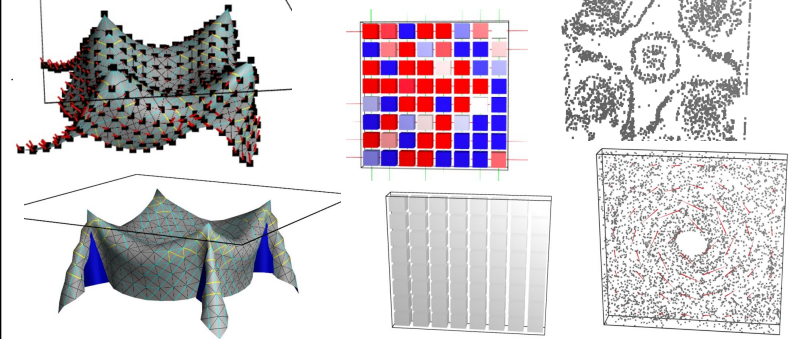


# Animation, Motion Capture, & Inverse Kinematics

## Homework 2 Questions?

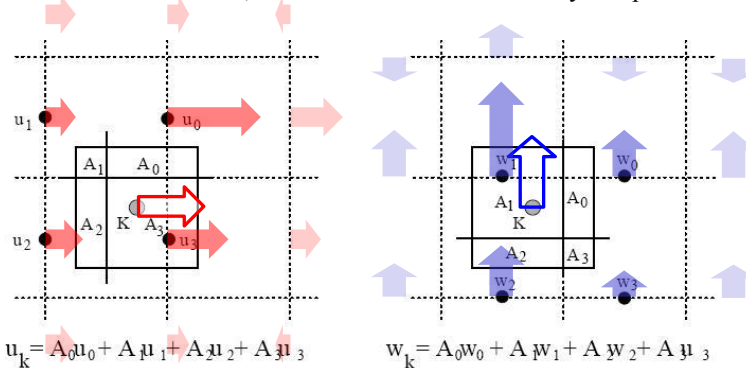
- Self-intersecting cloth?
- All at once? Or left to right, top to bottom?
- “Off by 1” (or “off by 1/2”)
- Pressure after divergence correction?
- Euler or something better?



## Velocity Interpolation

*Original image from  
Foster & Metaxas, 1996*

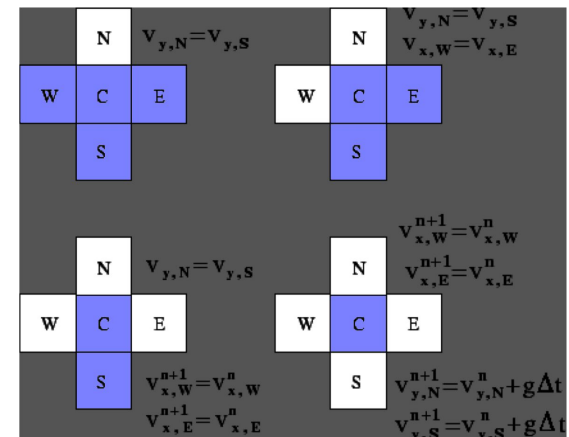
- In 2D: For each axis, find the 4 closest face velocity samples:



- (In 3D... Find 8 closest face velocities in each dimension)

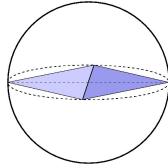
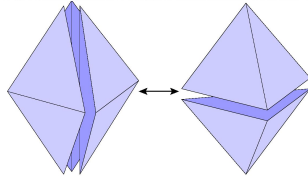
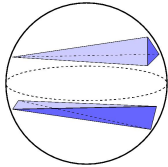
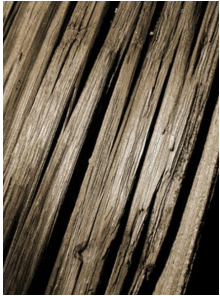
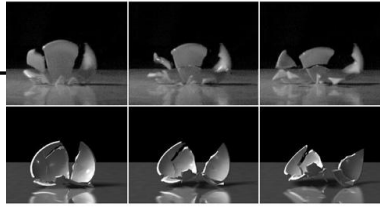
## Handling Free Surface with MAC

