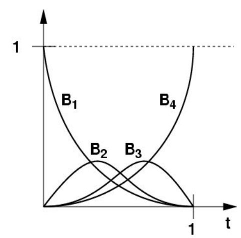
Cubic Bézier Curve

- de Casteljau's algorithm for constructing Bézier curves



Cubic Bézier Curve





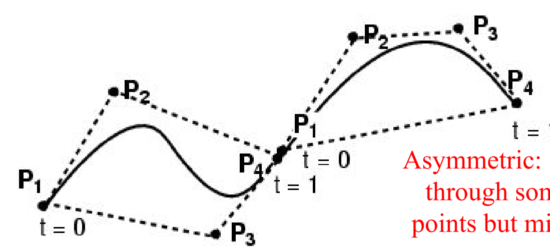
$$Q(t) = (1-t)^3 P_1 + 3t(1-t)^2 P_2 + 3t^2(1-t) P_3 + t^3 P_4$$

$$Q(t) = \mathbf{GBT(t)} \qquad B_{Bezier} = \begin{pmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Bernstein Polynomials

$$B_1(t) = (1-t)^3;\ B_2(t) = 3t(1-t)^2;\ B_3(t) = 3t^2(1-t);\ B_4(t) = t^3$$

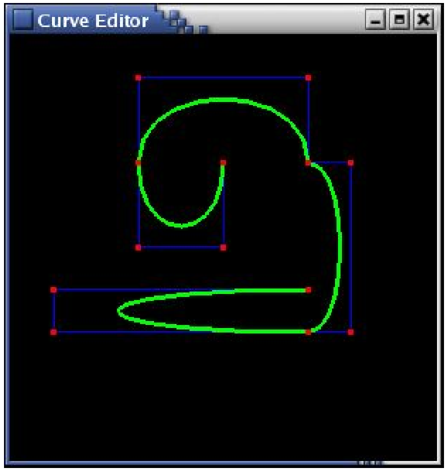
Connecting Cubic Bézier Curves



Asymmetric: Curve goes through some control points but misses others

- How can we guarantee C^0 continuity?
- How can we guarantee G^1 continuity?
- How can we guarantee C^1 continuity?
- Can't guarantee higher C^2 or higher continuity

Connecting Cubic Bézier Curves



- Where is this curve
  - C^0 continuous?
  - G^1 continuous?
  - C^1 continuous?
- What's the relationship between:
  - the # of control points, and
  - the # of cubic Bézier subcurves?

Higher-Order Bézier Curves

- > 4 control points
- Bernstein Polynomials as the basis functions

$$B_i^n(t) = \frac{n!}{i!(n-i)!} t^i (1-t)^{n-i}, \qquad 0 \le i \le n$$

- Every control point affects the entire curve
  - Not simply a local effect
  - More difficult to control for modeling

Questions?

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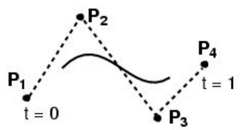
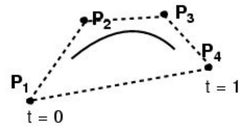
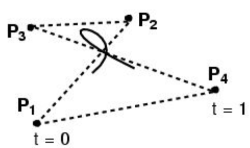
Cubic BSplines

- ≥ 4 control points
- Locally cubic
- Curve is not constrained to pass through any control points



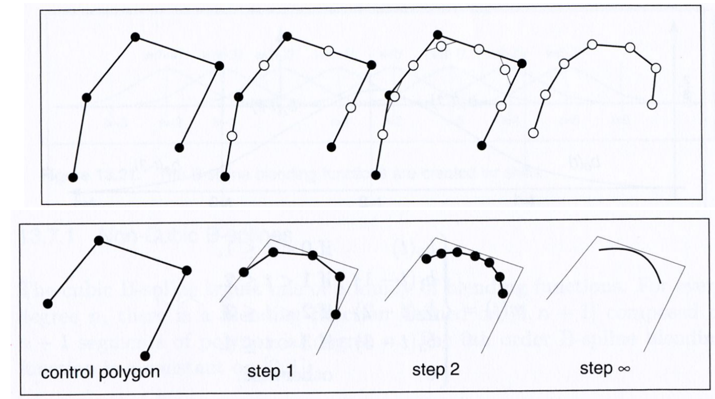




A BSpline curve is also bounded by the convex hull of its control points.

