Local vs. Global Illumination & Radiosity

An early application of radiative heat transfer in stables.

Reading for Today


Other Reading for Today


Last Time?

- Ray Casting & Ray-Object Intersection
- Recursive Ray Tracing
- Distributed Ray Tracing
Shadows

- one shadow ray per intersection per point light source

point light source

no shadow rays

one shadow ray

Soft Shadows

- multiple shadow rays to sample area light source

area light source

penumbra

umbra

penumbra

lots of shadow rays

Antialiasing – Supersampling

- multiple rays per pixel

point light

jaggies

w/ antialiasing

area light
Reflection

- one reflection ray per intersection

Glossy Reflection

- multiple reflection rays

Motion Blur

- Sample objects temporally

Depth of Field

- multiple rays per pixel

$\theta$
Ray Tracing Algorithm Analysis

- Ray casting
- Lots of primitives
- Recursive
- Distributed Ray Tracing Effects
  - Soft shadows
  - Anti-aliasing
  - Glossy reflection
  - Motion blur
  - Depth of field

\[ \text{cost} = \text{height} \times \text{width} \times \text{num primitives} \times \text{intersection cost} \times \text{size of recursive ray tree} \times \text{num shadow rays} \times \text{num supersamples} \times \text{num glossy rays} \times \text{num temporal samples} \times \text{num focal samples} \times \ldots \]

\text{can we reduce this?}

these can serve double duty

HW3: Raytracing & Epsilon

- intersect sphere2 \( t = 0.01 \)
- intersect sphere1 \( t = 14.3 \)
- intersect light \( t = 25.2 \)
- intersect light \( t = 26.9 \)

Solution: advance the ray start position \( \epsilon \) distance along the ray direction OR ignore all intersections \( < \epsilon \) (rather than \( < 0 \))

What’s a good value for \( \epsilon \)? Depends on hardware precision & scene dimensions

Today

- Local Illumination
  - BRDF
  - Ideal Diffuse Reflectance
  - Ideal Specular Reflectance
  - The Phong Model
- Why is Global Illumination Important?
- Radiosity Matrix
- Calculating the Form Factors
- Advanced Radiosity

BRDF

- Ratio of light coming from one direction that gets reflected in another direction
- Bidirectional Reflectance Distribution Function
  - 4D
  - \( R(\theta_i, \varphi_i; \theta_o, \varphi_o) \)
- Note: BRDF for isotropic materials is 3D
Incoming Radiance

- The amount of light received by a surface depends on incoming angle
  - Bigger at normal incidence (Winter/Summer difference)
- By how much?
  - $dB = dA \cos \theta$
  - Same as: $\mathbf{l} \cdot \mathbf{n}$
    (dot product with normal)

Ideal Diffuse Reflectance

- Assume surface reflects equally in all directions (a.k.a. Lambertian)
- An ideal diffuse surface is, at the microscopic level, a very rough surface
- Examples: chalk, clay, some paints

Ideal Specular Reflectance

- Assume surface reflects only in mirror direction
  - View dependent
- Microscopic surface elements are oriented in the same direction as the surface
- Examples: mirrors, highly polished metals

Non-Ideal Reflectors

- Real materials tend to be neither ideal diffuse nor ideal reflective
- Highlight is blurry, looks glossy
Non-Ideal Reflectors

- Most light reflects in the ideal reflected direction
- Microscopic surface variations will reflect light just slightly offset
- How much light is reflected?

![Diagram of light reflection](image1)

The Phong Model

- An empirical (observational) model
- How much light is reflected “specularly”? Depends on the angle $\alpha$, between the ideal reflection direction $r$ and the viewer direction $l$

$$L_o = k_s (\cos \alpha)^q \frac{L_i}{r^2}$$

$q$: specular reflection exponent

![Diagram of Phong model](image2)

Surface variations in Phong specular exponent

The Phong Model

- Sum of three components: diffuse reflection + specular reflection + “ambient”.

![Diagram of light interaction](image3)

Ambient Illumination

- In a typical room, everything receives at least a little bit of light
- Ambient illumination represents the reflection of all indirect illumination

$$L(\omega_r) = k_a$$

- This is a total hack!
Questions?

Today

- Local Illumination
- Why is Global Illumination Important?
  - The Cornell Box
  - Radiosity vs. Ray Tracing
- Radiosity Matrix
- Calculating the Form Factors
- Advanced Radiosity

Why Global Illumination?

