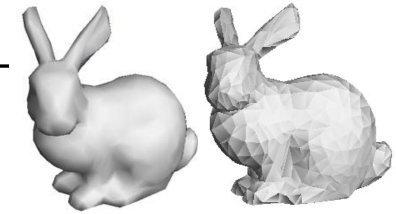
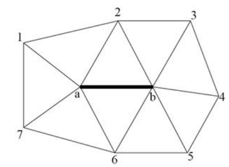
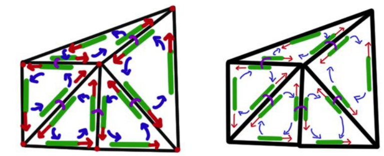
Subdivision Surfaces

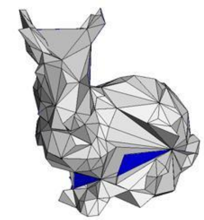
Homework 1:

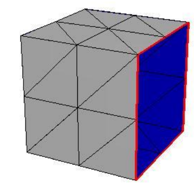
- Questions/Comments?

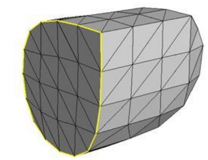






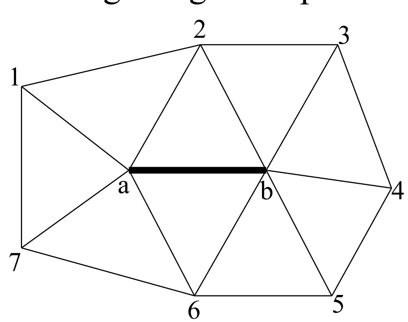






Questions on Homework?

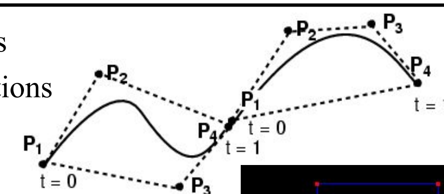
- What's an illegal edge collapse?

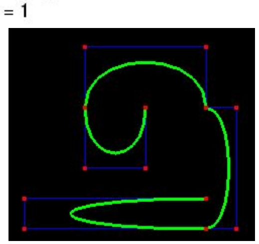


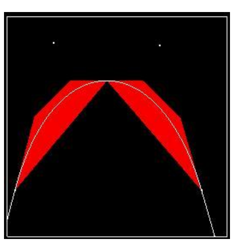
- To be legal, the ring of vertex neighbors *must be unique* (have no duplicates)!

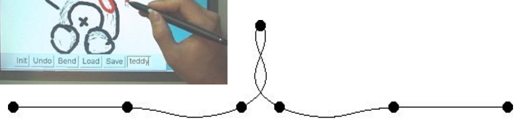
Last Time?

- Curves & Surfaces
- Continuity Definitions
  - C^0, G^1, C^1, … C^∞
- Interpolation vs. Approximation Splines
- Cubic Bezier & BSpline









Today

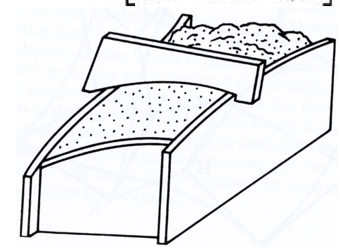
- Spline Surfaces / Patches
  - Tensor Product
  - Bezier Patches
  - Trimming Curves
- Misc. Mesh/Surface Vocabulary
- "Subdivision Surfaces in Character Animation"
- "Piecewise Smooth Surface Reconstruction"
- Subdivision Surface "Zoo"
- Interpolating Subdivision

Tensor Product

- Of two vectors:

$$\begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \otimes \begin{bmatrix} b_1 & b_2 & b_3 & b_4 \end{bmatrix} = \begin{bmatrix} a_1b_1 & a_2b_1 & a_3b_1 \\ a_1b_2 & a_2b_2 & a_3b_2 \\ a_1b_3 & a_2b_3 & a_3b_3 \\ a_1b_4 & a_2b_4 & a_3b_4 \end{bmatrix}$$

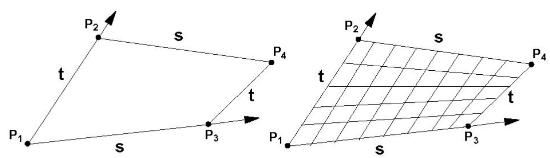
- Similarly, we can define a surface as the tensor product of two curves....



Farin, Curves and Surfaces for Computer Aided Geometric Design

Bilinear Patch

Bi-lerp a (typically non-planar) quadrilateral

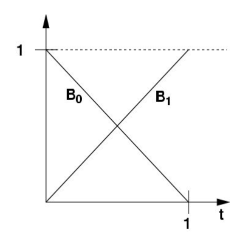


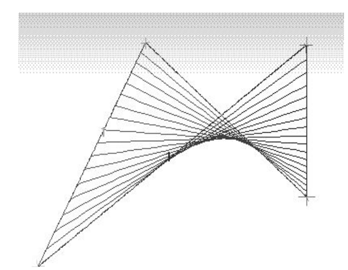
Notation: $\mathbf{L}(P_1, P_2, \alpha) \equiv (1-\alpha)P_1 + \alpha P_2$

$$Q(s,t) = \mathbf{L}(\mathbf{L}(P_1, P_2, t), L(P_3, P_4, t), s)$$

Bilinear Patch

- Smooth version of quadrilateral with non-planar vertices...





