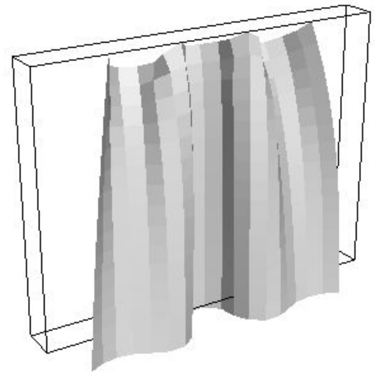
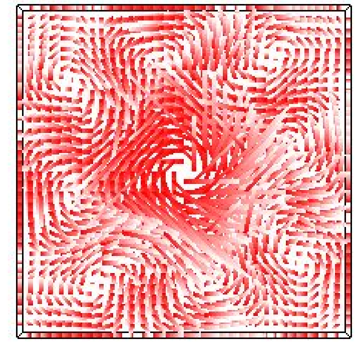
Rigid Body Dynamics, Collision Response, & Deformation

HW2: Cloth & Fluid Simulation

Questions?

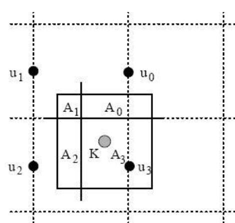


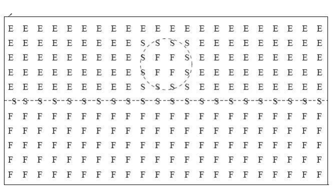


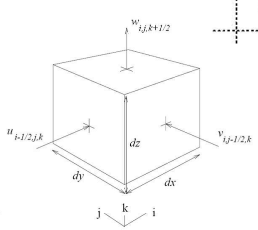
Last Time?

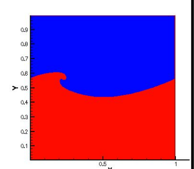
- Navier-Stokes Equations
- Conservation of Momentum & Mass
- Incompressible Flow

$$
\begin{aligned}
\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} &= -\frac{\partial p}{\partial x} + g_x + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \\
\frac{\partial v}{\partial t} + \frac{\partial vu}{\partial x} + \frac{\partial v^2}{\partial y} + \frac{\partial vw}{\partial z} &= -\frac{\partial p}{\partial y} + g_y + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) \\
\frac{\partial w}{\partial t} + \frac{\partial wu}{\partial x} + \frac{\partial wv}{\partial y} + \frac{\partial w^2}{\partial z} &= -\frac{\partial p}{\partial z} + g_z + \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)
\end{aligned}
$$









At each Timestep:

- Identify which cells are Empty, Full, or on the Surface
- Compute new velocities
- Adjust the velocities to maintain an incompressible flow
- Move the particles
  - Interpolate the velocities at the faces
- Render the geometry and repeat!

Adjusting the Velocities

- Calculate the *divergence* of the cell (the extra in/out flow)
- The divergence is used to update the *pressure* within the cell
- Adjust each face velocity uniformly to bring the divergence to zero
- Iterate across the entire grid until divergence is < ε

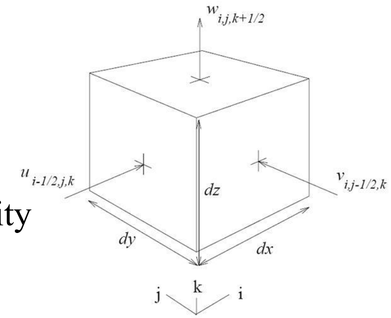
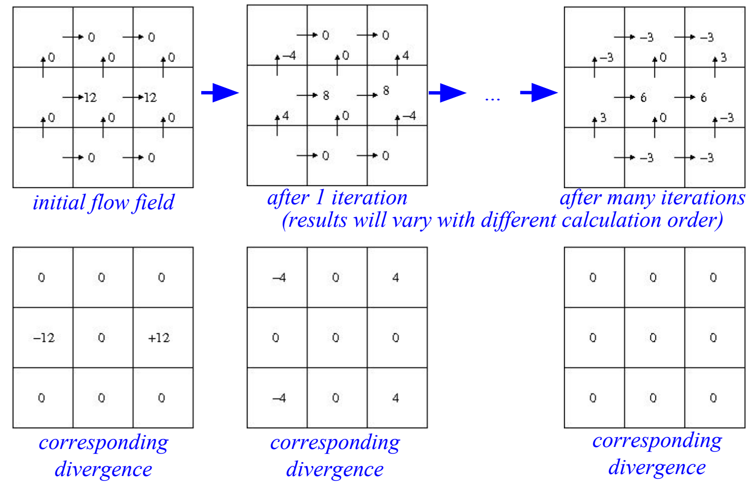


