

# Fracture & Tetrahedral Models

## Pop Worksheet!

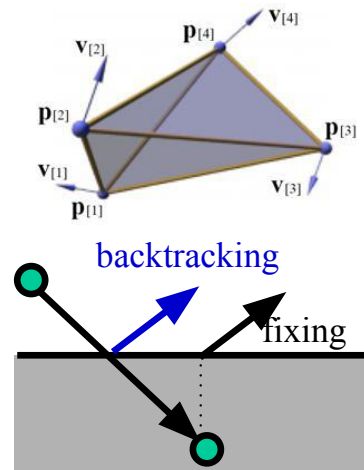
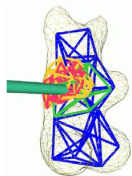
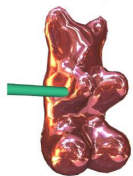
Teams of 2. Hand in to Jeramey after we discuss.

What are the horizontal and face velocities after 1, 2, and many iterations of divergence adjustment for an incompressible fluid?

	→ 0	→ 0	→ 0
↑ 0	↑ 0	↑ 0	↑ 0
	→ 0	→ 0	→ 0
↑ 0	↑ 0	↑ 0	↑ 0
	→ 12	→ 12	→ 0

## Last Time?

- Rigid Body
- Collision Response
- Finite Element Method
  - Stress/Strain
- Deformation
  - Level of Detail



## Today

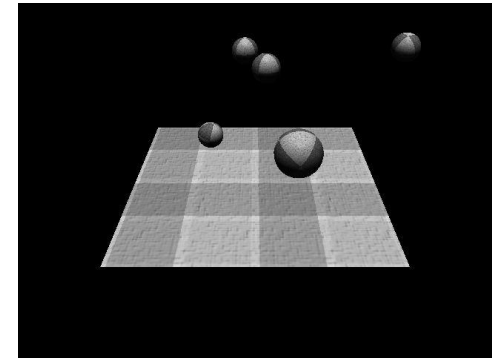
- Continuing from Last Time...
  - Collision Response
  - Non-Rigid Objects
  - Finite Element Method
- Useful & Related Term Definitions
- “Graphical Modeling and Animation of Brittle Fracture”
- “Dynamic Real-Time Deformations using Space & Time Adaptive Sampling”
  - Level-of-Detail

## Advanced Collisions

- What about Friction?
- Rolling objects?
- What if the contact between two objects is not a single point?
- What if more than two objects collide simultaneously?

## Rigid Body Dynamics

- Physics
  - Velocity
  - Acceleration
  - Angular Momentum
- Collisions
- Friction



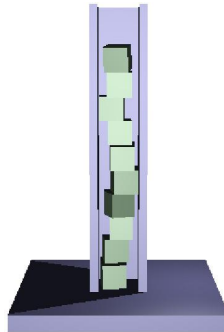
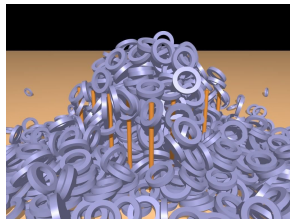
from: Darren Lewis

<http://www-cs-students.stanford.edu/~dalewis/cs448a/rigidbody.html>

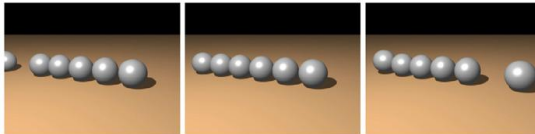
## Collisions

Victor J. Milenkovic & Harald Schmidl  
*Optimization-Based Animation*  
SIGGRAPH 2001

- We know how to simulate bouncing really well
- But resting collisions are harder to manage



Guendelman, Bridson  
& Fedkiw  
*Nonconvex Rigid  
Bodies with Stacking*  
SIGGRAPH 2003



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## Simulation of Non-Rigid Objects

- We modeled string & cloth using mass-spring systems. Can we do the same?
- Yes...
- But a more physically accurate model uses *volumetric elements*:

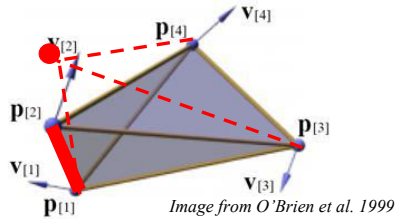
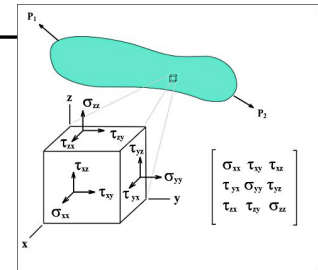


Image from O'Brien et al. 1999

## Strain & Stress

- Stress
  - the internal distribution of forces within a body that balance and react to the loads applied to it
  - *normal stress & shear stress*
- Strain
  - material deformation caused by stress.
  - measured by the change in length of a line or by the change in angle between two lines



[http://en.wikipedia.org/wiki/Image:Stress\\_tensor.png](http://en.wikipedia.org/wiki/Image:Stress_tensor.png)

$$\varepsilon = \frac{\Delta l}{l_0}$$

## Finite Element Method

- To solve the continuous problem (deformation of all points of the object)
  - Discretize the problem
  - Express the interrelationship
  - Solve a big linear system
- More principled than Mass-Spring

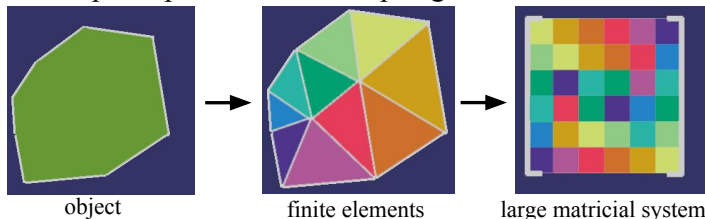


Diagram from DeBunne et al. 2001

## Today

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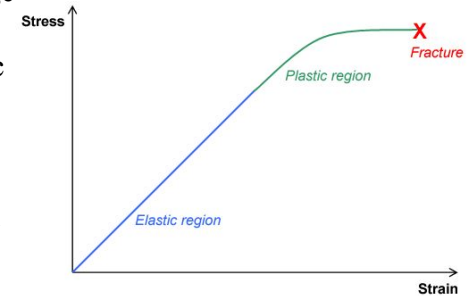
## Misc. Definitions

- *Isotropic*: is a property which does not depend on the direction.
- *Anisotropic*: is a property which is directionally dependent.



## Misc. Definitions

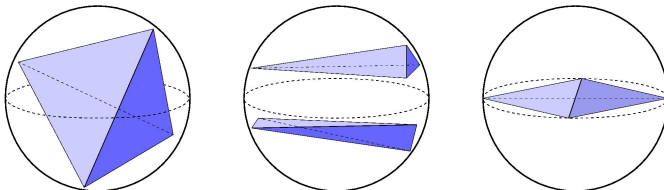
- *Elastic Deformation*: Once the forces are no longer applied, the object returns to its original shape.
- *Plastic Deformation*: An object in the plastic deformation range will first have undergone elastic deformation, which is reversible, so the object will return part way to its original shape.



<http://en.wikipedia.org/wiki/Image:Stress-strain1.png>

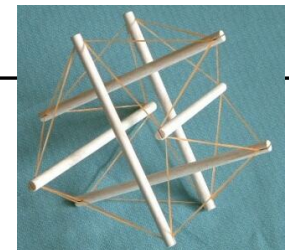
## Misc. Definitions

- *Degenerate/Ill-conditioned Element*:  
a.k.a. how “equilateral” are the elements?
  - Ratio of volume<sup>2</sup> to surface area<sup>3</sup>
  - Smallest *solid* angle
  - Ratio of volume to volume of smallest circumscribed sphere

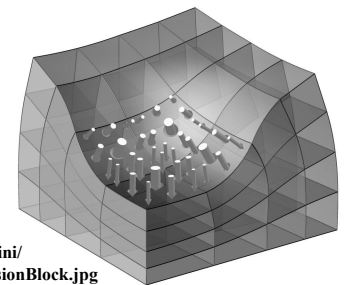


## Misc. Definitions

- *Tension*: The direction of the force of tension is parallel to the string, away from the object exerting the stretching force.
- *Compression*: resulting in reduction of volume