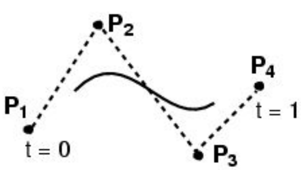
Cubic BSplines

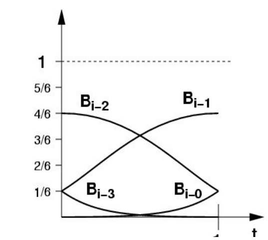
- Iterative method for constructing BSplines



Shirley, Fundamentals of Computer Graphics

Cubic BSplines



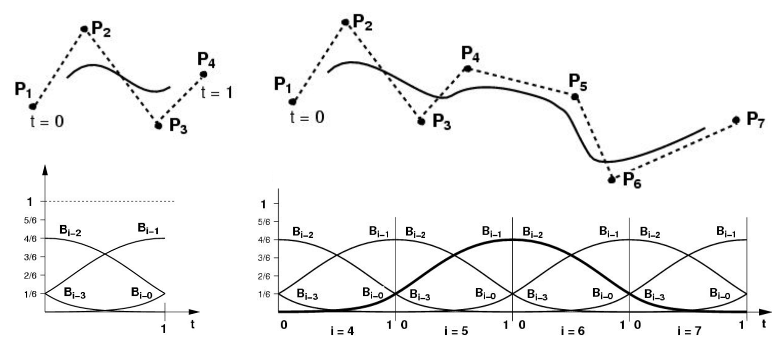


$$Q(t) = \frac{(1-t)^3}{6} P_{i-3} + \frac{3t^3 - 6t^2 + 4}{6} P_{i-2} + \frac{-3t^3 + 3t^2 + 3t + 1}{6} P_{i-1} + \frac{t^3}{6} P_i$$

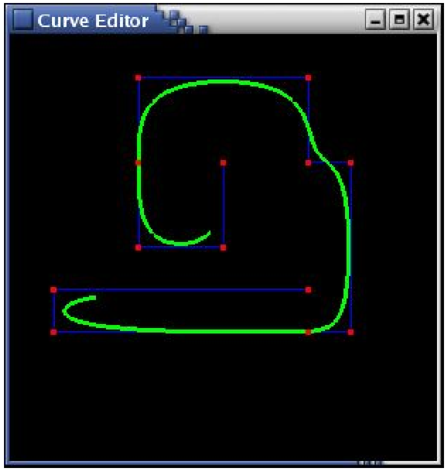
$$Q(t) = \mathbf{GBT(t)} \qquad B_{B-Spline} = \frac{1}{6}\begin{pmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 0 & 4 \\ -3 & 3 & 3 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Connecting Cubic BSpline Curves

- Can be chained together
- Better control locally (windowing)

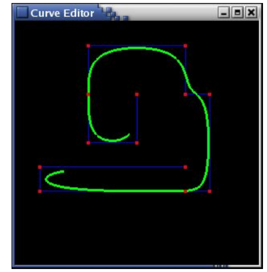


Connecting Cubic BSpline Curves

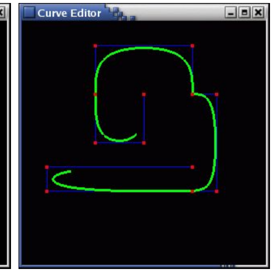


- What's the relationship between
  - the # of control points, and
  - the # of cubic BSpline subcurves?