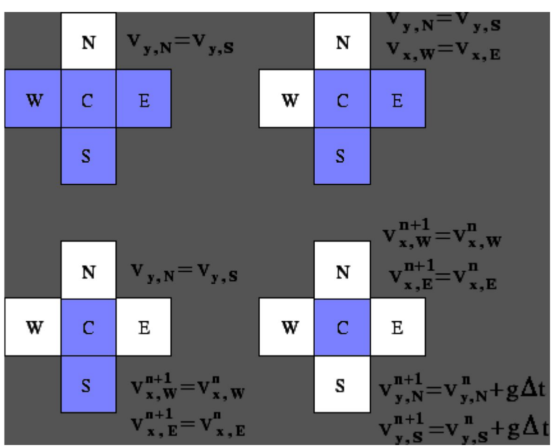
Image from Foster & Metaxas, 1996

Calculating/Eliminating Divergence



Handing Free Surface with MAC

- Divergence in surface cells:
  - Is divided equally amongst neighboring empty cells
  - Or other similar strategies?
- Zero out the divergence & pressure in empty cells



At each Timestep:

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- Move the particles
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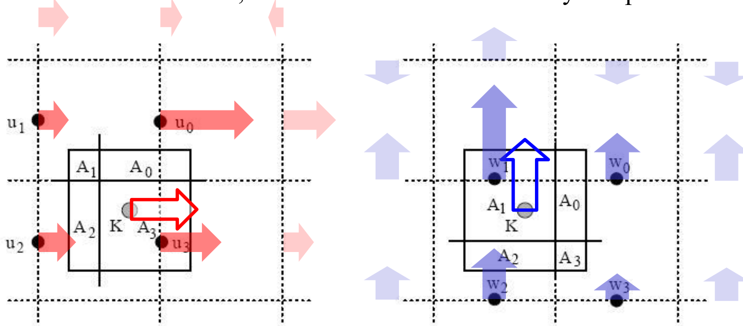
- Render the geometry and repeat!

Velocity Interpolation

Original image from Foster & Metaxas, 1996

- In 2D: For each axis, find the 4 closest face velocity samples:

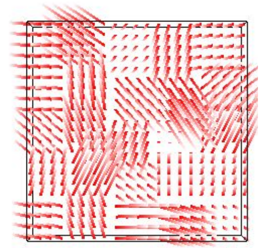


$u_k = A_0 u_0 + A_1 u_1 + A_2 u_2 + A_3 u_3$ $w_k = A_0 w_0 + A_1 w_1 + A_2 w_2 + A_3 u_3$

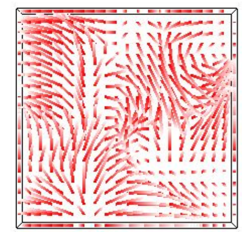
- (In 3D… Find 8 closest face velocities in each dimension)

Correct Velocity Interpolation

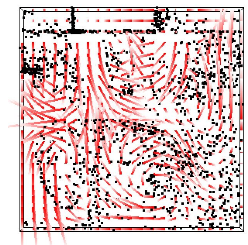
- NOTE: The complete implementation isn't particularly elegant… Storing velocities at face midpoints (req'd for conservation of mass) makes the index math messy!