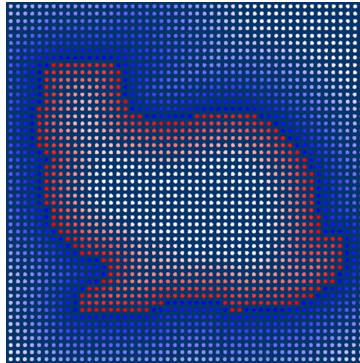
<http://fig.cox.miami.edu/~cmallery/255/255chem/tensegrity.sticks.jpg>



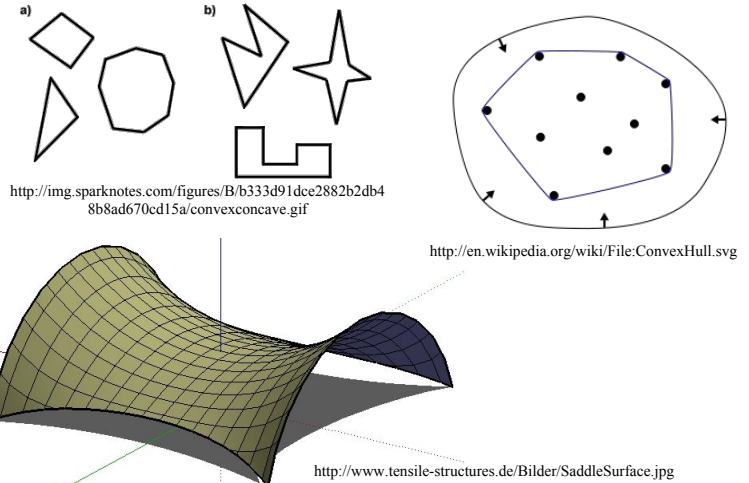
<http://www.aero.polimi.it/~merlini/SolidMechanics-FiniteElasticity/CompressionBlock.jpg>

## Implicit Surfaces

- For a sphere:  
 $H(x,y,z) = x^2 + y^2 + z^2 - r^2$
- If  $H(x,y,z) = 0$ ,  
on surface
- If  $H(x,y,z) > 0$ ,  
outside surface
- If  $H(x,y,z) < 0$ ,  
inside surface



## Convex vs. Non-Convex



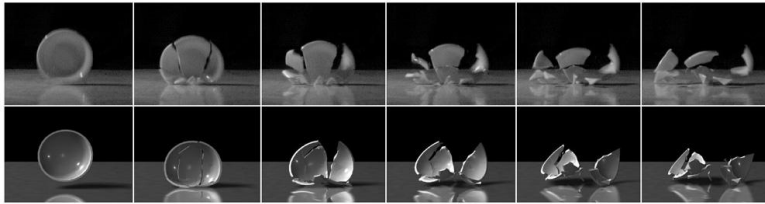
## Questions?

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## Reading for Today:

- James O'Brien & Jessica Hodgins "*Graphical Modeling and Animation of Brittle Fracture*" SIGGRAPH 1999.



- Fracture threshold
- Material properties
- Remeshing
- Parameter tuning
- need connectivity info!

## Fracture Opening Modes

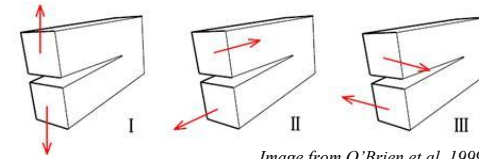
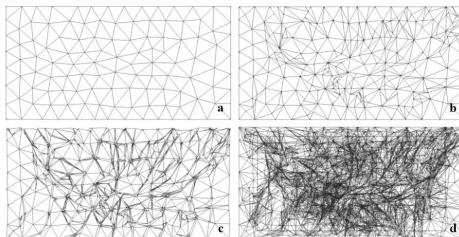
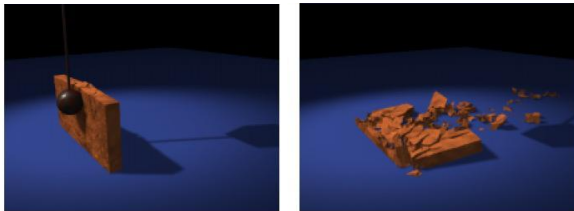