BSpline Curve Control Points

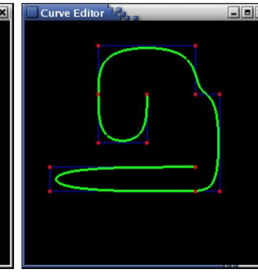


Default BSpline



BSpline with Discontinuity

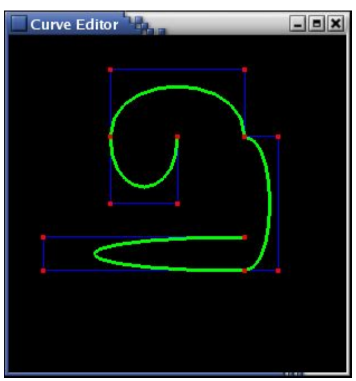
Repeat interior control point



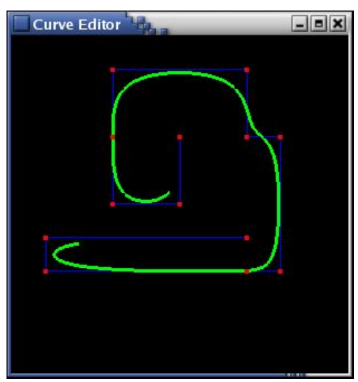
BSpline which passes through end points

Repeat end points

Bézier is not the same as BSpline



Bézier

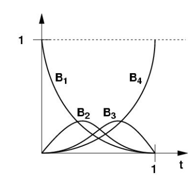


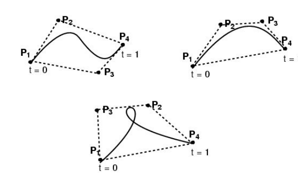
BSpline

Bézier is not the same as BSpline

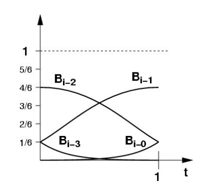
- Relationship to the control points is different

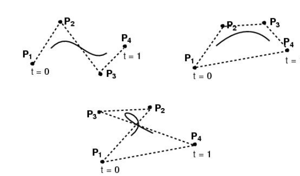
Bézier





BSpline

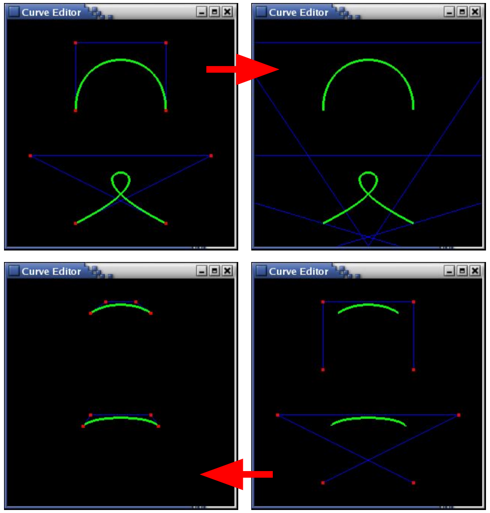




Converting between Bézier & BSpline

original control points as Bézier

new BSpline control points to match Bézier



new Bézier control points to match BSpline

original control points as BSpline

Converting between Bézier & BSpline

- Using the basis functions:

$$B_{Bezier} = \begin{pmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$B_{B-Spline} = \frac{1}{6} \begin{pmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 0 & 4 \\ -3 & 3 & 3 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

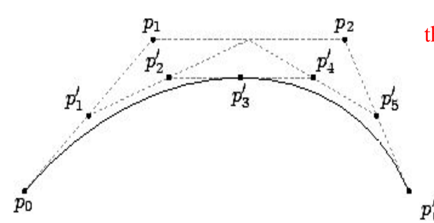
$$Q(t) = \mathbf{GBT(t)} = \text{Geometry } \mathbf{G} \cdot \text{Spline Basis } \mathbf{B} \cdot \text{Power Basis } \mathbf{T(t)}$$

NURBS (generalized BSplines)