No interpolation (just use the left/bottom face velocity)
Note the discontinuities in velocity at cell boundaries



Correct Interpolation
Note that the velocity perpendicular to the outer box is zero

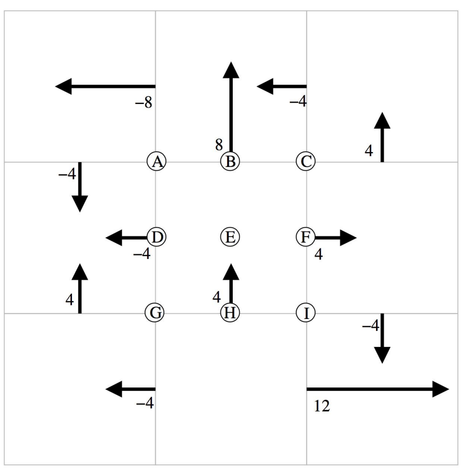


Buggy Interpolation
Note the clumping particles, and the discontinuities at some of the cell borders

Pop Worksheet!

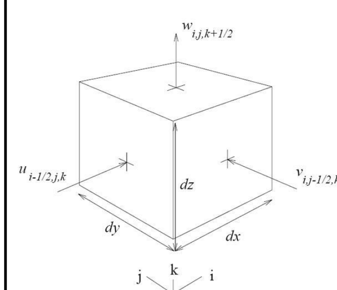
Teams of 2. Hand in to Jeramey after we discuss.

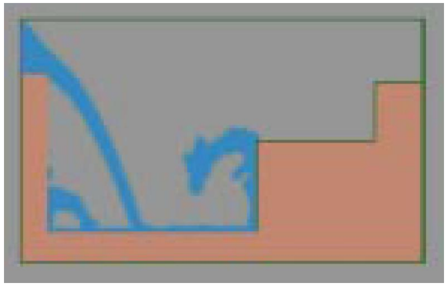
Determine the interpolated 2D velocity (x = horizontal, y = vertical) at the lettered white query dots in this diagram.



Reading for Today

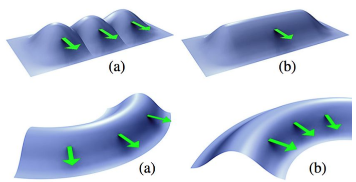
- "Realistic Animation of Liquids", Foster & Metaxas, 1996

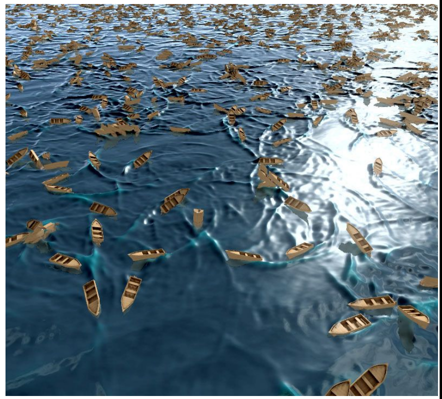




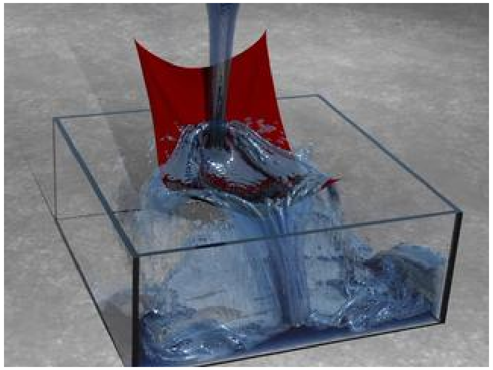
Optional Reading for Today

- Cem Yuksel, Donald H. House, and John Keyser, "Wave Particles", SIGGRAPH 2007





Optional Reading for Today



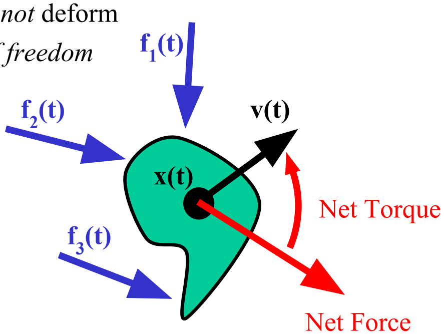
"Coupling Water and Smoke to Thin Deformable and Rigid Shells", Guendelman, Selle, Losasso, & Fedkiw, SIGGRAPH 2005.