- Simulate all light inter-reflections (indirect lighting)
  - in a room, a lot of the light is indirect: it is reflected by walls.
- How have we dealt with this so far?
  - Ambient term to fake some uniform indirect light

Why Radiosity?

- Sculpture by John Ferren
- Diffuse panels

Henrik Wann Jensen
Radiosity vs. Ray Tracing

Original sculpture by John Ferren lit by daylight from behind.

Ray traced image. A standard ray tracer cannot simulate the interreflection of light between diffuse surfaces.

Image rendered with radiosity, note color bleeding effects.

Reading for Tuesday

Goral, Torrance, Greenberg & Battaile
Modeling the Interaction of Light Between Diffuse Surfaces
SIGGRAPH '84

The Cornell Box

- Careful calibration and measurement allows for comparison between physical scene & simulation

Visualizing Inter-reflections...

photograph

simulation

direct illumination (0 bounces)

1 bounce

2 bounces

Note: image brightness not constant between images

images by Micheal Callahan
http://www.cs.utah.edu/~shirley/classes/cs684_98/students/callahan/bounce/

photograph

simulation

Light Measurement Laboratory
Cornell University, Program for Computer Graphics
Radiosity vs. Ray Tracing

- Ray tracing is an *image-space* algorithm
  - If the camera is moved, we have to start over
- Radiosity is computed in *object-space*
  - View-independent
    (just don't move the light)
  - Can pre-compute complex lighting to allow interactive walkthroughs

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Radiosity Overview

- Surfaces are assumed to be perfectly Lambertian (diffuse)
  - reflect incident light in all directions with equal intensity
- The scene is divided into a set of small areas, or patches.
- The radiosity, $B_i$, of patch $i$ is the total rate of energy leaving a surface. The radiosity over a patch is constant.
- Units for radiosity: Watts / steradian * meter$^2$

Discrete Radiosity Equation

Discretize the scene into $n$ patches, over which the radiosity is constant

$$B_i = E_i + \rho_i \sum_{j=1}^{n} F_{ij} B_j$$

The equation is recursive, but it can be solved iteratively.
Radiosity in Matrix Form

\[ B_i = E_i + \rho \sum_{j=1}^{n} F_{ij} B_j \]

*n* simultaneous equations with *n* unknown \( B_i \) values can be written in matrix form:

\[
\begin{bmatrix}
1 - \rho_1 F_{11} & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1n} \\
-\rho_2 F_{21} & 1 - \rho_2 F_{22} & \cdots & \cdots \\
\vdots & \vdots & \ddots & \vdots \\
-\rho_n F_{n1} & \cdots & \cdots & 1 - \rho_n F_{nn}
\end{bmatrix}
\begin{bmatrix}
B_1 \\
B_2 \\
\vdots \\
B_n
\end{bmatrix}
= \begin{bmatrix}
E_1 \\
E_2 \\
\vdots \\
E_n
\end{bmatrix}
\]

A solution yields a single radiosity value \( B_i \) for each patch in the environment, a view-independent solution.

Solving the Radiosity Matrix

- Initialize all radiosity values to 0
- Each iteration, update the radiosity of each patch by *gathering* the contribution of radiosities from all other patches:

\[
\begin{bmatrix}
B_1 \\
B_2 \\
\vdots \\
B_n
\end{bmatrix}
= \begin{bmatrix}
E_1 \\
E_2 \\
\vdots \\
E_n
\end{bmatrix}
+ \begin{bmatrix}
\rho_1 F_{11} & \rho_1 F_{12} & \cdots & \rho_1 F_{1n} \\
\rho_2 F_{21} & \rho_2 F_{22} & \cdots & \cdots \\
\vdots & \vdots & \ddots & \vdots \\
\rho_n F_{n1} & \cdots & \cdots & \rho_n F_{nn}
\end{bmatrix}
\begin{bmatrix}
B_1 \\
B_2 \\
\vdots \\
B_n
\end{bmatrix}
\]

A solution yields a single radiosity value \( B_i \) for each patch in the environment, a view-independent solution.

Interpolating Vertex Radiosities

- \( B_i \) radiosity values are constant over the extent of a patch.
- How are they mapped to the vertex radiosities (intensities) needed by the renderer?
  - Average the radiosities of patches that contribute to the vertex
  - Vertices on the edge of a surface are assigned values extrapolation

Questions?

