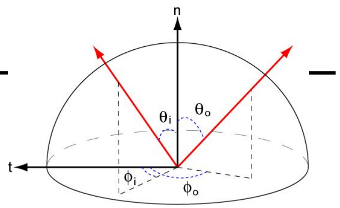
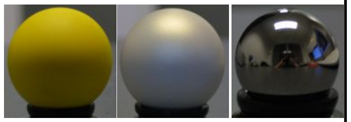
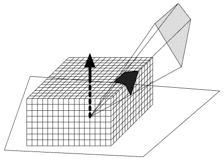
The Rendering Equation
& Monte Carlo Ray Tracing

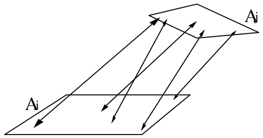
Last Time?

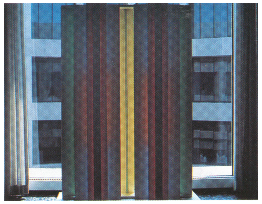


- Local Illumination
  - BRDF
  - Ideal Diffuse Reflectance
  - Ideal Specular Reflectance
  - The Phong Model
- Radiosity Equation/Matrix
- Calculating the Form Factors

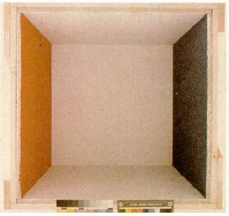




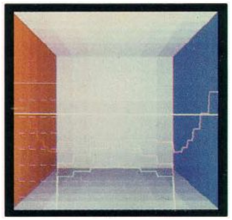




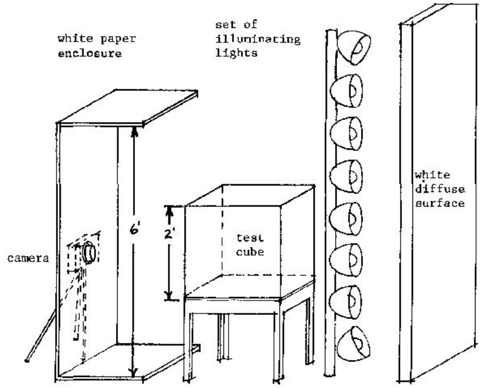
Reading for Friday:



photograph



simulation



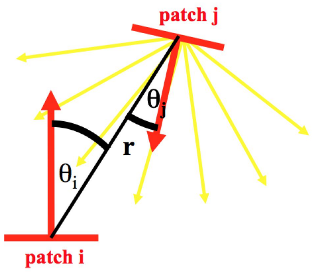
Goral, Torrance, Greenberg & Battaile
Modeling the Interaction of Light Between Diffuse Surfaces
SIGGRAPH '84

Leftover From Last Time…

- Calculating the Form Factors
- Advanced Radiosity
  - Progressive Radiosity
  - Adaptive Subdivision
  - Discontinuity Meshing
  - Hierarchical Radiosity

Calculating the Form Factor F_{ij}

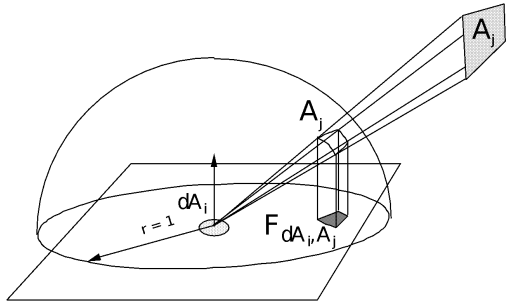
- F_{ij} = fraction of light energy leaving patch j that arrives at patch i



$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi r^2} V_{ij}\, dA_j\, dA_i$$

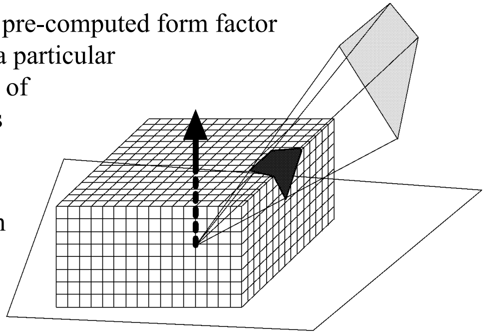
Form Factor Determination

The Nusselt analog: the form factor of a patch is equivalent to the fraction of the unit circle that is formed by taking the projection of the patch onto the hemisphere surface and projecting it down onto the circle.