Today

- Rigid Body Dynamics
- Collision Response
- Non-Rigid Objects
- Finite Element Method
- Deformation
- Level-of-Detail

Rigid Body Dynamics

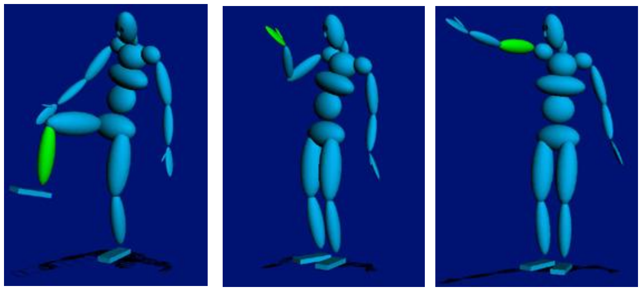
- How do we simulate this object's motion over time?
- We could discretize the object into many particles…
  - But a rigid body does *not* deform
  - Only a few *degrees of freedom*
- Instead, we use only one particle at the center of mass
  - Body has velocity *and* angular velocity
  - Compute net force & net torque



Nice Reference Material: http://www.pixar.com/companyinfo/research/pbm2001/

Degree of Freedom (DOF)

- Rotations:

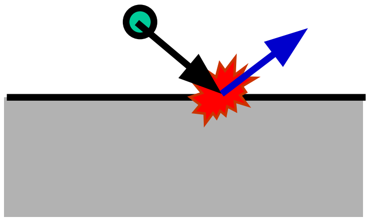


1 DOF: knee | 2 DOF: wrist | 3 DOF: arm

- Translations count too… ➔ 6 DOF

Collisions

- Detection
- Response
- Overshooting problem (when we enter the solid)

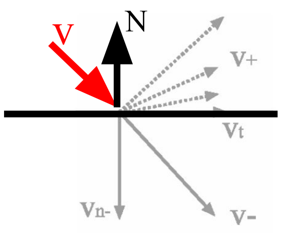


Collision Response

- tangential velocity v_t unchanged
- normal velocity v_n reflects:

$$v = v_t + v_n$$

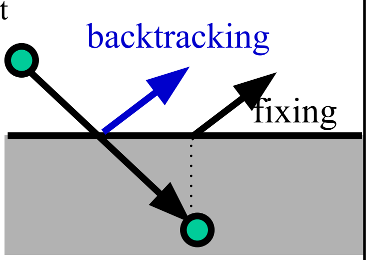
$$v \leftarrow v_t - \varepsilon v_n$$



- coefficient of restitution
  - 1 for elastic
  - 0 for plastic
- change of velocity = $-(1+\epsilon)v$
- change of momentum *Impulse* = $-m(1+\epsilon)v$

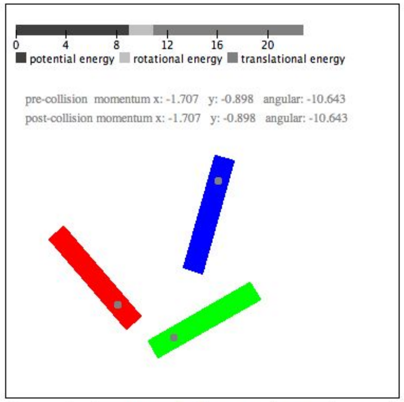
Collisions - Overshooting

- Usually, we detect collision when it's too late: we're already inside
- Solutions: back up
  - Compute intersection point
  - Compute response there
  - Advance for remaining fractional time step
- Other solution: Quick and dirty fixup
  - Just project back to object closest point



Energy & Rigid Body Collisions

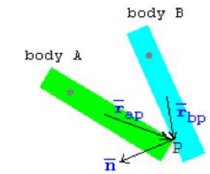
- Total Energy = Kinetic Energy + Potential Energy + Rotational Energy
- Total Energy stays constant if there is no damping and no friction
- Rotational Energy is constant between collisions



http://www.myphysicslab.com/collision.html

Collision Between Two Objects

- Suppose a vertex on body A is colliding into an edge of body B at point P. Define the following variables:



