Rigid Body Dynamics, Collision Response, & Deformation

Last Time?

- Navier-Stokes Equations
- Conservation of Momentum & Mass
- Incompressible Flow

At each Timestep:

- Identify which cells are Empty, Full, or on the Surface
- Compute new velocities
- Adjust the velocities to maintain an incompressible flow
- Move the particles
  - Interpolate the velocities at the faces
- Render the geometry and repeat!
Adjusting the Velocities

- Calculate the divergence of the cell (the extra in/out flow)
- The divergence is used to update the pressure within the cell
- Adjust each face velocity uniformly to bring the divergence to zero
- Iterate across the entire grid until divergence is < $\varepsilon$

Calculating/Eliminating Divergence

Handing Free Surface with MAC

At each Timestep:

- Identify which cells are Empty, Full, or on the Surface
- Compute new velocities
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  - Interpolate the velocities at the faces
- Render the geometry and repeat!
In 2D: For each axis, find the 4 closest face velocity samples:

\[ u_k = A_0 u_0 + A_1 u_1 + A_2 u_2 + A_3 u_3 \]

(In 3D... Find 8 closest face velocities in each dimension)

**Velocity Interpolation**

**Correct Velocity Interpolation**

**NOTE:** The complete implementation isn’t particularly elegant... Storing velocities at face midpoints (req’d for conservation of mass) makes the index math messy!

No interpolation (just use the left/bottom face velocity)

Correct Interpolation
Note that the velocity perpendicular to the outer box is zero

Buggy Interpolation
Note the clumping particles, and the discontinuities at some of the cell borders

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**Pop Worksheet!**

Teams of 2. Hand in to Jeramey after we discuss.

Determine the interpolated 2D velocity (x = horizontal, y = vertical) at the lettered white query dots in this diagram.

**Reading for Today**

• “Realistic Animation of Liquids”, Foster & Metaxas, 1996
Today

• Rigid Body Dynamics
• Collision Response
• Non-Rigid Objects
• Finite Element Method
• Deformation
• Level-of-Detail

Rigid Body Dynamics

• How do we simulate this object’s motion over time?
• We could discretize the object into many particles…
  – But a rigid body does not deform
  – Only a few degrees of freedom
• Instead, we use only one particle at the center of mass
  – Body has velocity and angular velocity
  – Compute net force & net torque

Net Force

Net Torque

f_1(t)

f_2(t)

f_3(t)

x(t)

v(t)
Degree of Freedom (DOF)

- Rotations:
  - 1 DOF: knee
  - 2 DOF: wrist
  - 3 DOF: arm

- Translations count too… → 6 DOF

Collisions

- Detection
- Response
- Overshooting problem (when we enter the solid)

Collision Response

- tangential velocity $v_t$ unchanged
- normal velocity $v_n$ reflects:
  $$v = v_t + v_n$$
  $$v \leftarrow v_t - \mathcal{E}v_n$$
- coefficient of restitution
  - 1 for elastic
  - 0 for plastic
- change of velocity = $-(1+\mathcal{E})v$
- change of momentum $\text{Impulse} = -m(1+\mathcal{E})v$

Collisions - Overshooting

- Usually, we detect collision when it’s too late: we’re already inside
- Solutions: back up
  - Compute intersection point
  - Compute response there
  - Advance for remaining fractional time step
- Other solution: Quick and dirty fixup
  - Just project back to object closest point

Collisions

- Detection
- Response
- Overshooting problem (when we enter the solid)
Energy & Rigid Body Collisions

• Total Energy = Kinetic Energy + Potential Energy + Rotational Energy
• Total Energy stays constant if there is no damping and no friction
• Rotational Energy is constant between collisions

Collision Between Two Objects

• Suppose a vertex on body A is colliding into an edge of body B at point P. Define the following variables:

\[ m_A, m_B = \text{mass of bodies A, B} \]
\[ \bar{r}_{AP} = \text{distance vector from center of mass of body A to point P} \]
\[ \bar{r}_{BP} = \text{distance vector from center of mass of body B to point P} \]
\[ \omega_{A1}, \omega_{B1} = \text{initial pre-collision angular velocity of bodies A, B} \]
\[ \omega_{A2}, \omega_{B2} = \text{final post-collision angular velocity of bodies A, B} \]
\[ v_{A1}, v_{B1} = \text{initial pre-collision velocities of center of mass bodies A, B} \]
\[ v_{A2}, v_{B2} = \text{final post-collision velocities of center of mass bodies A, B} \]
\[ v_{i1}, v_{i2} = \text{initial pre-collision velocity of impact point on body A} \]
\[ v_{n1}, v_{n2} = \text{initial pre-collision velocity of impact point on body B} \]
\[ \vec{n} = \text{normal (perpendicular) vector to edge of body B} \]
\[ \epsilon = \text{elasticity (0 = inelastic, 1 = perfectly elastic)} \]

Center of Mass & Moment of Inertia

• Center of Mass: mean location of all mass in the system
• Moment of Inertia: a measure of an object's resistance to changes to its rotation
• If a solid cylinder & a hollow tube have the same radius & the same mass, which will reach the bottom of the ramp first?

Advanced Collisions

• What about Friction?
• What if the contact between two objects is not a single point?
• What if more than two objects collide simultaneously?
Rigid Body Dynamics

- Physics
  - Velocity
  - Acceleration
  - Angular Momentum
- Collisions
- Friction

Collisions

- We know how to simulate bouncing really well
- But resting collisions are harder to manage

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Simulation of Non-Rigid Objects

- We modeled string & cloth using mass-spring systems. Can we do the same?
- Yes…
- But a more physically accurate model uses *volumetric elements*:
Strain & Stress

- Stress
  - the internal distribution of forces within a body that balance and react to the loads applied to it
  - normal stress & shear stress

- Strain
  - material deformation caused by stress.
  - measured by the change in length of a line or by the change in angle between two lines

 Finite Element Method

- To solve the continuous problem (deformation of all points of the object)
  - Discretize the problem
  - Express the interrelationship
  - Solve a big linear system

- More principled than Mass-Spring

Finite Element Method

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Level of Detail

- Interactive shape deformation
- Use high-resolution model only in areas of extreme deformation

Gilles Debunne, Mathieu Desbrun, Marie-Paule Cani, & Alan H. Barr
Dynamic Real-Time Deformations using Space & Time Adaptive Sampling
SIGGRAPH 2001
### Multi-Resolution Deformation
- Use Voronoi diagrams to match parent & child vertices.
- Interpolate values for inactive interface vertices from active parent/child vertices
- *Need to avoid interference of vibrations between simulations at different resolutions*


### Pre-computation & Simulation
- FEM matrix pre-computed
- Level of detail coupling pre-computed for rest topology
- What to do if connectivity of elements changes?
  - Cloth is cut or torn
  - Surgery simulation

### Multiple Materials
Mueller, Dorsey, McMillan, Jagnow, & Cutler
*Stable Real-Time Deformations*  
Symposium on Computer Animation 2002

### Tree Stump
Images from Cutler et al. 2002
Reading for Tuesday: *(pick one)*

- James O’Brien & Jessica Hodgins  “Graphical Modeling and Animation of Brittle Fracture”  
  SIGGRAPH 1999.

- Fracture threshold
- Remeshing  
  – need connectivity info!
- Material properties
- Parameter tuning

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