  - But will this help us model smooth surfaces?
  - Do we have control of the derivative at the edges?

Ruled Surfaces in Art & Architecture

http://www.bergenwood.no/wp-content/media/images/frozenmusic.jpg

Chiras Iulia
Astri Isabella
Matiss Shteinerts

Antoni Gaudi
Children's School
Barcelona

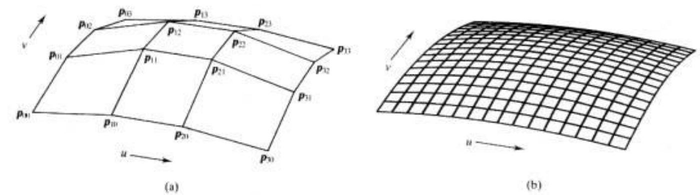
http://www.lonelyplanetimages.com/images/399954

Bicubic Bezier Patch

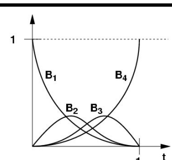
Notation: $\mathbf{CB}(P_1, P_2, P_3, P_4, \alpha)$ is Bézier curve with control points P_i evaluated at α

Define "Tensor-product" Bézier surface

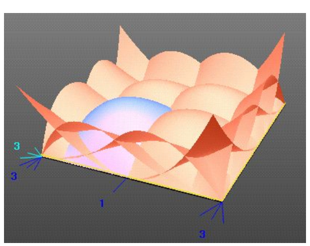
$$
\begin{aligned}
Q(s,t) = \mathbf{CB}(\quad & \mathbf{CB}(P_{00}, P_{01}, P_{02}, P_{03}, t), \\
& \mathbf{CB}(P_{10}, P_{11}, P_{12}, P_{13}, t), \\
& \mathbf{CB}(P_{20}, P_{21}, P_{22}, P_{23}, t), \\
& \mathbf{CB}(P_{30}, P_{31}, P_{32}, P_{33}, t), \\
& s)
\end{aligned}
$$



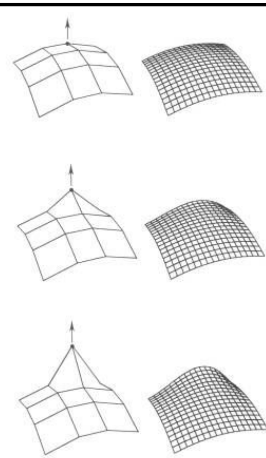
Editing Bicubic Bezier Patches



Curve Basis Functions

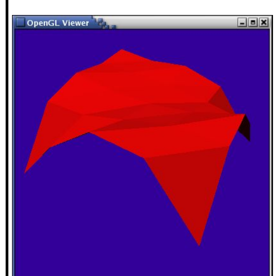


Surface Basis Functions

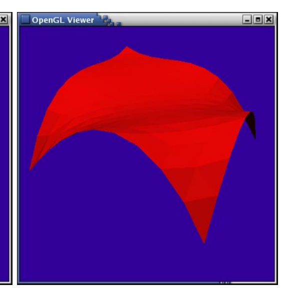


Bicubic Bezier Patch Tessellation

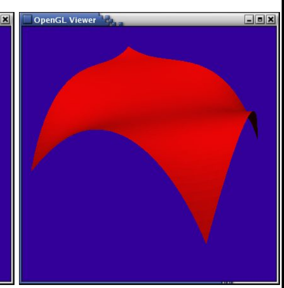
- Given 16 control points and a tessellation resolution, we can create a triangle mesh



resolution:
5x5 vertices



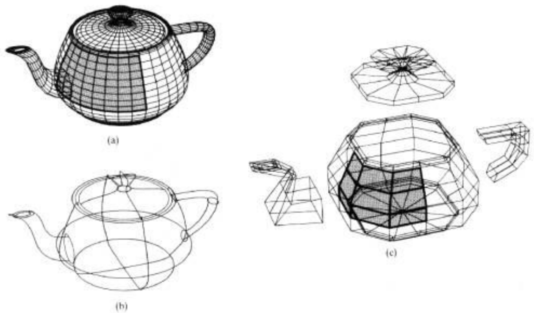
resolution:
11x11 vertices



resolution:
41x41 vertices

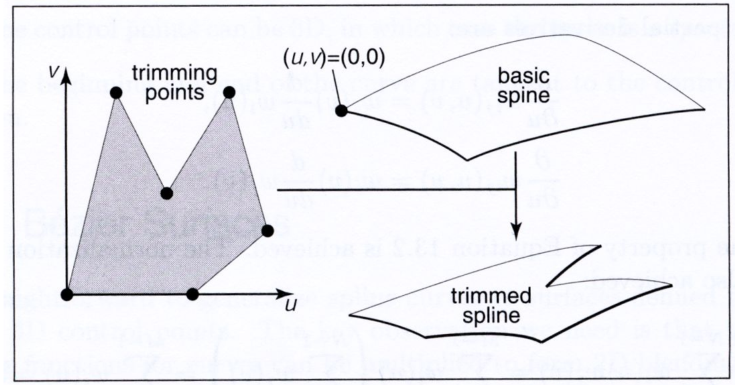
Modeling with Bicubic Bezier Patches

- Original Teapot specified with Bezier Patches



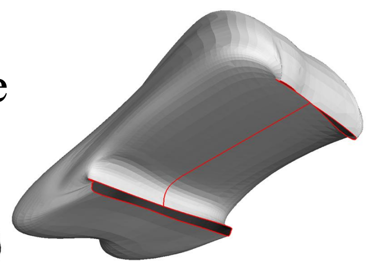
- But it’s not "watertight": it has intersecting surfaces at spout & handle, no bottom, a hole at the spout tip, a gap between lid & base