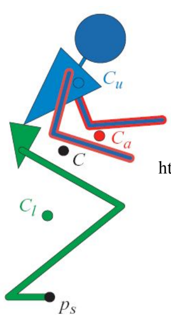
vectors involved in collision

m_a, m_b = mass of bodies A, B
$\overline{\mathbf{r}}_{ap}$ = distance vector from center of mass of body A to point P
$\overline{\mathbf{r}}_{bp}$ = distance vector from center of mass of body B to point P
ω_{a1}, ω_{b1} = initial pre-collision angular velocity of bodies A, B
ω_{a2}, ω_{b2} = final post-collision angular velocity of bodies A, B
$\overline{\mathbf{v}}_{a1}, \overline{\mathbf{v}}_{b1}$ = initial pre-collision velocities of center of mass bodies A, B
$\overline{\mathbf{v}}_{a2}, \overline{\mathbf{v}}_{b2}$ = final post-collision velocities of center of mass bodies A, B
$\overline{\mathbf{v}}_{ap1}$ = initial pre-collision velocity of impact point on body A
$\overline{\mathbf{v}}_{bp1}$ = initial pre-collision velocity of impact point on body B
$\overline{\mathbf{n}}$ = normal (perpendicular) vector to edge of body B
e = elasticity (0 = inelastic, 1 = perfectly elastic)

http://www.myphysicslab.com/collision.html

Center of Mass & Moment of Inertia

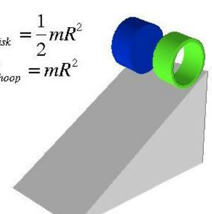
- Center of Mass: mean location of all mass in the system
- Moment of Inertia: a measure of an object's resistance to changes to its rotation
- If a solid cylinder & a hollow tube have the same radius & the same mass, which will reach the bottom of the ramp first?



Liu & Popović

http://en.wikipedia.org/wiki/Fosbury_Flop

$$I_{disk} = \frac{1}{2} mR^2$$
$$I_{hoop} = mR^2$$



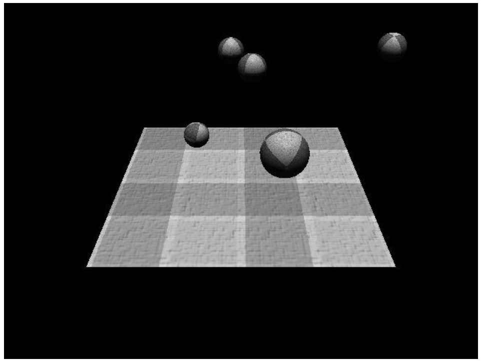
http://solomon.physics.sc.edu/~tedeschi/demo/demo12.html
http://hyperphysics.phy-astr.gsu.edu/hbase/hoocyl2.html

Advanced Collisions

- What about Friction?
- What if the contact between two objects is not a single point?
- What if more than two objects collide simultaneously?

Rigid Body Dynamics

- Physics
  - Velocity
  - Acceleration
  - Angular Momentum
- Collisions
- Friction

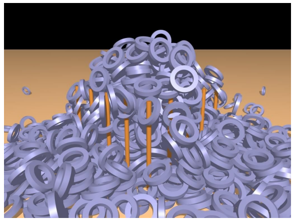


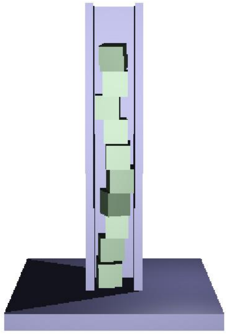
from: Darren Lewis
http://www-cs-students.stanford.edu/~dalewis/cs448a/rigidbody.html

Collisions

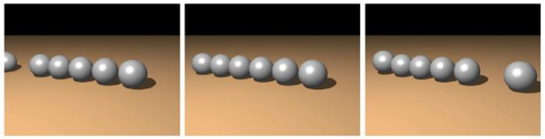
Victor J. Milenkovic & Harald Schmidl
Optimization-Based Animation
SIGGRAPH 2001

- We know how to simulate bouncing really well
- But resting collisions are harder to manage





Guendelman, Bridson & Fedkiw
Nonconvex Rigid Bodies with Stacking
SIGGRAPH 2003



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- Finite Element Method
- Deformation
- Level-of-Detail