- Divergence in surface cells:
  - Is divided equally amongst neighboring empty cells
  - Or other similar strategies?
- Zero out the divergence & pressure in empty cells



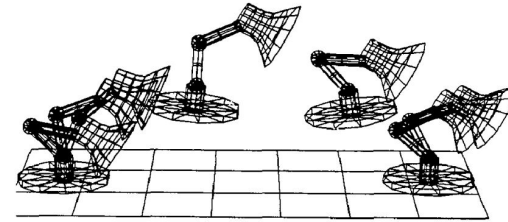
## Last Time?

- Tetrahedral Meshing
- Haptics
- Anisotropic Materials
- Fracture



## Today: How do we Animate?

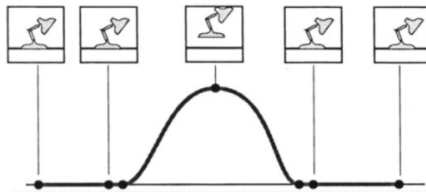
- Keyframing
- Procedural Animation
- Physically-Based Animation
- Motion Capture
- Skeletal Animation
- Forward and Inverse Kinematics



"Spacetime Constraints", Witkin & Kass, SIGGRAPH 1988

## Keyframing

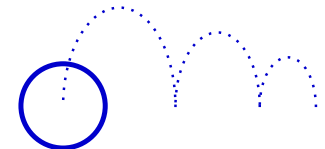
- Use spline curves to automate the in betweening
  - Good control
  - Less tedious than drawing *every* frame
- Creating a good animation still requires considerable skill and talent



ACM © 1987 "Principles of traditional animation applied to 3D computer animation"

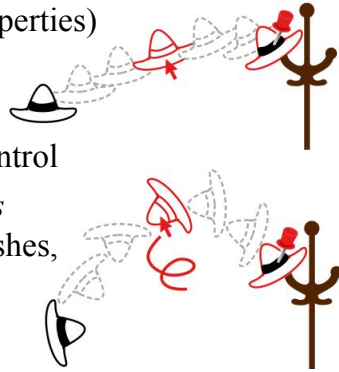
## Procedural Animation

- Describes the motion algorithmically, as a function of small number of parameters
- Example: a clock with second, minute and hour hands
  - express the clock motions in terms of a "seconds" variable
  - the clock is animated by varying the seconds parameter
- Example: A bouncing ball
  - $\text{Abs}(\sin(\omega t + \theta_0)) * e^{-kt}$



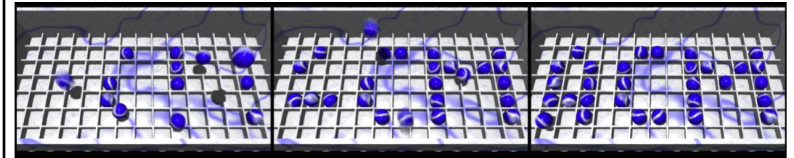
# Physically-Based Animation

- Assign physical properties to objects (masses, forces, inertial properties)
- Simulate physics by solving equations
- Realistic, but difficult to control
- Used for *secondary motions* (hair, cloth, scattering, splashes, breaking, smoke, etc.) that respond to primary *user controlled* animation



“Interactive Manipulation of Rigid Body Simulations”  
SIGGRAPH 2000, Popović, Seitz, Erdmann, Popović & Witkin

“Sampling Plausible Solutions to Multi-body Constraint Problems”  
Chenney & Forsyth,  
SIGGRAPH 2000