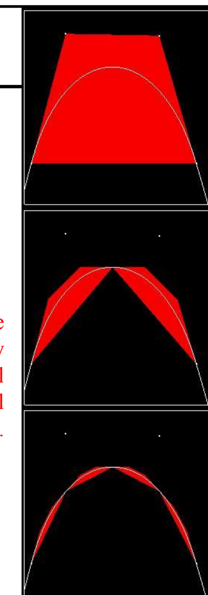
- BSpline: uniform cubic BSpline
- NURBS: Non-Uniform Rational BSpline
  - non-uniform = different spacing between the blending functions, a.k.a. knots
  - rational = ratio of polynomials (instead of cubic)

Neat Bezier Spline Trick

- A Bezier curve with 4 control points:
  - $P_0 \quad P_1 \quad P_2 \quad P_3$
- Can be split into 2 new Bezier curves:
  - $P_0 \quad P'_1 \quad P'_2 \quad P'_3$
  - $P'_3 \quad P'_4 \quad P'_5 \quad P_3$



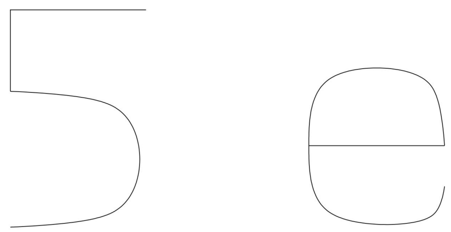
A Bézier curve is bounded by the convex hull of its control points.



Pop Worksheet!

Teams of 2. NOT EITHER OF PEOPLE YOU WORKED WITH LAST WEEK!
Hand in to Jeramey after we discuss.

- What is the minimum number of cubic Bezier curve segments needed to approximately reproduce the two curves below? Sketch the positions of the control vertices.



- Repeat for cubic BSplines curve segments.

Today

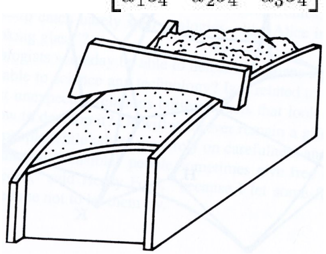
- Limitations of Polygonal Models
  - Interpolating Color & Normals in OpenGL
  - Some Modeling Tools & Definitions
- What's a Spline?
  - Interpolation Curves vs. Approximation Curves
  - Linear Interpolation
- Bézier Spline
- BSpline (NURBS)
- Extending to Surfaces – Tensor Product

Spline Surface via Tensor Product

- Of two vectors:

$$\begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \otimes \begin{bmatrix} b_1 & b_2 & b_3 & b_4 \end{bmatrix} = \begin{bmatrix} a_1b_1 & a_2b_1 & a_3b_1 \\ a_1b_2 & a_2b_2 & a_3b_2 \\ a_1b_3 & a_2b_3 & a_3b_3 \\ a_1b_4 & a_2b_4 & a_3b_4 \end{bmatrix}$$

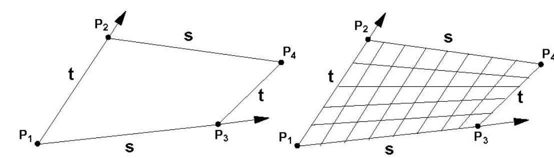
- Similarly, we can define a surface as the tensor product of two curves....



Farin, Curves and Surfaces for Computer Aided Geometric Design

Bilinear Patch

Bi-lerp a (typically non-planar) quadrilateral

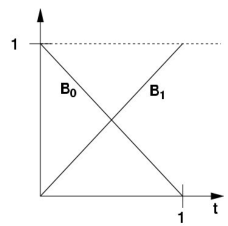


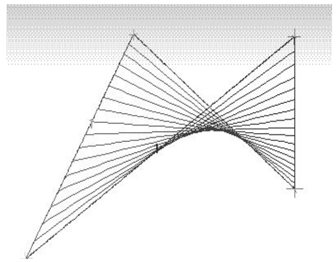
Notation: $\mathbf{L}(P_1, P_2, \alpha) \equiv (1-\alpha)P_1 + \alpha P_2$

$$Q(s,t) = \mathbf{L}(\mathbf{L}(P_1, P_2, t), L(P_3, P_4, t), s)$$

Bilinear Patch

- Smooth version of quadrilateral with non-planar vertices...