Image from O'Brien et al. 1999

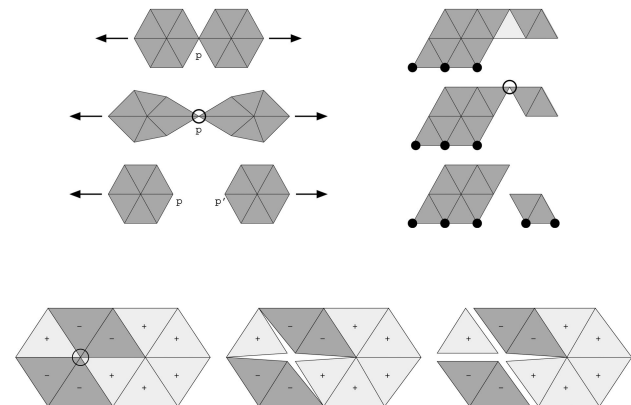
Figure 6: Three loading modes that can be experienced by a crack. Mode I: Opening, Mode II: In-Plane Shear, and Mode III: Out-of-Plane Shear. Adapted from Anderson [1].

## Local Mesh Refinement



Images from  
O'Brien et al. 1999

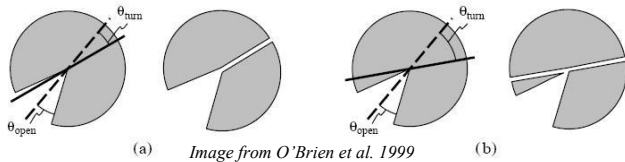
## Managing Fracture Adjacency



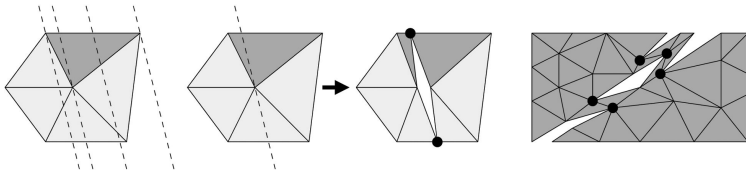


## Fracture Propagation Difficulties

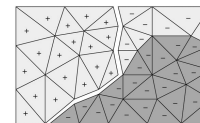
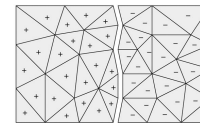
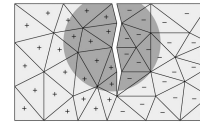
- Need to track direction of fracture propagation?



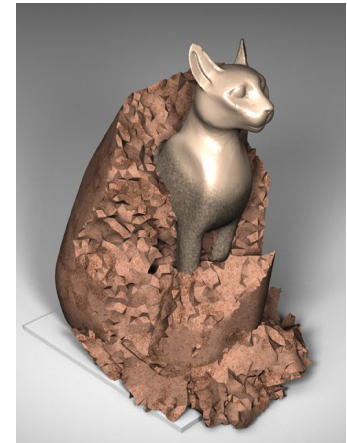
- Need to track crack tip?



## Controlling Speed of Propagation



(no remeshing)



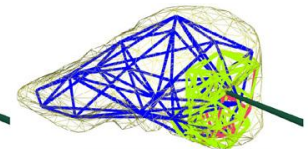
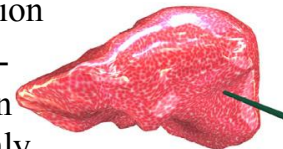
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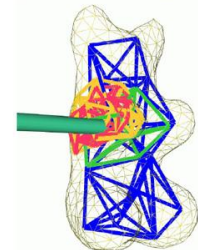
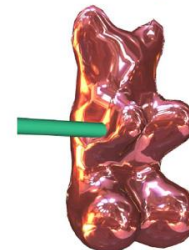
## Level of Detail

Gilles Debunne, Mathieu Desbrun,  
Marie-Paule Cani, & Alan H. Barr  
*Dynamic Real-Time Deformations using  
Space & Time Adaptive Sampling*  
SIGGRAPH 2001