Hemicube Algorithm

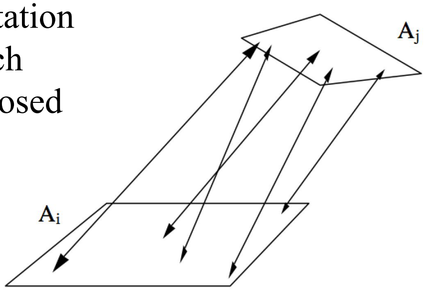
- A hemicube is constructed around the center of each patch
- Faces of the hemicube are divided into "pixels"
- Each patch is projected (rasterized) onto the faces of the hemicube
- Each pixel stores its pre-computed form factor The form factor for a particular patch is just the sum of the pixels it overlaps
- Patch occlusions are handled similar to z-buffer rasterization



Form Factor from Ray Casting

- Cast *n* rays between the two patches
  - Compute visibility (what fraction of rays do not hit an occluder)
  - Integrate the point-to-point form factor
- Permits the computation of the patch-to-patch form factor, as opposed to point-to-patch

Use this for HW3!



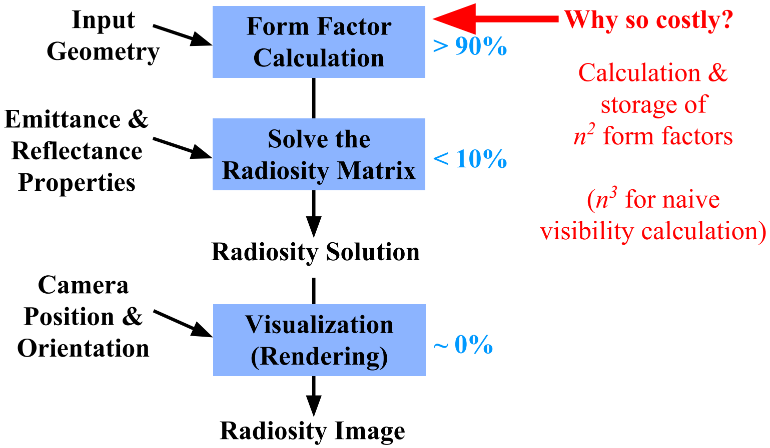
Questions?

Lightscape `http://www.lightscape.com`

Leftover From Last Time…

- Calculating the Form Factors
- Advanced Radiosity
  - Progressive Radiosity
  - Adaptive Subdivision
  - Discontinuity Meshing
  - Hierarchical Radiosity

Stages in a Radiosity Solution



Calculation & storage of n^2 form factors

(n^3 for naive visibility calculation)

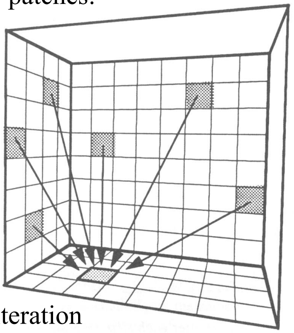
Solving the Radiosity Matrix

- Initialize all radiosity values to 0
- Each iteration, update the radiosity of each patch by *gathering* the contribution of radiosities from all other patches:

$$\begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_i \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_i \\ \vdots \\ E_n \end{bmatrix} + \begin{bmatrix} & & & \\ & & & \\ & & & \\ \rho_i F_{i1} & \rho_i F_{i2} & \cdots & \rho_i F_{in} \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_i \\ \vdots \\ B_n \end{bmatrix}$$

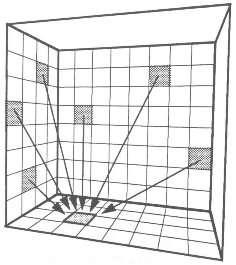
Radiosity values on iteration ***t+1***

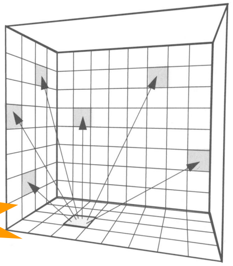
Radiosity values on iteration ***t***



- Radiosity values only increase on each iteration
- This method is fundamentally a Gauss-Seidel relaxation

Progressive Refinement





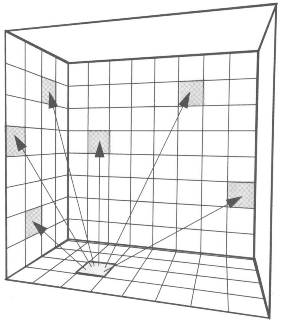
- Goal: Provide frequent and timely updates to the user during computation
- Key Idea: Update the entire image at every iteration, rather than a single patch
- How? Instead of summing the light received by one patch, distribute the radiance of the patch with the most *undistributed radiance*.