  – But will this help us model smooth surfaces?
  – Do we have control of the derivative at the edges?

Ruled Surfaces in Art & Architecture

http://www.bergenwood.no/wp-content/media/images/frozenmusic.jpg

Chiras Iulia
Astri Isabella
Matiss Shteinerts

http://www.lonelyplanetimages.com/images/399954

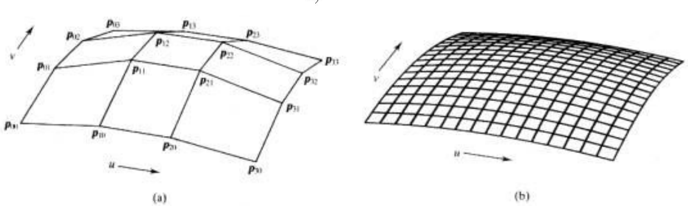
Antoni Gaudi
Children's School
Barcelona

Bicubic Bezier Patch

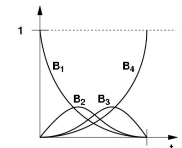
Notation: $\mathbf{CB}(P_1, P_2, P_3, P_4, \alpha)$ is Bézier curve with control points P_i evaluated at α

Define "Tensor-product" Bézier surface

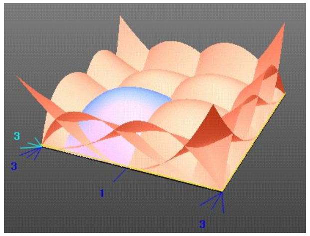
$$
\begin{aligned}
Q(s,t) = \mathbf{CB}(\quad & \mathbf{CB}(P_{00}, P_{01}, P_{02}, P_{03}, t), \\
& \mathbf{CB}(P_{10}, P_{11}, P_{12}, P_{13}, t), \\
& \mathbf{CB}(P_{20}, P_{21}, P_{22}, P_{23}, t), \\
& \mathbf{CB}(P_{30}, P_{31}, P_{32}, P_{33}, t), \\
& s)
\end{aligned}
$$



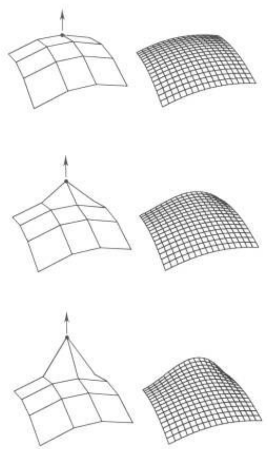
Editing Bicubic Bezier Patches



Curve Basis Functions

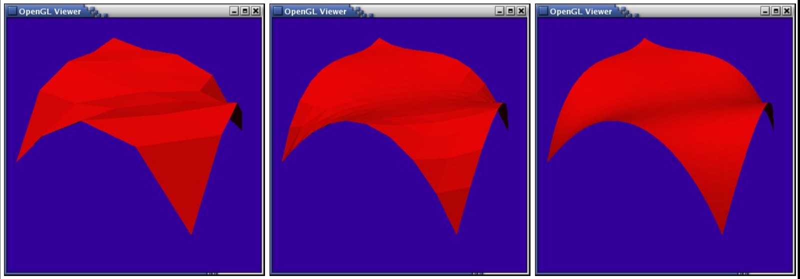


Surface Basis Functions



Bicubic Bezier Patch Tessellation

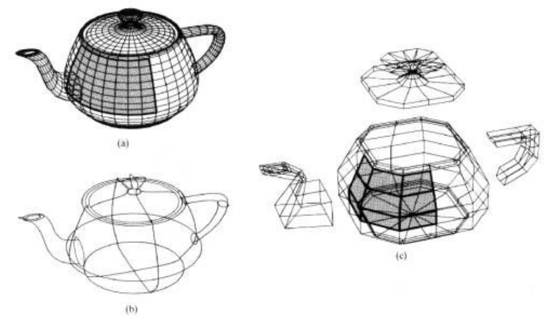
- Given 16 control points and a tessellation resolution, we can create a triangle mesh



resolution: 5x5 vertices | resolution: 11x11 vertices | resolution: 41x41 vertices