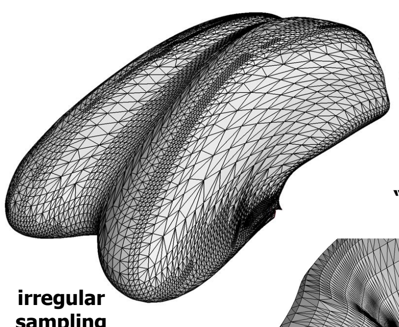
Trimming Curves for Patches

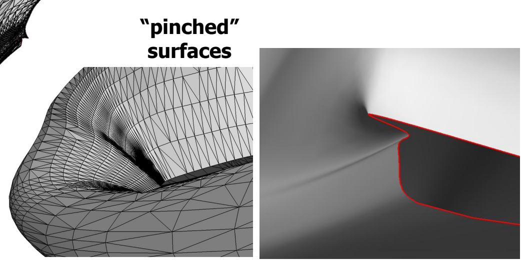


Shirley, Fundamentals of Computer Graphics

Spline-Based Modeling Headache







Questions?

- Bezier Patches?

or

- Triangle Mesh?

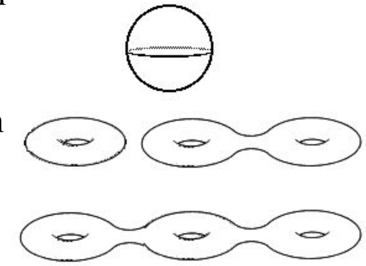
Henrik Wann Jensen

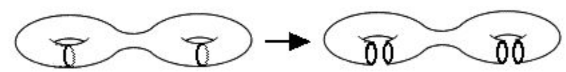
Today

- Spline Surfaces / Patches
- Misc. Mesh/Surface Vocabulary
- "Subdivision Surfaces in Character Animation"
- "Piecewise Smooth Surface Reconstruction"
- Subdivision Surface "Zoo"
- Interpolating Subdivision

Misc. Mesh/Surface Vocabulary

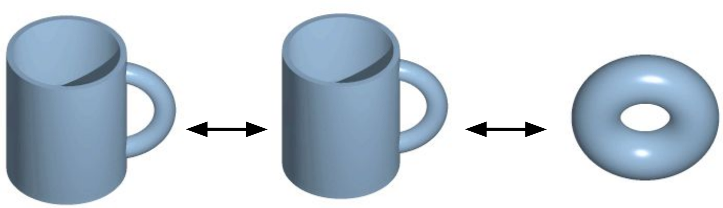
- *Genus:* The maximum number of disjoint simple closed curves which can be cut from an orientable surface of genus g without disconnecting it is g.





Misc. Mesh/Surface Vocabulary

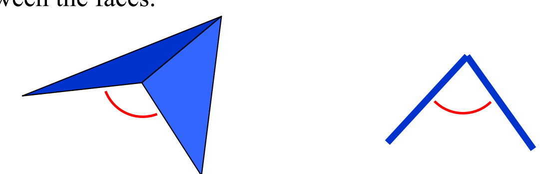
- *Homeomorphic/Topological equivalence*: a continuous stretching and bending of the object into a new shape



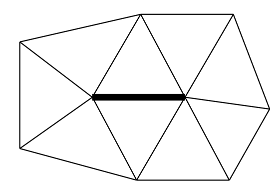
http://en.wikipedia.org/wiki/Image:Mug_and_Torus_morph.gif

Misc. Mesh/Surface Vocabulary

- *Dihedral Angle*:
  - the angle between the planes of two triangular faces
  - "looking down the edge" between two faces, the angle between the faces.

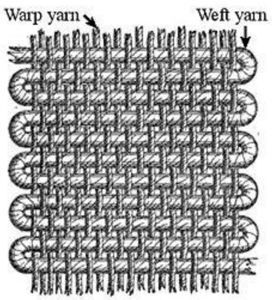


- *Valence (a.k.a. degree)*: the number of edges incident to the vertex.



Misc. Mesh/Surface Vocabulary

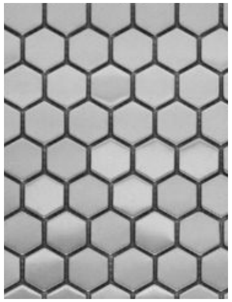
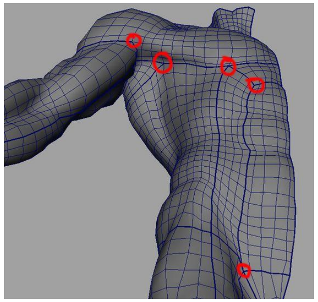
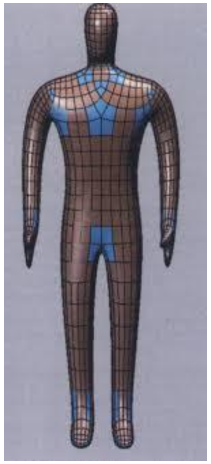
- *Warp & weft:* Yarns used in weaving. Because the weft does not have to be stretched in the way that the warp is, it can generally be less strong.