Simulation of Non-Rigid Objects

- We modeled string & cloth using mass-spring systems. Can we do the same?
- Yes…
- But a more physically accurate model uses *volumetric elements:*

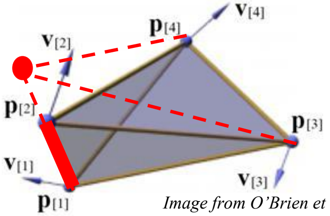
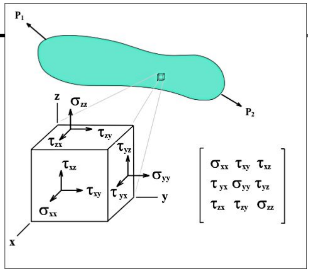


Image from O'Brien et al. 1999

Strain & Stress

- Stress
  - the internal distribution of forces within a body that balance and react to the loads applied to it
  - *normal stress & shear stress*
- Strain
  - material deformation caused by stress.
  - measured by the change in length of a line or by the change in angle between two lines



http://en.wikipedia.org/wiki/Image:Stress_tensor.png

$$\varepsilon = \frac{\Delta_l}{l_0}$$

Finite Element Method

- To solve the continuous problem (deformation of all points of the object)
  - Discretize the problem
  - Express the interrelationship
  - Solve a big linear system
- More principled than Mass-Spring

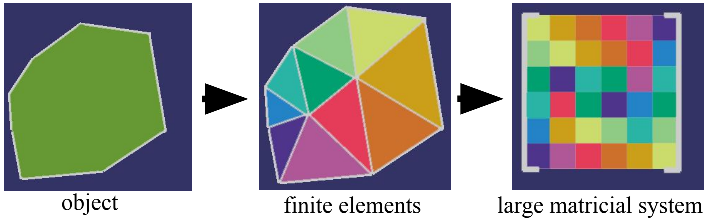


Diagram from Debunne et al. 2001

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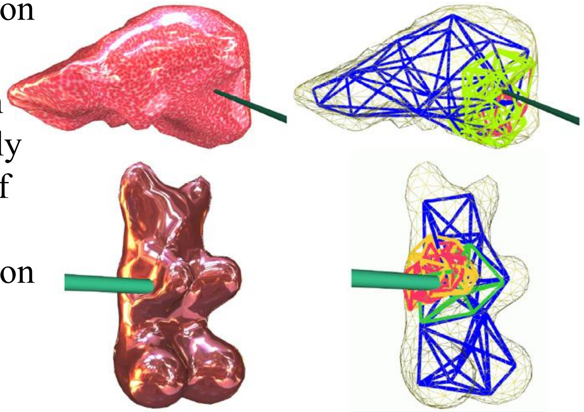
Level of Detail

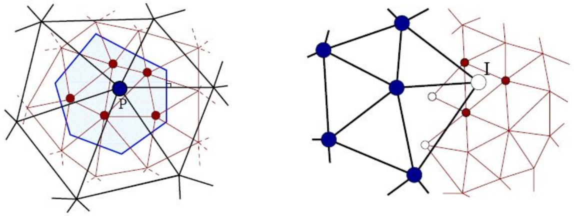
Gilles Debunne , Mathieu Desbrun, Marie-Paule Cani, & Alan H. Barr
Dynamic Real-Time Deformations using Space & Time Adaptive Sampling
SIGGRAPH 2001

- Interactive shape deformation
- Use high-resolution model only in areas of extreme deformation



Multi-Resolution Deformation

- Use Voronoi diagrams to match parent & child vertices.
- Interpolate values for inactive interface vertices from active parent/child vertices

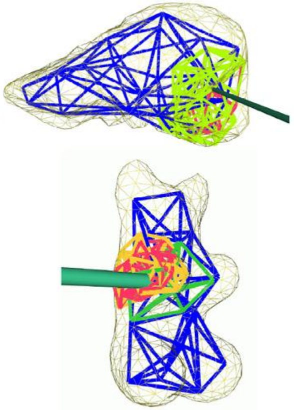


- *Need to avoid interference of vibrations between simulations at different resolutions*