- Interactive shape deformation

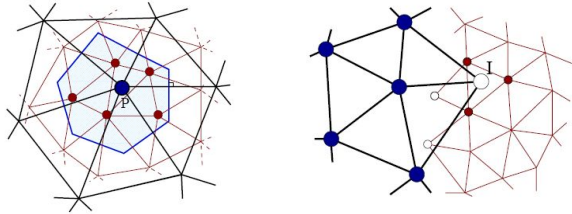


- Use high-resolution model only in areas of extreme deformation



## Multi-Resolution Deformation

- Use Voronoi diagrams to match parent & child vertices.
- Interpolate values for inactive interface vertices from active parent/child vertices

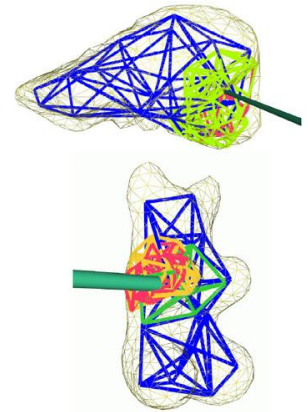


- *Need to avoid interference of vibrations between simulations at different resolutions*

DeBunne et al. "Dynamic Real-Time Deformations using Space & Time Adaptive Sampling", 2001

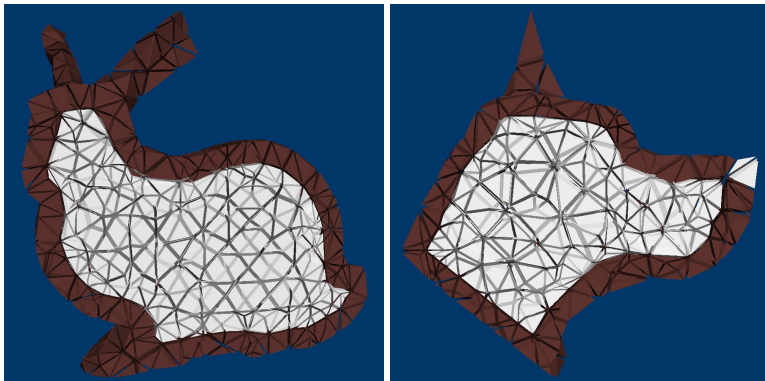
## Pre-computation & Simulation

- FEM matrix pre-computed
- Level of detail coupling pre-computed for rest topology
- What to do if connectivity of elements changes?
  - Cloth is cut or torn
  - Surgery simulation



## Multiple Materials

Mueller, Dorsey, McMillan,  
Jagnow, & Cutler  
*Stable Real-Time Deformations*  
Symposium on Computer  
Animation 2002



## Tree Stump



Images from Cutler et al. 2002





Image from Cutler et al. 2002



Image from Cutler et al. 2002

## Haptic Device

- “3D mouse” + force feedback
- 6 DOF (position & orientation)
- *requires 1000 Hz refresh*  
(visual only requires ~30 Hz)

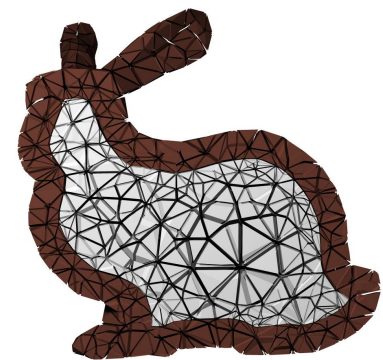


Sensable's Phantom  
<http://www.sensable.com/>

## 3D Mesh Simplification



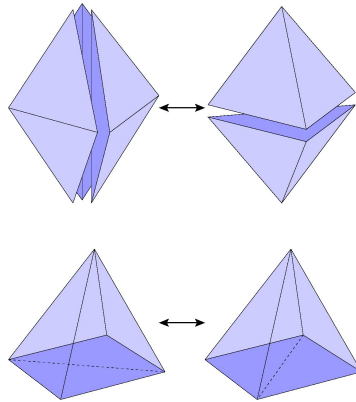
1,050K tetras  
(133K faces)



10K tetras  
(3K faces)

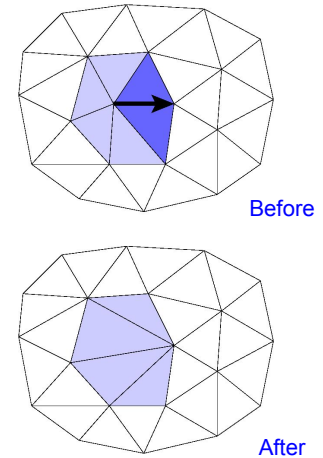
## 3D Mesh Operations

- **Tetrahedral Swaps**
  - Choose the configuration with the best local element shape
- Edge Collapse
- Vertex Smoothing
- Vertex Addition