Use this for HW3!

Reordering the Solution for PR

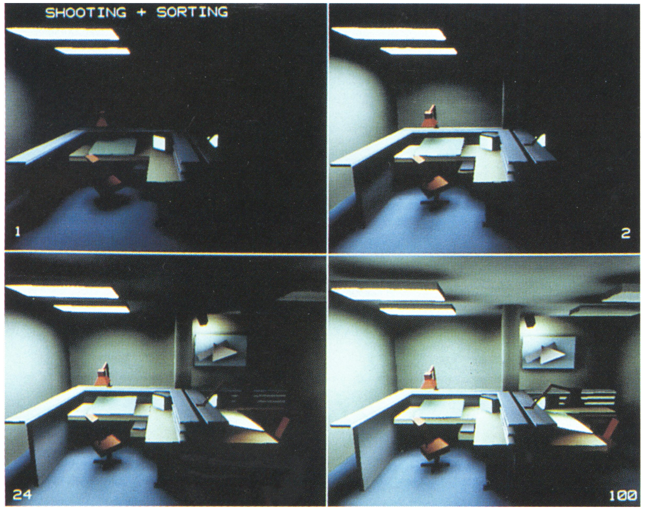
Shooting: the radiosity of all patches is updated for each iteration:

$$\begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ \vdots \\ B_n \end{bmatrix} + \begin{bmatrix} \cdots & \rho_1 F_{1i} & \cdots \\ \cdots & \rho_2 F_{2i} & \\ & & \\ & & \\ \cdots & \rho_n F_{ni} & \cdots \end{bmatrix} \begin{bmatrix} \vdots \\ B_i \\ \vdots \end{bmatrix}$$

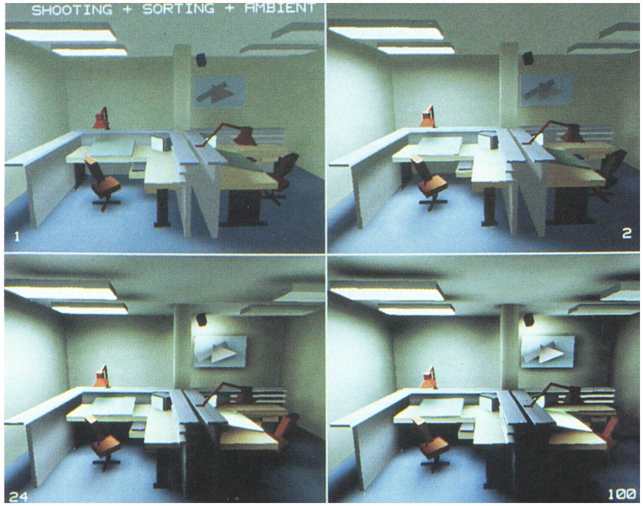


This method is fundamentally a Southwell relaxation

Progressive Refinement w/out Ambient Term



Progressive Refinement with Ambient Term



Questions?

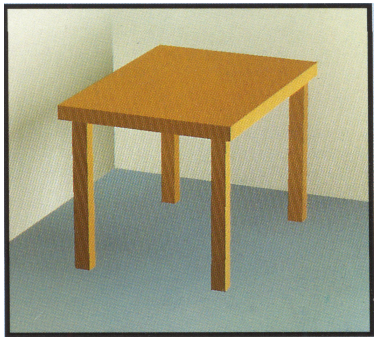
Lightscape `http://www.lightscape.com`

Leftover From Last Time…

- Calculating the Form Factors
- Advanced Radiosity
  - – Progressive Radiosity
  - – Adaptive Subdivision
  - – Discontinuity Meshing
  - – Hierarchical Radiosity

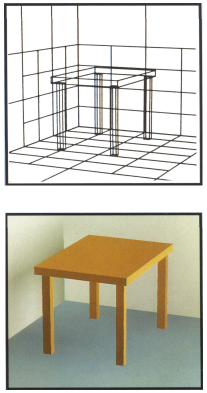
Increasing the Accuracy of the Solution

What's wrong with this picture?