http://en.wikipedia.org/wiki/Weft

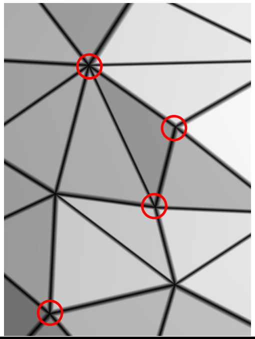
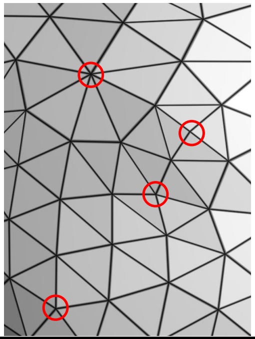
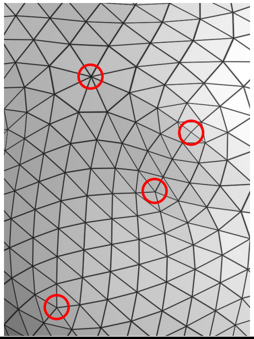
Misc. Mesh/Surface Vocabulary

- Extraordinary Vertex
  - Quad mesh: vertices w/ valence $\neq 4$
  - Hex mesh: vertices w/ valence $\neq 3$
  - Tri mesh: vertices w/ valence $\neq 6$

Misc. Mesh/Surface Vocabulary

- Extraordinary Vertex
  - Quad mesh: vertices w/ valence $\neq 4$
  - Hex mesh: vertices w/ valence $\neq 3$
  - Tri mesh: vertices w/ valence $\neq 6$

Today

- Spline Surfaces / Patches
- Misc. Mesh/Surface Vocabulary
- “Subdivision Surfaces in Character Animation”
- “Piecewise Smooth Surface Reconstruction”
- Subdivision Surface “Zoo”
- Interpolating Subdivision

Reading for Today

- DeRose, Kass, & Truong, "Subdivision Surfaces in Character Animation", SIGGRAPH 1998

Figure 5: Geri's hand as a piecewise smooth Catmull-Clark surface. Infinitely sharp creases are used between the skin and the finger nails.

Subdivision Surfaces in Character Animation

- Catmull Clark Subdivision Rules
- Semi-sharp vs. Infinitely-sharp creases
- Mass-Spring Cloth *(next week)*
- Hierarchical Mesh for Collision
- Texturing Subdivision Surfaces

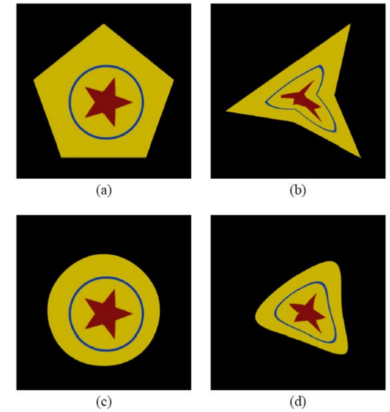


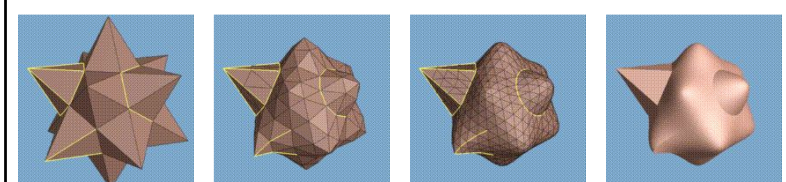
Figure 11: (a) A texture mapped regular pentagon comprised of 5 triangles; (b) the pentagonal model with its vertices moved; (c) A subdivision surface whose control mesh is the same 5 triangles in (a), and where boundary edges are marked as creases; (d) the subdivision surface with its vertices positioned as in (b).

Today

- Spline Surfaces / Patches
- Misc. Mesh/Surface Vocabulary
- "Subdivision Surfaces in Character Animation"
- "Piecewise Smooth Surface Reconstruction"
- Subdivision Surface "Zoo"
- Interpolating Subdivision

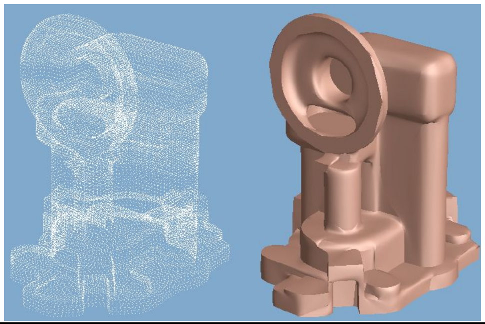
Reading for Today

- Hoppe et al., "Piecewise Smooth Surface Reconstruction" SIGGRAPH 1994



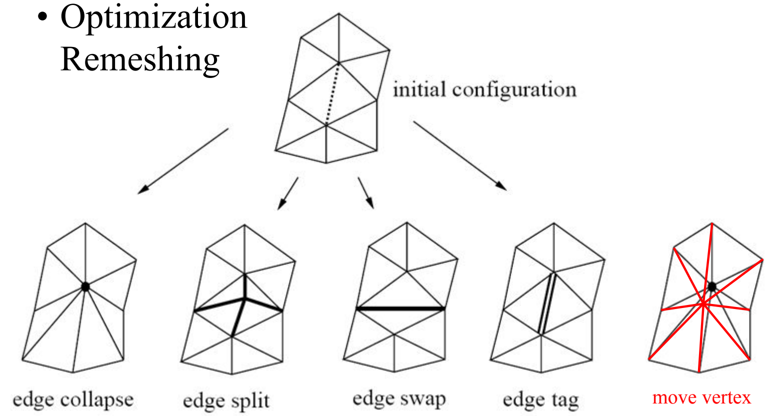
Piecewise Smooth Surface Reconstruction

- From input: scanned mesh points
  - Estimate topological type (genus)
  - Mesh optimization (a.k.a. simplification)
  - Smooth surface optimization