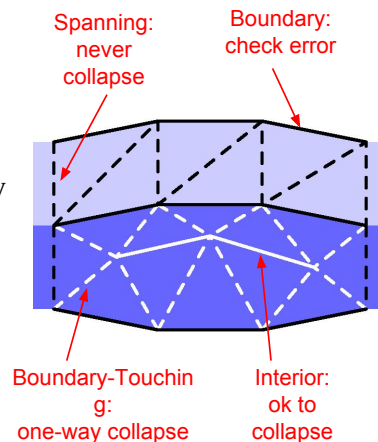
## 3D Mesh Operations

- Tetrahedral Swaps
- **Edge Collapse**
  - Delete a vertex & the elements around the edge
- Vertex Smoothing
- Vertex Addition



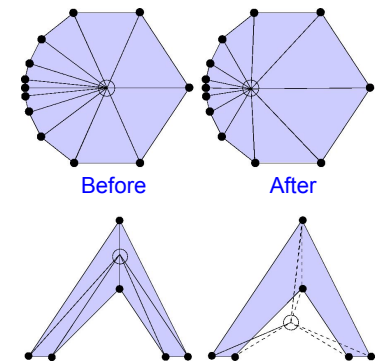
## Prioritizing Edge Collapses

- Preserve topology
  - Thin layers should not pinch together
- Collapse weight
  - Edge length + boundary error
- No negative volumes
- Local element quality does not significantly worsen



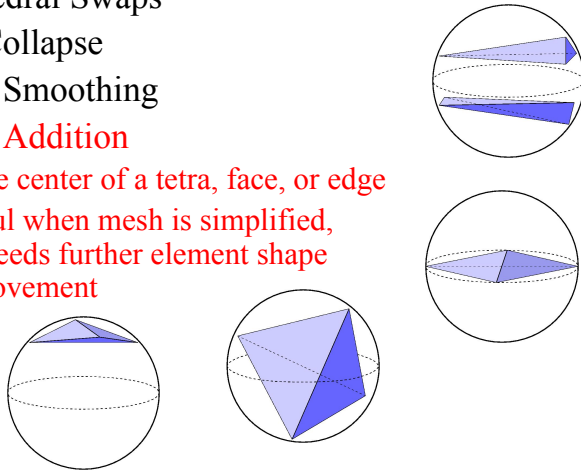
## 3D Mesh Operations

- Tetrahedral Swaps
- Edge Collapse
- **Vertex Smoothing**
  - Move a vertex to the centroid of its neighbors
  - Convex or concave, but avoid negative-volume elements
- Vertex Addition

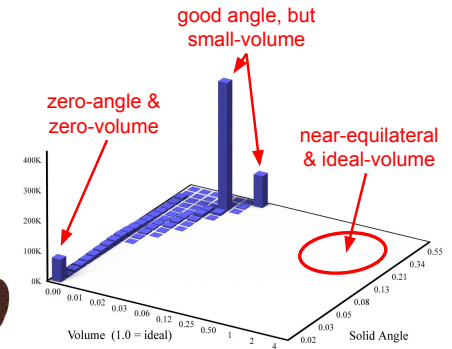
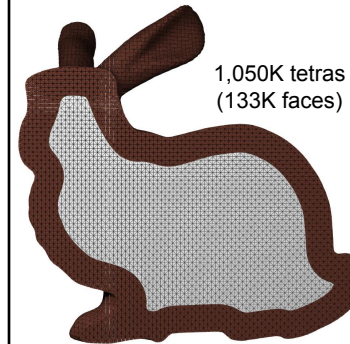


## 3D Mesh Operations

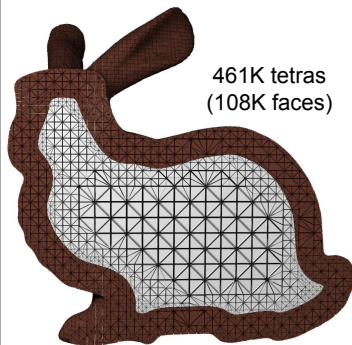
- Tetrahedral Swaps
- Edge Collapse
- Vertex Smoothing
- **Vertex Addition**
  - At the center of a tetra, face, or edge
  - Useful when mesh is simplified, but needs further element shape improvement