Debunne et al. "Dynamic Real-Time Deformations using Space & Time Adaptive Sampling", 2001

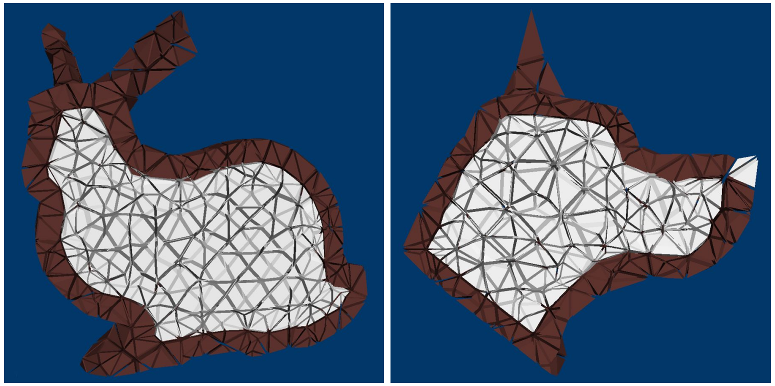
Pre-computation & Simulation

- FEM matrix pre-computed
- Level of detail coupling pre-computed for rest topology
- What to do if connectivity of elements changes?
  - Cloth is cut or torn
  - Surgery simulation

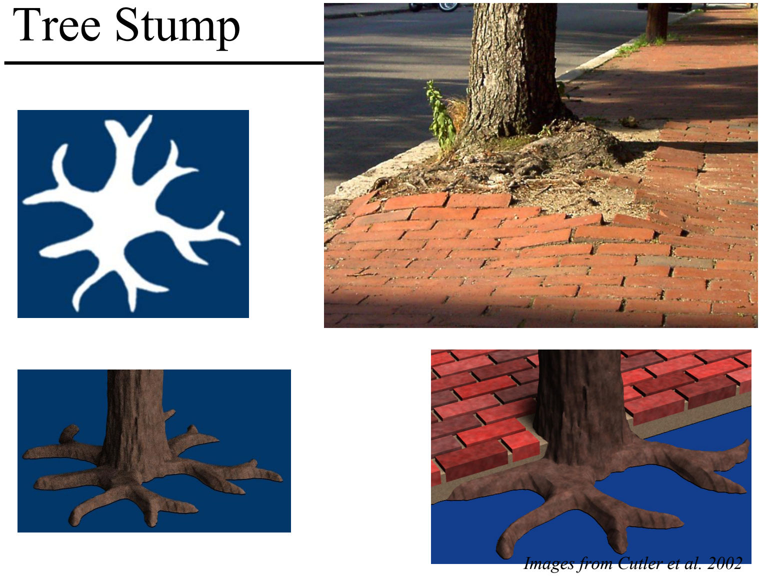


Multiple Materials

Mueller, Dorsey, McMillan, Jagnow, & Cutler
Stable Real-Time Deformations
Symposium on Computer Animation 2002



Tree Stump



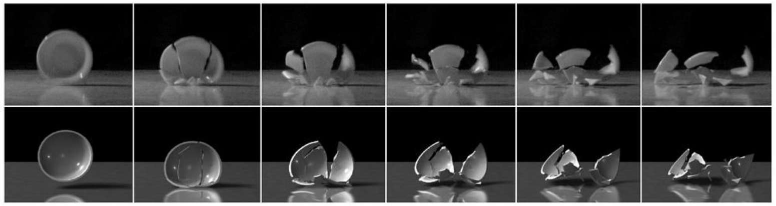
Images from Cutler et al. 2002

Image from Cutler et al. 2002

Image from Cutler et al. 2002

Reading for Tuesday: *(pick one)*

- James O'Brien & Jessica Hodgins *"Graphical Modeling and Animation of Brittle Fracture"* SIGGRAPH 1999.



- Fracture threshhold
- Remeshing
  - need connectivity info!
- Material properties
- Parameter tuning

Level of Detail

Gilles Debunne , Mathieu Desbrun, Marie-Paule Cani, & Alan H. Barr
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SIGGRAPH 2001

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