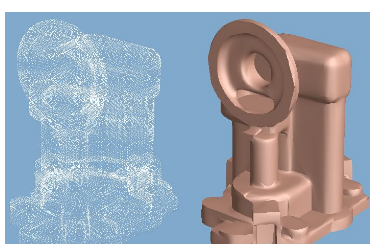
Piecewise Smooth Surface Reconstruction

- Optimization Remeshing



Piecewise Smooth Surface Reconstruction

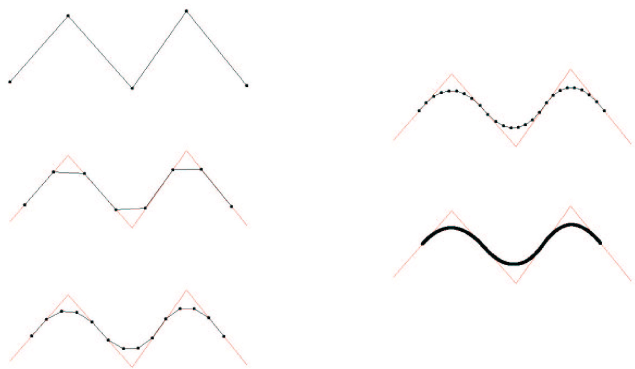
- Crease subdivision masks *decouple* behavior of surface on either side of crease
- Crease rules cannot model a cone
- Optimization can be done locally
  - subdivision control points have only local influence
- Results
  - Noise?
  - Applicability?
  - Limitations?
  - Running Time



Today

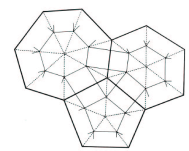
- Spline Surfaces / Patches
- Misc. Mesh/Surface Vocabulary
- “Subdivision Surfaces in Character Animation”
- “Piecewise Smooth Surface Reconstruction”
- Subdivision Surface “Zoo”
  - Doo Sabin (anything!)
  - Loop, Butterfly, $\sqrt{3}$ (triangles only)
  - Catmull Clark (turns everything into quads)
  - … many others!
- Interpolating Subdivision

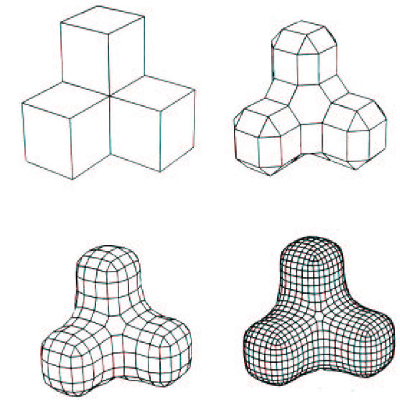
Chaikin's Algorithm



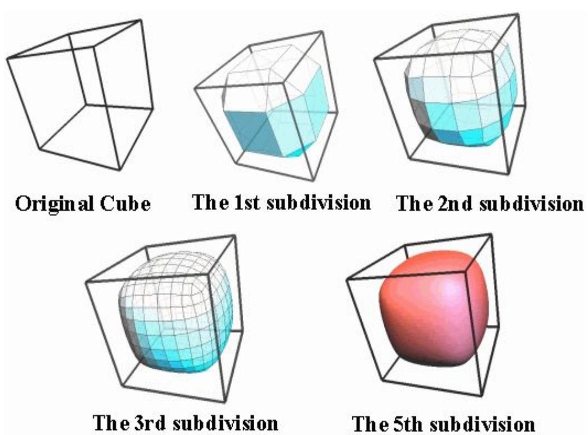
Doo-Sabin Subdivision

Idea: introduce a new vertex for each face
At the midpoint of old vertex, face centroid



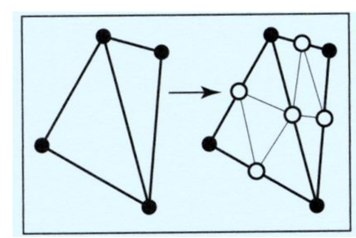


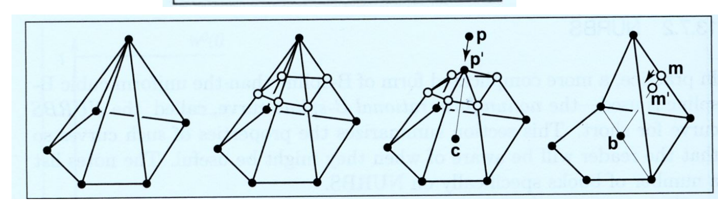
Doo-Sabin Subdivision



http://www.ke.ics.saitama-u.ac.jp/xuz/pic/doo-sabin.gif

Loop Subdivision





Shirley, Fundamentals of Computer Graphics

Loop Subdivision

Subdivision Rules. The masks for the Loop scheme are shown in Figure 4.3. For boundaries and edges tagged as *crease* edges, special rules are used. These rules produce a cubic spline curve along the boundary/crease. The curve only depends on control points on the boundary/crease.