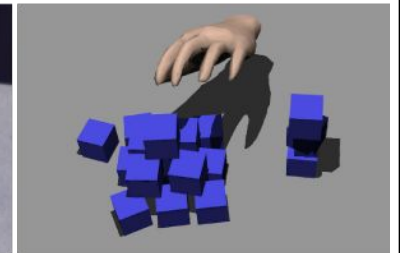
# Motion Capture

- Optical markers, high-speed cameras, triangulation → 3D position
- Captures style, subtle nuances and realism at high-resolution
- You must observe someone do something
- Difficult (or impossible?) to *edit* mo-cap data



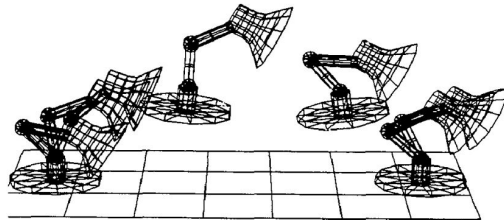
# Reading for Today:

- “Real-Time Hand-Tracking with a Color Glove”  
SIGGRAPH 2009,  
Wang & Popović



## Today: How do we Animate?

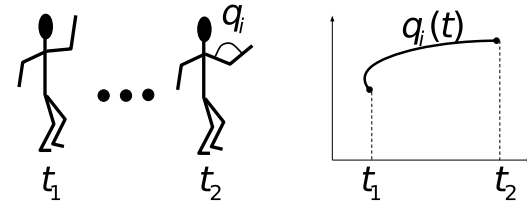
- Keyframing
- Procedural Animation
- Physically-Based Animation
- Motion Capture
- **Skeletal Animation**
- **Forward and Inverse Kinematics**



“Spacetime Constraints”, Witkin & Kass, SIGGRAPH 1988

## Articulated Models

- Articulated models:
  - rigid parts
  - connected by joints
- They can be animated by specifying the joint angles as functions of time.

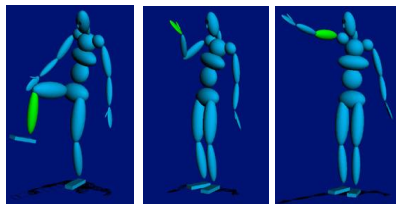
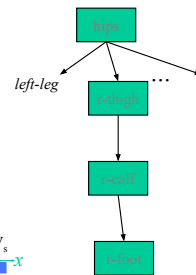


## Skeleton Hierarchy

- Each bone transformation described relative to the parent in the hierarchy:

$$x_h, y_h, z_h, q_h, f_h, s_h$$

$$q, f_t, s_t$$

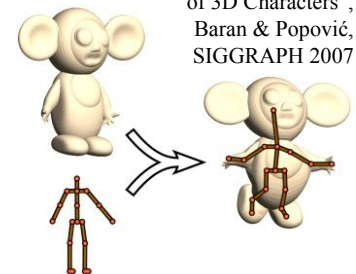


1 DOF: knee    2 DOF: wrist    3 DOF: arm

## Skeletal Animation Challenges

- Skinning
  - Complex deformable skin, muscle, skin motion
- Hierarchical controls
  - Smile control, eye blinking, etc.
  - Keyframes for these higher-level controls
- A huge amount of time is spent building the 3D models, its skeleton, and its controls

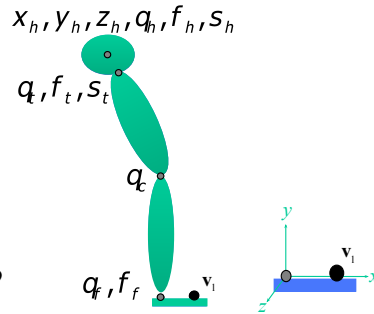
“Automatic Rigging and Animation of 3D Characters”, Baran & Popović, SIGGRAPH 2007



Maya tutorial

## Forward Kinematics

- Given skeleton parameters  $p$ , and the position of the effector in local coordinates  $V_l$ , what is the position of the effector in the world coordinates  $V_w$ ?



