Subdivision Surfaces

Questions on Homework?

• What’s an illegal edge collapse?

• To be legal, the ring of vertex neighbors must be unique (have no duplicates)!

Last Time?

• Curves & Surfaces
• Continuity Definitions – C^0, G^1, C^1, … C^∞
• Interpolation vs. Approximation Splines
• Cubic Bezier & BSpline

Homework 1:

• Questions/Comments?
Today

- Spline Surfaces / Patches
  - Tensor Product
  - Bezier Patches
  - Trimming Curves
- Misc. Mesh/Surface Vocabulary
- “Subdivision Surfaces in Character Animation”
- “Piecewise Smooth Surface Reconstruction”
- Subdivision Surface “Zoo”
- Interpolating Subdivision

Tensor Product

- Of two vectors:
  \[
  \begin{bmatrix}
  a_1 & a_2 & a_3 \\
  b_1 & b_2 & b_3 & b_4
  \end{bmatrix}
  \]

- Similarly, we can define a surface as the tensor product of two curves.

Bilinear Patch

Bi-lerp a (typically non-planar) quadrilateral

\[L(P_1, P_2, \alpha) = (1 - \alpha)P_1 + \alpha P_2\]

\[Q(s, t) = L(L(P_1, P_2, t), L(P_3, P_4, t), s)\]

Notation:

Farin, Curves and Surfaces for Computer Aided Geometric Design

Bilinear Patch

- Smooth version of quadrilateral with non-planar vertices...

- But will this help us model smooth surfaces?
- Do we have control of the derivative at the edges?
Ruled Surfaces in Art & Architecture

Antoni Gaudi
Children’s School
Barcelona

Bicubic Bezier Patch

Notation: $\mathbf{CB}(P_1, P_2, P_3, P_4, \alpha)$ is Bézier curve with control points $P_i$ evaluated at $\alpha$

Define “Tensor-product” Bézier surface

$$Q(s, t) = \mathbf{CB}( \mathbf{CB}(P_{00}, P_{01}, P_{02}, P_{03}, t), \mathbf{CB}(P_{10}, P_{11}, P_{12}, P_{13}, t), \mathbf{CB}(P_{20}, P_{21}, P_{22}, P_{23}, t), \mathbf{CB}(P_{30}, P_{31}, P_{32}, P_{33}, t), t, s)$$

Editing Bicubic Bezier Patches

• Given 16 control points and a tessellation resolution, we can create a triangle mesh

Bicubic Bezier Patch Tessellation

resolution: 5x5 vertices
resolution: 11x11 vertices
resolution: 41x41 vertices
Modeling with Bicubic Bezier Patches

- Original Teapot specified with Bezier Patches

- But it’s not "watertight": it has intersecting surfaces at spout & handle, no bottom, a hole at the spout tip, a gap between lid & base

Trimming Curves for Patches

Shirley, Fundamentals of Computer Graphics

Spline-Based Modeling Headache

- Irregular sampling
- “pinched” surfaces

Questions?

- Bezier Patches?
  - or
- Triangle Mesh?
Today

• Spline Surfaces / Patches
• Misc. Mesh/Surface Vocabulary
• “Subdivision Surfaces in Character Animation”
• “Piecewise Smooth Surface Reconstruction”
• Subdivision Surface “Zoo”
• Interpolating Subdivision

Genus:
The maximum number of disjoint simple closed curves which can be cut from an orientable surface of genus g without disconnecting it is g.

Homeomorphic/Topological equivalence:
a continuous stretching and bending of the object into a new shape

Dihedral Angle:
– the angle between the planes of two triangular faces
– “looking down the edge” between two faces, the angle between the faces.

Valence (a.k.a. degree):
the number of edges incident to the vertex.

Misc. Mesh/Surface Vocabulary

• **Warp & weft**: Yarns used in weaving. Because the weft does not have to be stretched in the way that the warp is, it can generally be less strong.

http://en.wikipedia.org/wiki/Weft

Misc. Mesh/Surface Vocabulary

• Extraordinary Vertex
  – Quad mesh: vertices w/ valence ≠ 4
  – Hex mesh: vertices w/ valence ≠ 3
  – Tri mesh: vertices w/ valence ≠ 6

Today

• Spline Surfaces / Patches
• Misc. Mesh/Surface Vocabulary
• “Subdivision Surfaces in Character Animation”
• “Piecewise Smooth Surface Reconstruction”
• Subdivision Surface “Zoo”
• Interpolating Subdivision
Reading for Today

- DeRose, Kass, & Truong, "Subdivision Surfaces in Character Animation", SIGGRAPH 1998

Figure 5: Geri’s hand as a piecewise smooth Catmull-Clark surface. Infinitely sharp creases are used between the skin and the finger nails.

Today

- Spline Surfaces / Patches
- Misc. Mesh/Surface Vocabulary
- “Subdivision Surfaces in Character Animation”
- “Piecewise Smooth Surface Reconstruction”
- Subdivision Surface “Zoo”
- Interpolating Subdivision

Reading for Today

- Hoppe et al., “Piecewise Smooth Surface Reconstruction” SIGGRAPH 1994

Subdivision Surfaces in Character Animation

- Catmull Clark Subdivision Rules
- Semi-sharp vs. Infinitely-sharp creases
- Mass-Spring Cloth (next week)
- Hierarchical Mesh for Collision
- Texturing Subdivision Surfaces

Figure 11: (a) A texture mapped regular pentagon comprised of 5 triangles; (b) the pentagonal model with its vertices moved; (c) A subdivision surface whose control mesh is the same 5 triangles in (a), and where boundary edges are marked as creases; (d) the subdivision surface with its vertices positioned as in (b).
Piecewise Smooth Surface Reconstruction