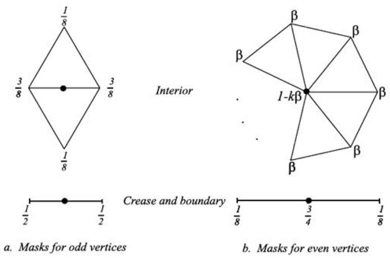
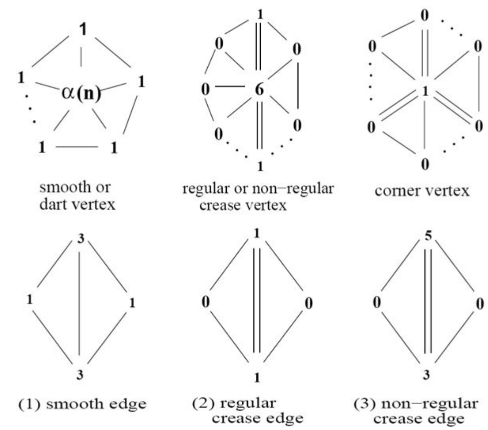


Figure 4.3: *Loop subdivision: in the picture above,* β *can be chosen to be either* $\frac{1}{n}(5/8 - (\frac{3}{8} + \frac{1}{4}\cos\frac{2\pi}{n})^2)$ *(original choice of Loop [16]), or, for* $n > 3$, $\beta = \frac{3}{8n}$ *as proposed by Warren [33]. For* $n = 3$, $\beta = 3/16$ *can be used.*

SIGGRAPH 2000 course notes
Subdivision for Modeling and Animation (page 70)

Adding creases to Loop Subdivision

- Vertex & edge masks
- Limit masks
  - Position
  - Tangent



Catmull Clark Subdivision

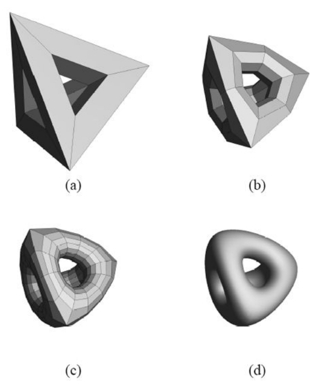


Figure 3: Recursive subdivision of a topologically complicated mesh: (a) the control mesh; (b) after one subdivision step; (c) after two subdivision steps; (d) the limit surface.

$$e_j^{i+1} = \frac{v^i + e_j^i + f_{j-1}^{i+1} + f_j^{i+1}}{4}, \quad (1)$$

where subscripts are taken modulo the valence of the central vertex v^0. (The valence of a vertex is the number of edges incident to it.) Finally, a vertex point v^i is computed as

$$v^{i+1} = \frac{n-2}{n} v^i + \frac{1}{n^2}\sum_j e_j^i + \frac{1}{n^2}\sum_j f_j^{i+1} \quad (2)$$

Vertices of valence 4 are called ordinary; others are called extraordinary.

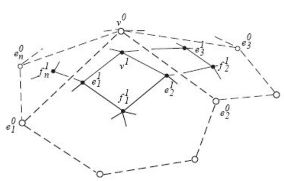
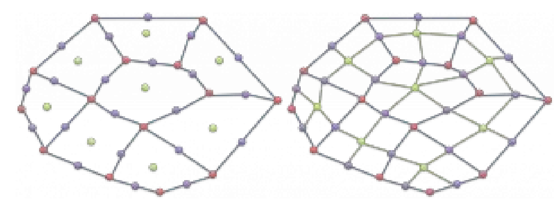


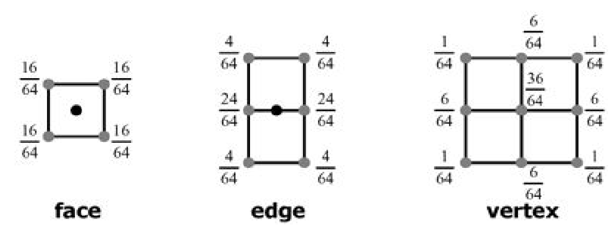
Figure 4: The situation around a vertex v^0 of valence n.

"Subdivision Surfaces in Character Animation",
DeRose, Kass & Truong, SIGGRAPH 1998

Catmull-Clark Subdivision



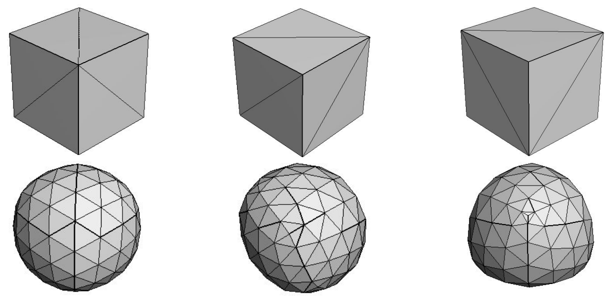
https://team.inria.fr/virtualplants/teaching/informatique-graphique-2016/tp4-instructions/



http://www.cl.cam.ac.uk/teaching/2005/AdvGraph/exercise2.html

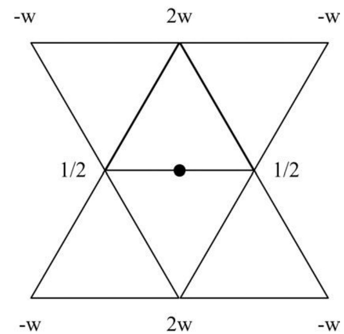
Catmull-Clark preferred by Artists

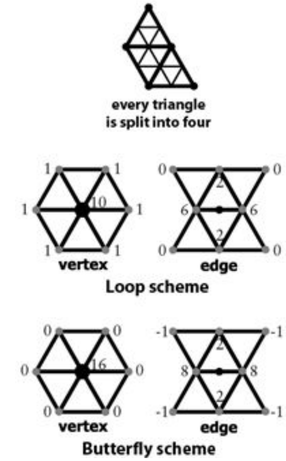
- Catmull-Clark is based on quadrilaterals
  - Like NURBS, specifically cubic bsplines
  - Implicit adjacency in subdivided microgeometry
  - Better than triangles for symmetric objects