30,000 patches.
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Calculating the Form Factor $F_{ij}$

- $F_{ij} = \text{fraction of light energy leaving patch } j \text{ that arrives at patch } i$
- Takes account of both:
  - geometry (size, orientation & position)
  - visibility (are there any occluders?)

Calculating the Form Factor $F_{ij}$

- $F_{ij} = \text{fraction of light energy leaving patch } j \text{ that arrives at patch } i$

\[
F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos \theta_i \cos \theta_j}{\pi r^2} V_{ij} \, dA_j \, dA_i
\]

Form Factor Determination

The Nusselt analog: the form factor of a patch is equivalent to the fraction of the unit circle that is formed by taking the projection of the patch onto the hemisphere surface and projecting it down onto the circle.
### Hemicube Algorithm

- A hemicube is constructed around the center of each patch
- Faces of the hemicube are divided into "pixels"
- Each patch is projected (rasterized) onto the faces of the hemicube
- Each pixel stores its pre-computed form factor
  - The form factor for a particular patch is just the sum of the pixels it overlaps
- Patch occlusions are handled similar to z-buffer rasterization

### Form Factor from Ray Casting

- Cast $n$ rays between the two patches
  - Compute visibility (what fraction of rays do not hit an occluder)
  - Integrate the point-to-point form factor
- Permits the computation of the patch-to-patch form factor, as opposed to point-to-patch

### Questions?

- [Lightscape](http://www.lightscape.com)

### Today

- Local Illumination
- Why is Global Illumination Important?
- Radiosity Equation/Matrix
- Calculating the Form Factors
- Advanced Radiosity
  - Progressive Radiosity
  - Adaptive Subdivision
  - Discontinuity Meshing
  - Hierarchical Radiosity
Stages in a Radiosity Solution

- **Input Geometry**
  - Form Factor Calculation
  - Why so costly? > 90%
  - Calculation & storage of \( n^2 \) form factors

- **Emittance & Reflectance Properties**
  - Solve the Radiosity Matrix
  - \(< 10\%\)
  - \((n^3 \text{ for naive visibility calculation})\)

- **Camera Position & Orientation**
  - Radiosity Solution
  - Visualization (Rendering)
  - ~ 0%

- **Radiosity Image**

Progressive Refinement

- **Goal**: Provide frequent and timely updates to the user during computation
- **Key Idea**: Update the entire image at every iteration, rather than a single patch
- **How?** Instead of summing the light received by one patch, distribute the radiance of the patch with the most undistributed radiance.

Reordering the Solution for PR

*Shooting*: the radiosity of all patches is updated for each iteration:

\[
\begin{bmatrix}
B_1 \\
B_2 \\
\vdots \\
B_n
\end{bmatrix} = 
\begin{bmatrix}
B_1 \\
B_2 \\
\vdots \\
B_n
\end{bmatrix} + 
\begin{bmatrix}
\cdots & \rho_1 F_1 & \cdots \\
\cdots & \rho_2 F_2 & \cdots \\
\cdots & \rho_n F_n & \cdots
\end{bmatrix}
\]

This method is fundamentally a Southwell relaxation

Progressive Refinement w/out Ambient Term
Progressive Refinement with Ambient Term

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Goral, Torrance, Greenberg & Battaile
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SIGGRAPH '84

photograph

simulation

Lightscape  http://www.lightscape.com
Increasing the Accuracy of the Solution

What’s wrong with this picture?

- Image quality is a function of patch size
- Compute a solution on a uniform initial mesh, then refine the mesh in areas that exceed some error tolerance:
  - shadow boundaries
  - other areas with a high radiosity gradient

Adaptive Subdivision of Patches

Coarse patch solution (145 patches)
Improved solution (1021 subpatches)
Adaptive subdivision (1306 subpatches)

Discontinuity Meshing

- Limits of umbra and penumbra
  - Captures nice shadow boundaries
  - Complex geometric computation to construct mesh

Optional Reading for Friday:

“Fast and Accurate Hierarchical Radiosity Using Global Visibility”
Durand, Drettakis, & Puech 1999
Hierarchical Radiosity

- Group elements when the light exchange is not important
  - Breaks the quadratic complexity
  - Control non trivial, memory cost

Practical Problems with Radiosity

- Meshing
  - memory
  - robustness

- Form factors
  - computation

- Diffuse limitation
  - extension to specular takes too much memory

Questions?

Lightscape  http://www.lightscape.com