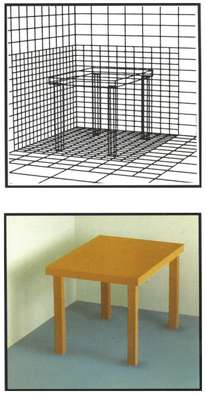


- Image quality is a function of patch size
- Compute a solution on a uniform initial mesh, then refine the mesh in areas that exceed some error tolerance:
  - – shadow boundaries
  - – other areas with a high radiosity gradient

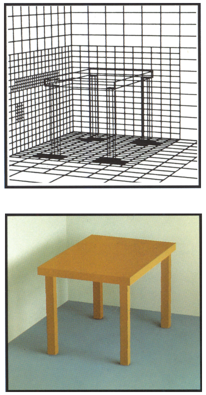
Adaptive Subdivision of Patches



Coarse patch solution
(145 patches)



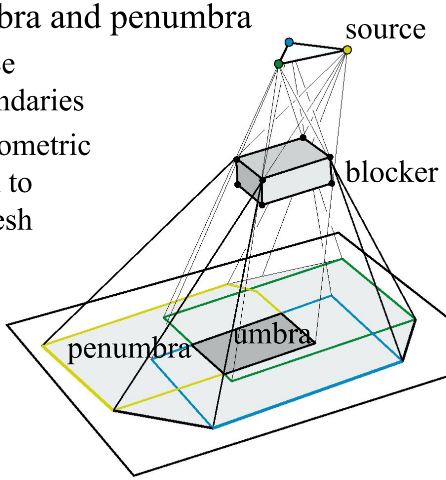
Improved solution
(1021 subpatches)



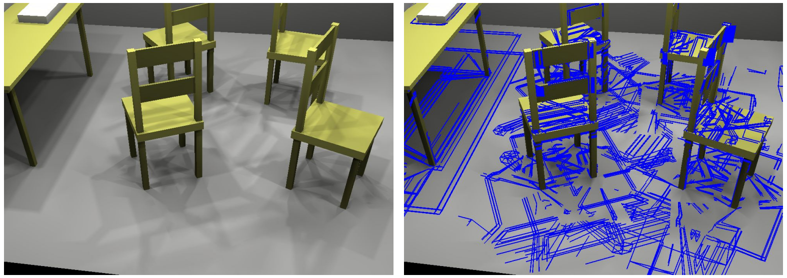
Adaptive subdivision
(1306 subpatches)

Discontinuity Meshing

- Limits of umbra and penumbra
  - Captures nice shadow boundaries
  - Complex geometric computation to construct mesh



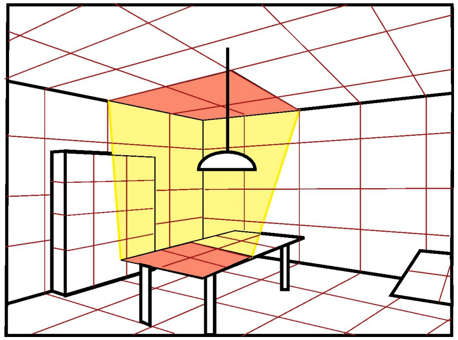
Optional Reading for Today:



"Fast and Accurate Hierarchical Radiosity Using Global Visibility"
Durand, Drettakis, & Puech 1999

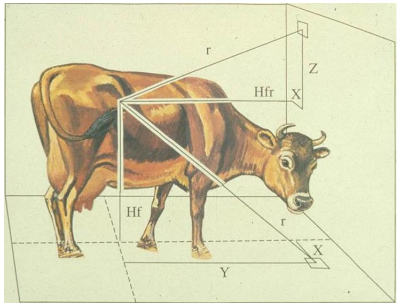
Hierarchical Radiosity

- Group elements when the light exchange is not important
  - Breaks the quadratic complexity
  - Control non trivial, memory cost



Practical Problems with Radiosity

- Meshing
  - memory
  - robustness
- Form factors
  - computation
- Diffuse limitation
  - extension to specular takes too much memory



Cow-cow form factor?

Questions?