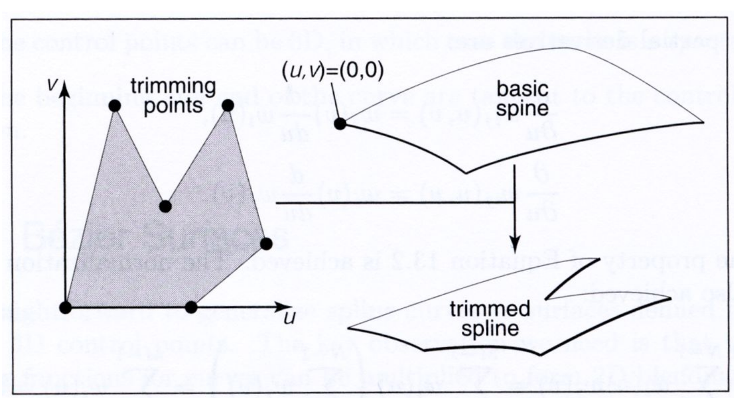
Modeling with Bicubic Bezier Patches

- Original Teapot specified with Bezier Patches



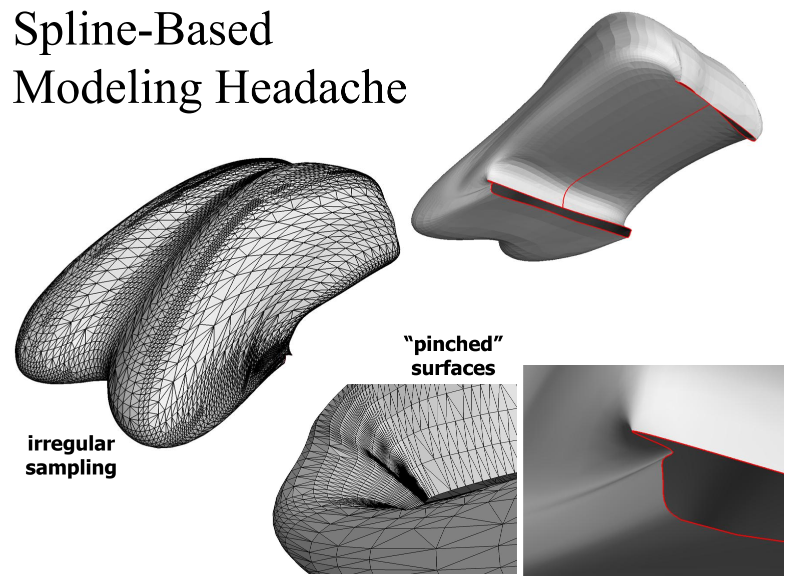
- But it's not "watertight": it has intersecting surfaces at spout & handle, no bottom, a hole at the spout tip, a gap between lid & base

Trimming Curves for Patches



Shirley, Fundamentals of Computer Graphics

Spline-Based Modeling Headache



Questions?

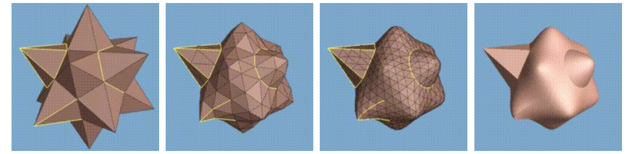
- Bezier Patches?

  or

- Triangle Mesh?

Readings for Friday *(pick one)*

- Hoppe et al., "Piecewise Smooth Surface Reconstruction" SIGGRAPH 1994



- DeRose, Kass, & Truong, "Subdivision Surfaces in Character Animation", SIGGRAPH 1998

Post a comment or question on the LMS discussion by 10am on Tuesday

Homework 1:

- Questions/Comments?