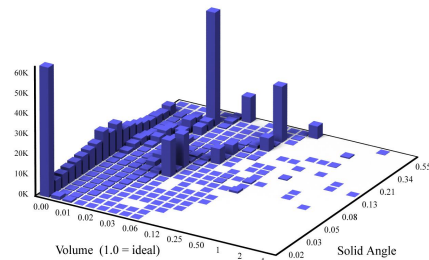
## Visualization of Tetrahedra Quality



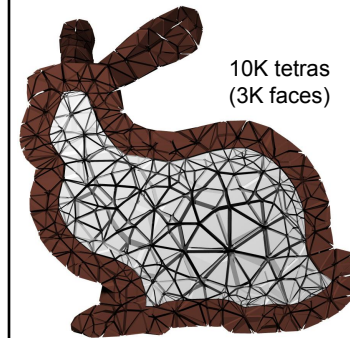
## Visualization of Tetrahedra Quality



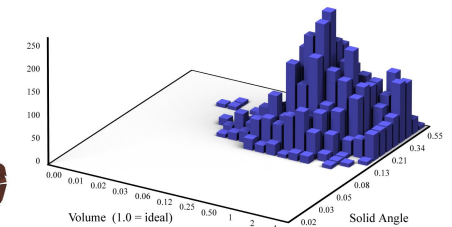
Octree or Adaptive Distance Field (ADF)



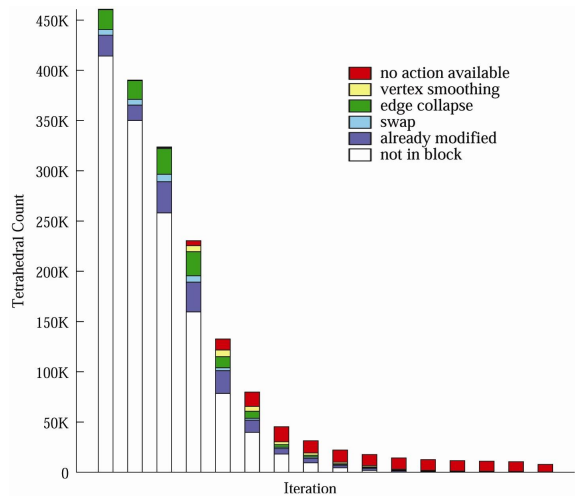
## Visualization of Tetrahedra Quality



After Simplification & Mesh Improvement

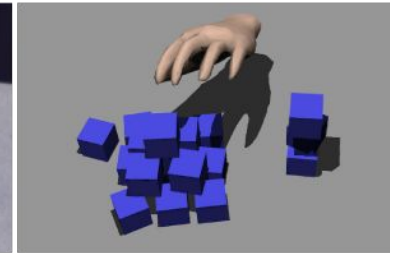


## Visualization of Simplification Algorithm



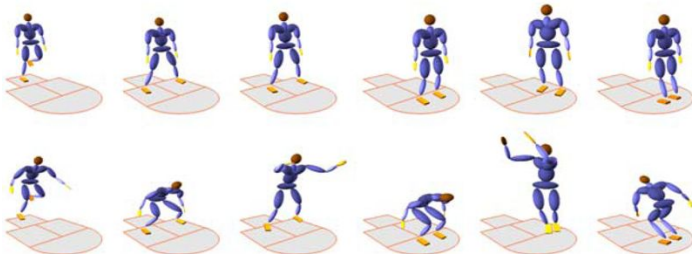
## Reading for Friday

- “Real-Time Hand-Tracking with a Color Glove”  
SIGGRAPH 2009,  
Wang & Popović



## Reading for Friday

“Synthesis of Complex  
Dynamic Character Motion  
from Simple Animation”,  
Liu & Popović, 2002



- Rapid prototyping of realistic character motion  
*from rough low-quality animations*
- Obey the laws of physics & stay within space  
of naturally-occurring movements