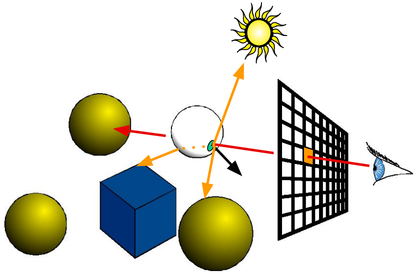
Lightscape `http://www.lightscape.com`

Today

- Does Ray Tracing Simulate Physics?
- The Rendering Equation
- Monte-Carlo Integration
- Sampling
- Monte-Carlo Ray Tracing vs. Path Tracing

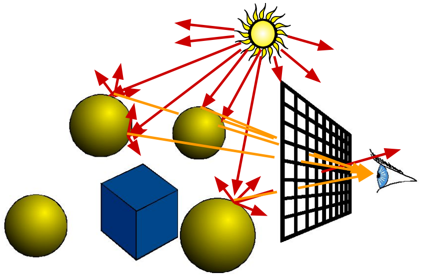
Does Ray Tracing Simulate Physics?

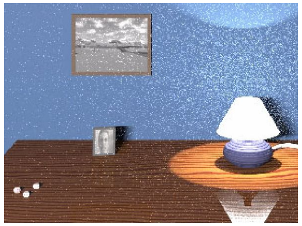
- No…. traditional ray tracing is also called "*backward" ray tracing*
- In reality, photons actually travel from the light to the eye



Forward Ray Tracing

- Start from the light source
  - But very, very low probability to reach the eye
- What can we do about it?
  - Always send a ray to the eye…. still not efficient

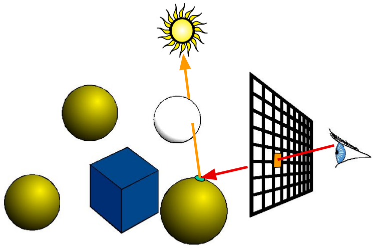




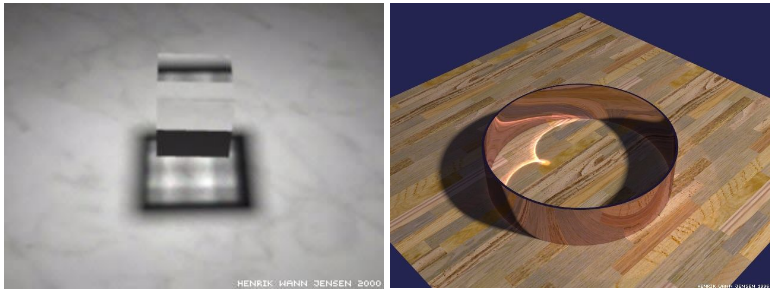
Henrik Wann Jensen

Transparent Shadows?

- What to do if the shadow ray sent to the light source intersects a transparent object?
  - Pretend it's opaque?
  - Multiply by transparency color? (ignores refraction & does not produce caustics)
- Unfortunately, ray tracing is full of dirty tricks



Is this Traditional Ray Tracing?

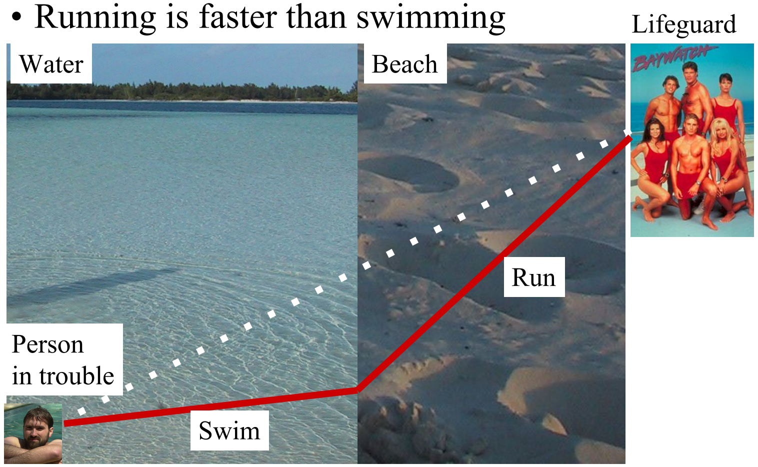


Images by Henrik Wann Jensen

No, Refraction and complex reflection for illumination are not handled properly in traditional (backward) ray tracing

Refraction and the Lifeguard Problem

- Running is faster than swimming

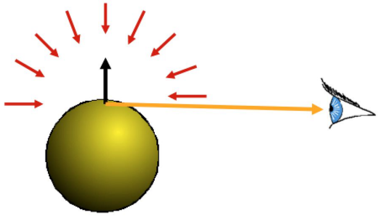


Today

- Does Ray Tracing Simulate Physics?
- The Rendering Equation
- Monte-Carlo Integration
- Sampling
- Monte-Carlo Ray Tracing vs. Path Tracing