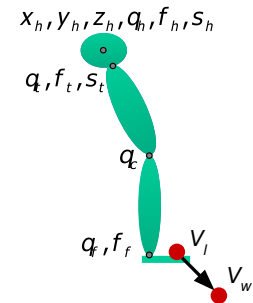
$$V_w = T(x_h, y_h, z_h) R(q_h, f_h, s_h) T_h R(q_t, f_t, s_t) T_t R(q_e) T_e R(q_p, f_p) V_l$$

$$V_w = S(p) V_l$$

$S(p)$  is "just" a 4x4 affine transformation matrix!

## Inverse Kinematics (IK)

- Given the position of the effector in local coordinates  $V_l$  and the *desired position*  $V_w$  in world coordinates, what are the skeleton parameters  $p$ ?
- Much harder requires solving the inverse of the non-linear function:

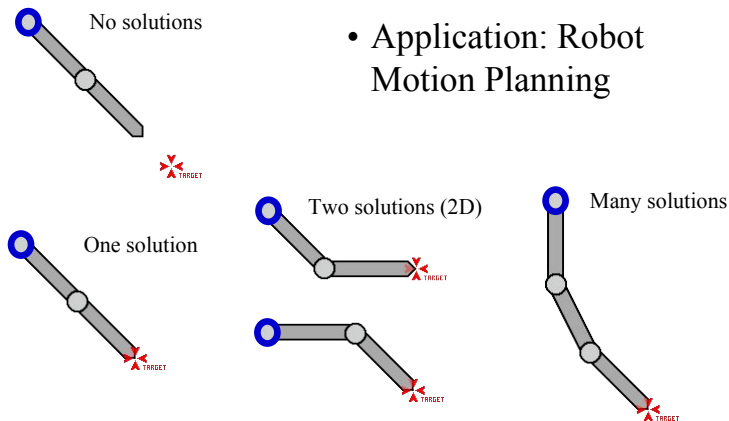


find  $p$  such that  $S(p)V_l = V_w$

Why is this hard? Why is it non-linear?

## Under-/Over- Constrained IK

- Application: Robot Motion Planning

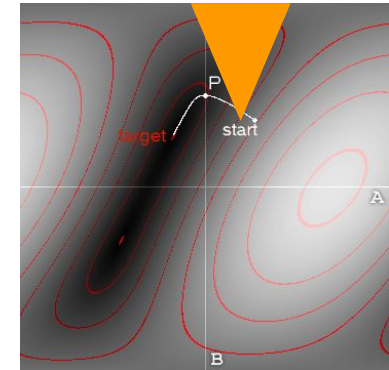


"The good-looking textured light-sourced bouncy fun smart and stretchy page"  
Hugo Elias, [http://freespace.virgin.net/hugo.elias/models/m\\_ik.htm](http://freespace.virgin.net/hugo.elias/models/m_ik.htm)

## Searching Configuration Space



pose space shaded by distance to target



- Use *gradient descent* to walk from starting configuration to target
- Angle restrictions & collisions can introduce local minima

"The good-looking textured light-sourced bouncy fun smart and stretchy page"  
Hugo Elias, [http://freespace.virgin.net/hugo.elias/models/m\\_ik.htm](http://freespace.virgin.net/hugo.elias/models/m_ik.htm)



## IK Challenge

- Find a “natural” skeleton configuration for a given collection of pose constraints
- A *vector constraint function*  $C(p) = 0$  collects all pose constraints
- A *scalar objective function*  $g(p)$  measures the quality of a pose,  $g(p)$  is minimum for most natural poses. Example  $g(p)$ :
  - deviation from natural pose
  - joint stiffness
  - power consumption