- From input: scanned mesh points
  - Estimate topological type (genus)
  - Mesh optimization (a.k.a. simplification)
  - Smooth surface optimization

Piecewise Smooth Surface Reconstruction

- Optimization
- Remeshing

- Crease subdivision masks *decouple* behavior of surface on either side of crease
- Crease rules cannot model a cone
- Optimization can be done locally
  - subdivision control points have only local influence

Piecewise Smooth Surface Reconstruction

- Results
  - Noise?
  - Applicability?
  - Limitations?
  - Running Time

Today

- Spline Surfaces / Patches
- Misc. Mesh/Surface Vocabulary
- “Subdivision Surfaces in Character Animation”
- “Piecewise Smooth Surface Reconstruction”
- Subdivision Surface “Zoo”
  - Doo Sabin (anything!)
  - Loop, Butterfly, $\sqrt{3}$ (triangles only)
  - Catmull Clark (turns everything into quads)
  - … many others!
- Interpolating Subdivision
Loop Subdivision

Subdivision Rules. The masks for the Loop scheme are shown in Figure 4.3. For boundaries and edges tagged as crease edges, special rules are used. These rules produce a cubic spline curve along the boundary/crease. The curve only depends on control points on the boundary/crease.

Adding creases to Loop Subdivision

• Vertex & edge masks
• Limit masks
  – Position
  – Tangent

SIGGRAPH 2000 course notes
Subdivision for Modeling and Animation (page 70)

Catmull-Clark Subdivision

\[ v^{j+1} = \frac{v^j + v_{1,j + 1} + v_{2,j + 1} + v_{3,j + 1}}{4}, \]

where subscripts are taken modulo the valence of the central vertex \( v^j \). (The valence of a vertex is the number of edges incident to it.) Finally, a vertex point \( v^j \) is computed as

\[ v^{j+1} = \frac{2}{r} v^j + \frac{1}{r} \sum_{i} \frac{1}{2} v_{i,j + 1} \]

Vertices of valence 4 are called ordinary; others are called extraordinary.

Catmull-Clark Subdivision

https://team.inria.fr/virtualplants/teaching/informatique-graphique-2016/tp4-instructions/

"Subdivision Surfaces in Character Animation",
DeRose, Kass & Truong, SIGGRAPH 1998
Catmull-Clark preferred by Artists

- Catmull-Clark is based on quadrilaterals
  - Like NURBS, specifically cubic bsplines
  - Implicit adjacency in subdivided microgeometry
  - Better than triangles for symmetric objects

Butterfly Subdivision

- Triangle-based subdivision
- Alternate scheme to Loop

√3 Subdivision

“3-Subdivision”, Kobbelt, SIGGRAPH 2000

Adaptive Subdivision (Loop): Need to close gaps between different levels of refinement

Loop: less localized refinement

√3: more localized refinement

Questions?

Justin Legakis
Pop Worksheet! Teams of 2. SOMEONE YOU HAVEN'T MET BEFORE! Hand in to Jeramey after we discuss.

Sketch the polygonal mesh after performing 2 iterations of subdivision (Loop/Butterfly, Catmull-Clark, and Doo-Sabin). If necessary, pre-process the mesh to allow use of the specified method.

Today

- Spline Surfaces / Patches
- Misc. Mesh/Surface Vocabulary
- “Subdivision Surfaces in Character Animation”
- “Piecewise Smooth Surface Reconstruction”
- Subdivision Surface “Zoo”
- Interpolating Subdivision

Interpolation vs. Approximation Curves

- Interpolation Curve – over constrained → lots of (undesirable?) oscillations
- Approximation Curve – more reasonable?

Interpolating Subdivision

- Chaikin:
- Doo-Sabin:

of the centroids of each edge/face
Interpolating Subdivision

- Interpolation vs. Approximation of control points
- Handle arbitrary topological type
- Reduce the “extraneous bumps & wiggles”

"Efficient, fair interpolation using Catmull-Clark surfaces", Halstead, Kass & DeRose, SIGGRAPH 1993

Interpolation of Catmull-Clark Surfaces

- Solve for a new control mesh (generally “bigger”) such that when Catmull-Clark subdivision is applied it interpolates the original control mesh

Vertex Position in Limit

- \( V_n \) stores the center vertex & surrounding edge & face vertices as a big column vector

\[
V_n^{i+1} = S_n V_n^i
\]

- When \( n = 4 \):
  \( (n = \text{valence}) \)

\[
V_n^\infty := \lim_{i \to \infty} S_n V_n^i
\]

Solve for New Positions

- Goal: Find the control mesh vertex positions, \( x \) (a column vector of 3D points), such that the position of the vertices in the limit match the input vertices, \( b \) (also a column vector of points)

- Use Least Squares to solve

\[
A x = b
\]

where \( A \) is a square matrix with the interpolation rules and connectivity of the mesh

- See paper for extension to match limit normals
Fairing

- Fairing: an additional part or structure added to an aircraft, tractor-trailer, etc. to smooth the outline and thus reduce drag
- Subdivide initial resolution twice so that all constrained vertex positions are independent

Figure 5: Top row: Original mesh, Interpolating mesh, Fairied interpolating mesh. Bottom row: Corresponding Catmull-Clark surfaces. Interpolation introduces wiggles which are removed by fairing.

Reading for Tuesday: (pick one)


- "Painting and Rendering Textures on Unparameterized Models", DeBry, Gibbs, Deleon, and Robins, SIGGRAPH 2002

Post a comment/question on the LMS discussion by 10am