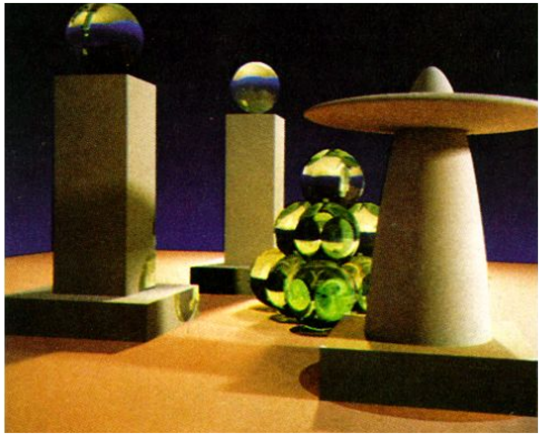
The Rendering Equation

- Clean mathematical framework for light-transport simulation
- At each point, outgoing light in one direction is the integral of incoming light in all directions multiplied by reflectance property

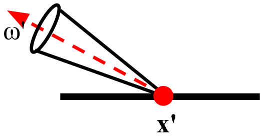


Reading for Tuesday:

- “The Rendering Equation”, Kajiya, SIGGRAPH 1986



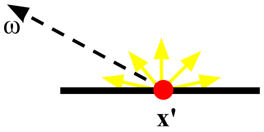
The Rendering Equation



$$L(x',\omega') = E(x',\omega') + \int \rho_{x'}(\omega,\omega')L(x,\omega)G(x,x')V(x,x')\, dA$$

L (x',ω') is the radiance from a point on a surface in a given direction ω'

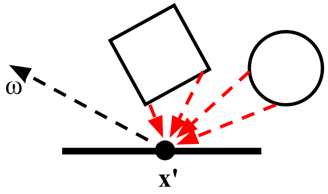
The Rendering Equation



$$L(x',\omega') = E(x',\omega') + \int \rho_{x'}(\omega,\omega')L(x,\omega)G(x,x')V(x,x')\, dA$$

E(x',ω') is the emitted radiance from a point: E is non-zero only if x' is emissive (a light *source*)

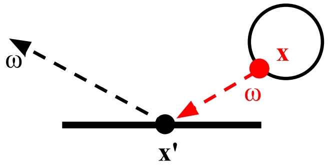
The Rendering Equation



$$L(x',\omega') = E(x',\omega') + \underbrace{\int \rho_{x'}(\omega,\omega')L(x,\omega)G(x,x')V(x,x')\ dA}$$

Sum the contribution from all of
the other surfaces in the scene

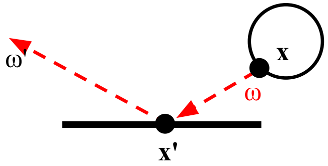
The Rendering Equation



$$L(x',\omega') = E(x',\omega') + \int \rho_{x'}(\omega,\omega')L(x,\omega)G(x,x')V(x,x')\ dA$$

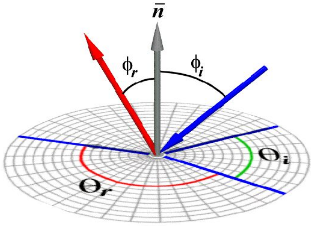
For each x, compute L(x, ω), the radiance
at point x in the direction ω (from x to x')

The Rendering Equation

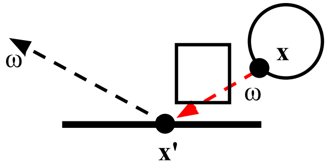


$$L(x',\omega') = E(x',\omega') + \int \rho_{x'}(\omega,\omega')L(x,\omega)G(x,x')V(x,x')\ dA$$

scale the contribution by
$\rho_{x'}(\omega,\omega')$, the reflectivity
(BRDF) of the surface at x'



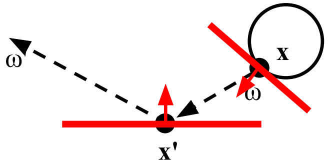
The Rendering Equation



$$L(x',\omega') = E(x',\omega') + \int \rho_{x'}(\omega,\omega')L(x,\omega)G(x,x')V(x,x')\ dA$$

For each x, compute V(x,x'),
the visibility between x and x':
1 when the surfaces are unobstructed
along the direction ω, 0 otherwise