Force: Newton (N) =  $\text{kg} \cdot \text{m} / \text{s}^2$   
 Work: Joule (J) =  $\text{N} \cdot \text{m} = \text{kg} \cdot \text{m}^2 / \text{s}^2$   
 Power: Watt (W) =  $\text{J} / \text{s} = \text{kg} \cdot \text{m}^2 / \text{s}^3$

## Questions?

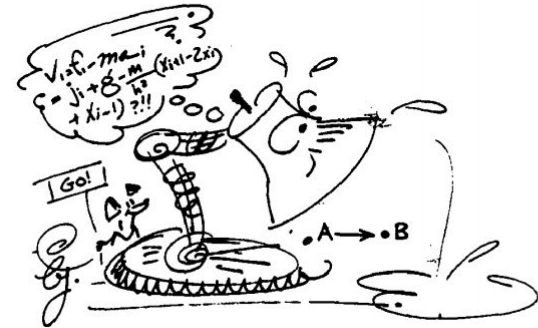
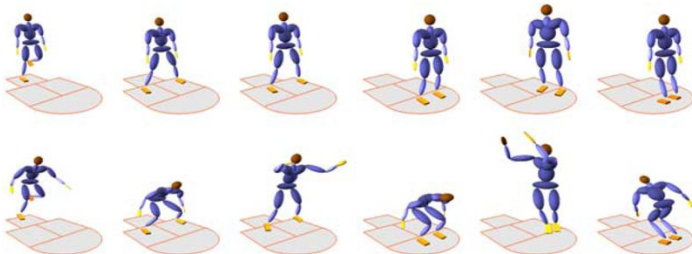


Figure 8: Spacetime constraints: a cartoonist's view. (c) 1988 by Laura Green, used by permission.

“Spacetime Constraints”, Witkin & Kass, SIGGRAPH 1988

## Reading for Today:

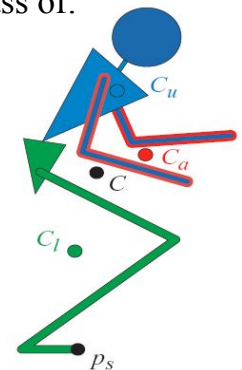
“Synthesis of Complex Dynamic Character Motion from Simple Animation”, Liu & Popović, 2002



- Rapid prototyping of realistic character motion from rough low-quality animations
- Obey the laws of physics & stay within space of naturally-occurring movements

## What's a Natural Pose?

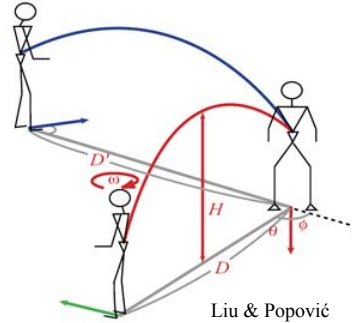
- Training database of ~50 “natural poses”
- For each, compute center of mass of:
  - Upper body
  - Arms
  - Lower body
- The relative COM of each generated pose is matched to most the most similar database example



Liu & Popović

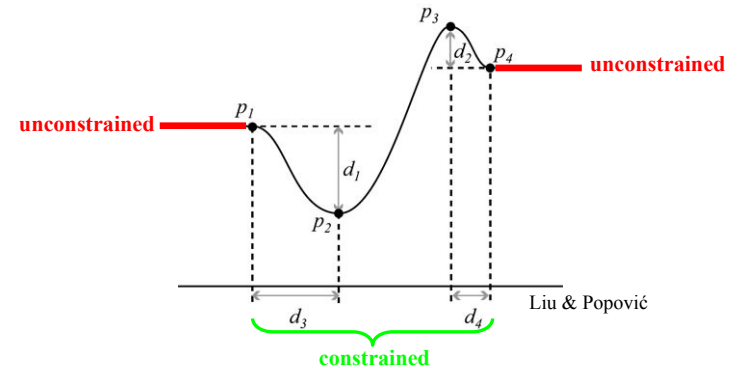
## Linear and Angular Momentum

- In unconstrained animation (no contacts), both linear & angular momentum should be conserved
- The center of mass should follow a parabolic trajectory according to gravity
- The joints should move such that the angular momentum of the whole body remains constant