Butterfly Subdivision

- Triangle-based subdivision
- Alternate scheme to Loop

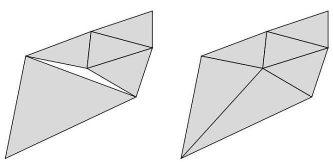




http://www.cl.cam.ac.uk/teaching/2005/AdvGraph/exercise2.html

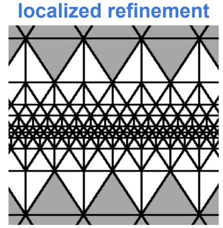
√3 Subdivision

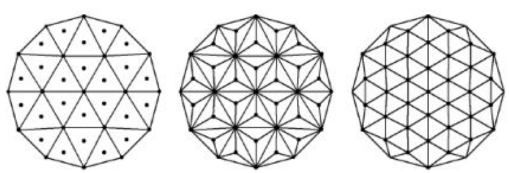
"3-Subdivision", Kobbelt, SIGGRAPH 2000



Adaptive Subdivision (Loop): Need to close gaps between different levels of refinement

Loop: less localized refinement



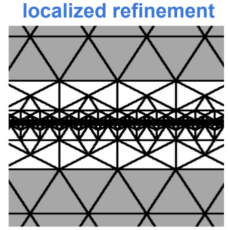


the split operation places a midvertex at the centre of each triangle

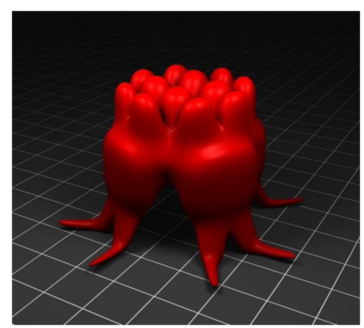
joining the midvertex to the vertices of the triangle realises the 1-to-3 split

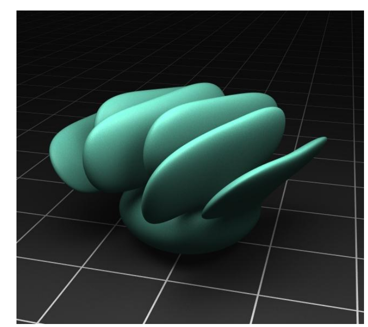
after smoothing each old vertex, edges are flipped to connect pairs of midvertices

√3: more localized refinement



Questions?



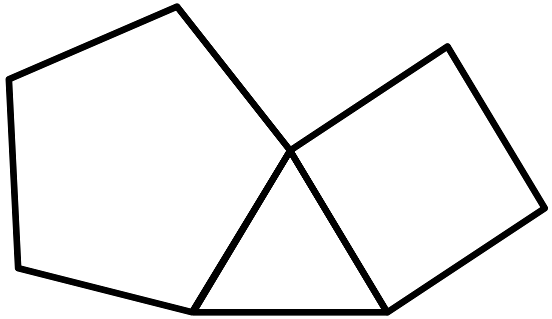


Justin Legakis

Pop Worksheet!

Teams of 2. SOMEONE YOU HAVEN'T MET BEFORE! Hand in to Jeramey after we discuss.

Sketch the polygonal mesh after performing 2 iterations of subdivision (Loop/Butterfly, Catmull-Clark, and Doo-Sabin). If necessary, pre-process the mesh to allow use of the specified method.

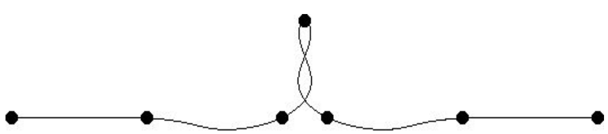


Today

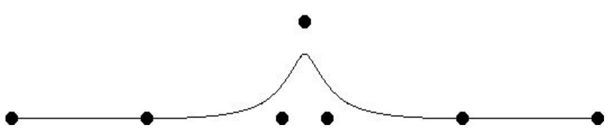
- Spline Surfaces / Patches
- Misc. Mesh/Surface Vocabulary
- "Subdivision Surfaces in Character Animation"
- "Piecewise Smooth Surface Reconstruction"
- Subdivision Surface "Zoo"
- Interpolating Subdivision

Interpolation vs. Approximation Curves

- Interpolation Curve – over constrained → lots of (undesirable?) oscillations



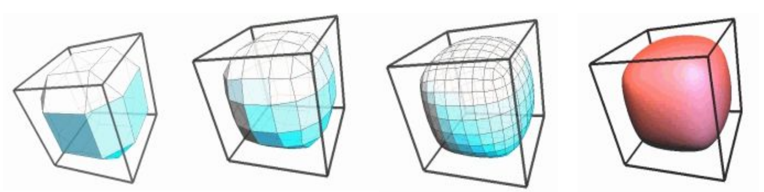
- Approximation Curve – more reasonable?