The Rendering Equation

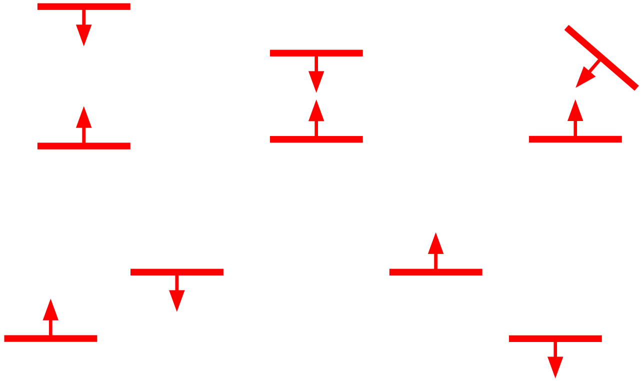


$$L(x',\omega') = E(x',\omega') + \int \rho_{x'}(\omega,\omega')L(x,\omega)G(x,x')V(x,x')\,dA$$

For each x, compute G(x, x'), which describes the on the geometric relationship between the two surfaces at x and x’

Intuition about G(x,x')?

- Which arrangement of two surfaces will yield the greatest transfer of light energy? Why?



Rendering Equation → Radiosity

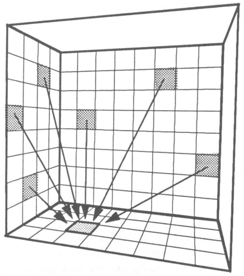
$$L(x',\omega') = E(x',\omega') + \int \rho_{x'}(\omega,\omega')L(x,\omega)G(x,x')V(x,x')\,dA$$

Radiosity assumption:
perfectly diffuse surfaces (not directional)

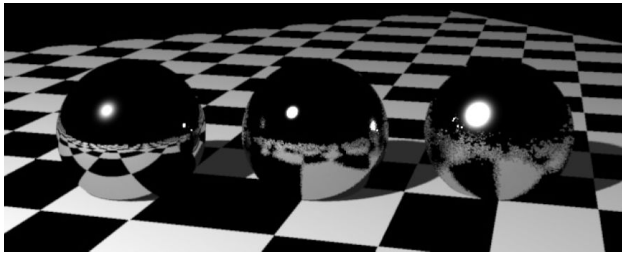
$$B_{x'} = E_{x'} + \rho_{x'} \int B_x \, G(x,x')V(x,x')$$

discretize

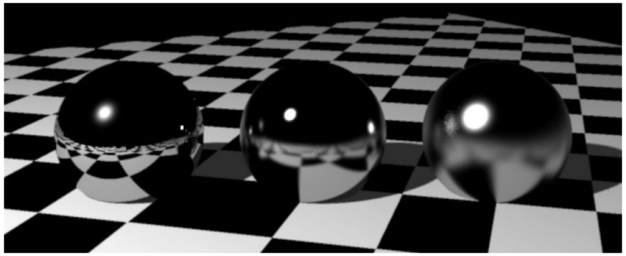
$$B_i = E_i + \rho_i \sum_{j=1}^{n} F_{ij} B_j$$



Questions?



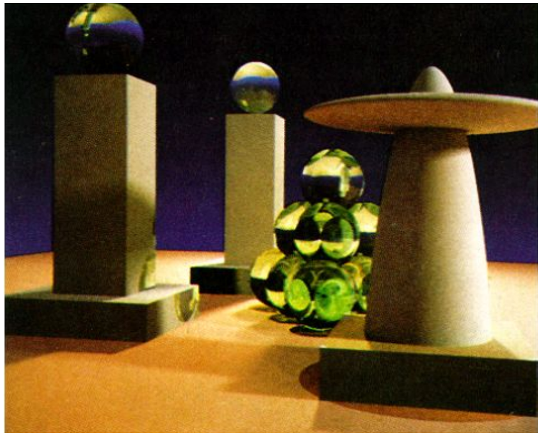
1 glossy sample per pixel



256 glossy samples per pixel

Reading for Tuesday: *(pick one)*

- "The Rendering Equation", Kajiya, SIGGRAPH 1986



Reading for Tuesday: *(pick one)*

"Implicit Visibility and Antiradiance for Interactive Global Illumination"
Dachsbacher,
Stamminger,
Drettakis, and
Durand
Siggraph 2007