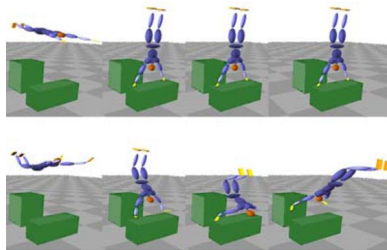
## During Constrained Motion

- During *constrained* motion (when in contact with the ground), the angular momentum follows a spline curve modeled after biomechanics data



## System Features

- Automatically detect point/line/plane constraints
- Divide animation into constrained portions (e.g., feet in contact with ground) and unconstrained portions (e.g., free flight)
- Linear and angular momentum constraints *without having to compute muscle forces*
- Minimize:
  - Mass displacement
  - Velocity of the degrees of freedom (DOF)
  - “Unbalance” (distance the COM is outside of ground constraints)



## Readings for Friday 2/24: (pick one)

“Artist-Directed Dynamics for 2D Animation”,  
Bai, Kaufman, Liu, & Popović, SIGGRAPH 2016

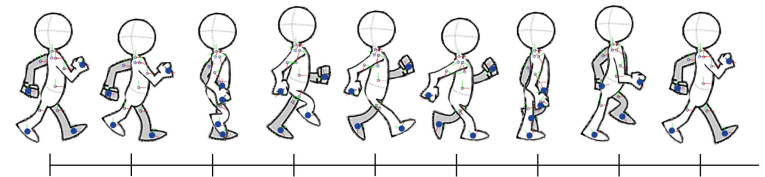
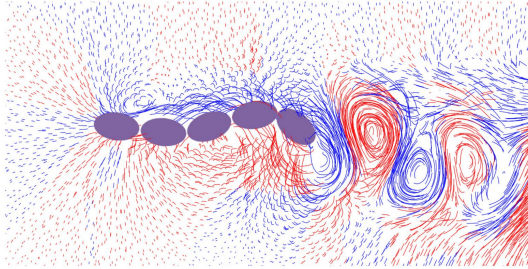


Figure 6: Keyframes used in the articulated character walk example. The artist only specifies keyframes for a subset of handles (handles at hands and feet) which are shown as blue dots. Nine keyframes are used to create a walking cycle. Their timing is visualized by the black lines at the bottom. The artworks are adapted from AngryAnimator.com (<http://www.angryanimator.com/>)

## Readings for Friday 2/24: *(pick one)*

“Articulated Swimming Creatures” Jie Tan, Yuting Gu, Greg Turk, and C. Karen Liu, SIGGRAPH 2011

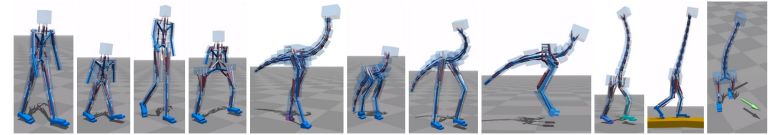


**Figure 8:** A five-link eel swims in a 2D fluid environment. In contrast to the simulation in 3D, an eel swimming in 2D fluid sheds only one single vortex street. Red traces show the counter-clockwise vortices while blue traces show the clockwise vortices.

<http://www.cc.gatech.edu/~jtan34/project/articulatedSwimmingCreatures.html>

## Readings for Friday 2/24: *(pick one)*

“Flexible Muscle-Based Locomotion for Bipedal Creatures”, Geijtenbeek, van de Panne, van der Stappen, SIGGRAPH Asia 2013



**Figure 1:** Physics-based simulation of locomotion for a variety of creatures driven by 3D muscle-based control. The synthesized controllers can locomote in real time at a range of speeds, be steered to a target heading, and can traverse variable terrain.