Interpolating Subdivision

- Chaikin:

- Doo-Sabin:



of the centroids of each edge/face

Interpolating Subdivision

- *Interpolation* vs. *Approximation* of control points
- Handle arbitrary topological type
- Reduce the "extraneous bumps & wiggles"

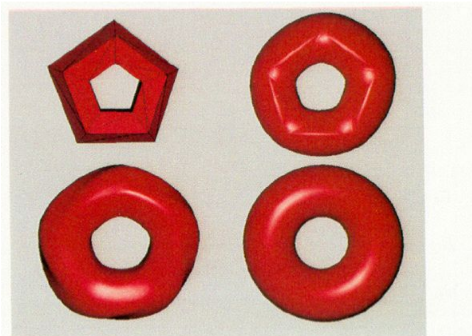
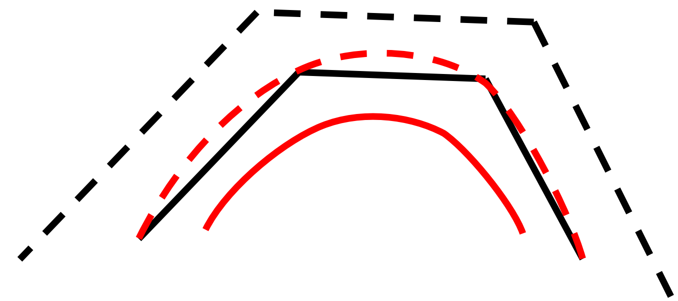


Figure 4: Interpolating a coarsely polygonized torus. Upper left: original mesh. Upper right: Shirman-Séquin interpolation[14]. Lower left: Interpolating Catmull-Clark surface. Lower right: Faired interpolating Catmull-Clark surface.

"Efficient, fair interpolation using Catmull-Clark surfaces",
Halstead, Kass & DeRose, SIGGRAPH 1993

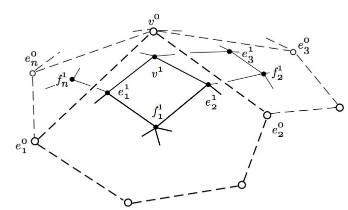
Interpolation of Catmull-Clark Surfaces

- Solve for a new control mesh (generally "bigger") such that when Catmull-Clark subdivision is applied it interpolates the original control mesh



Vertex Position in Limit

- V_n stores the center vertex & surrounding edge & face vertices as a big column vector



$$V_n^{i+1} = \mathbf{S}_n V_n^i$$

- When n = 4: (n = valence)

$$\mathbf{S}_4 = \frac{1}{16} * \begin{pmatrix} 9 & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{3}{2} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 6 & 6 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 6 & 1 & 6 & 1 & 0 & 1 & 1 & 0 & 0 \\ 6 & 0 & 1 & 6 & 1 & 0 & 1 & 1 & 0 \\ 6 & 1 & 0 & 1 & 6 & 0 & 0 & 1 & 1 \\ 4 & 4 & 4 & 0 & 0 & 4 & 0 & 0 & 0 \\ 4 & 0 & 4 & 4 & 0 & 0 & 4 & 0 & 0 \\ 4 & 0 & 0 & 4 & 4 & 0 & 0 & 4 & 0 \\ 4 & 4 & 0 & 0 & 4 & 0 & 0 & 0 & 4 \end{pmatrix}$$

$$V_n^\infty := \lim_{i \to \infty} \mathbf{S}_n^i V_n^1$$

Solve for New Positions

- Goal: Find the control mesh vertex positions, *x* (a column vector of 3D points), such that the position of the vertices in the limit match the input vertices, *b* (also a column vector of points)
- Use Least Squares to solve

$$\mathbf{A}x = b$$

where A is a square matrix with the interpolation rules and connectivity of the mesh

- *See paper for extension to match limit normals*

Fairing

- Fairing: an additional part or structure added to an aircraft, tractor-trailer, etc. to smooth the outline and thus reduce drag
- Subdivide initial resolution twice so that all constrained vertex positions are independent

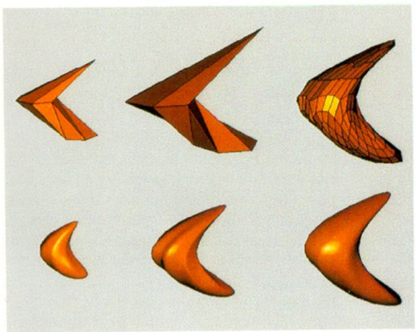
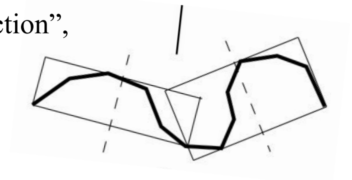


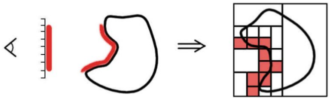
Figure 5: Top row: Original mesh, Interpolating mesh, Faired interpolating mesh. Bottom row: Corresponding Catmull-Clark surfaces. Interpolation introduces wiggles which are removed by fairing.

Reading for Tuesday: *(pick one)*

- "OBB-Tree: A Hierarchical Structure for Rapid Interference Detection", Gottschalk, Lin, Manocha, SIGGRAPH 1996.



- "Painting and Rendering Textures on Unparameterized Models", DeBry, Gibbs, Deleon, and Robins, SIGGRAPH 2002



Post a comment/question on the LMS discussion by 10am