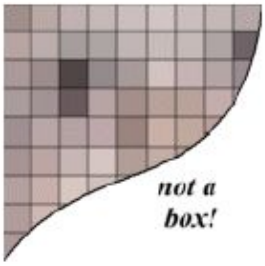
Subsurface Scattering & Complex Material Properties

Leftover from Last Time...

- What is a Pixel?
- Examples of Aliasing
- Sampling & Reconstruction
- Filters in Computer Graphics
- Anti-Aliasing for Texture Maps

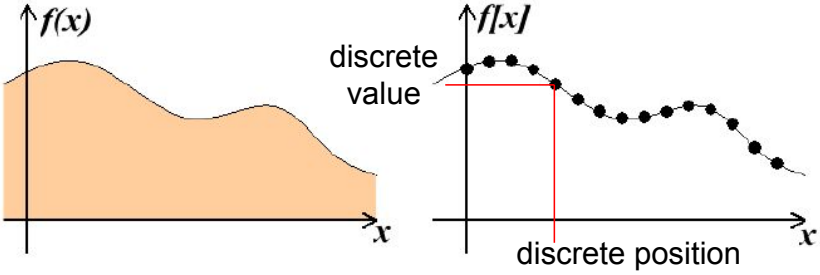
What is a Pixel?

- A pixel is not:
  - a box
  - a disk
  - a teeny tiny little light
- A pixel "looks different" on different display devices
- A pixel is a sample
  - it has no dimension
  - it occupies no area
  - it cannot be seen
  - it has a coordinate
  - it has a value



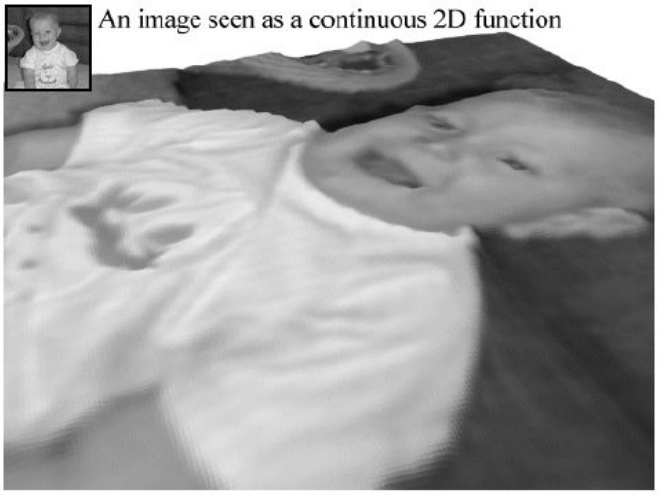
More on Samples

- Most things in the real world are *continuous*, yet everything in a computer is *discrete*
- The process of mapping a continuous function to a discrete one is called *sampling*
- The process of mapping a continuous variable to a discrete one is called *quantization*
- To represent or render an image using a computer, we must both sample and quantize



An Image is a 2D Function

- An *ideal image* is a continuous function I(x,y) of intensities.
- It can be plotted as a height field.
- In general an image cannot be represented as a continuous, analytic function.
- Instead we represent images as tabulated functions.
- How do we fill this table?



Sampling Grid

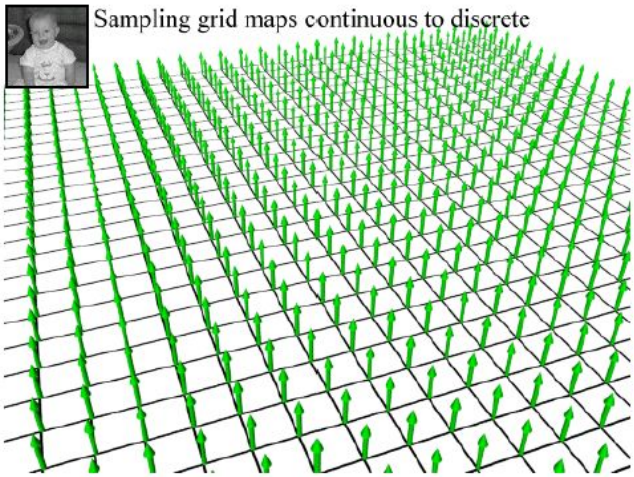
- We can generate the table values by multiplying the continuous image function by a sampling grid of Kronecker delta functions.

The definiton of the 2-D Kronecker delta is:

$$\delta(x,y) = \begin{cases} 1, & (x,y) = (0,0) \\ 0, & \text{otherwise} \end{cases}$$

And a 2-D sampling grid:

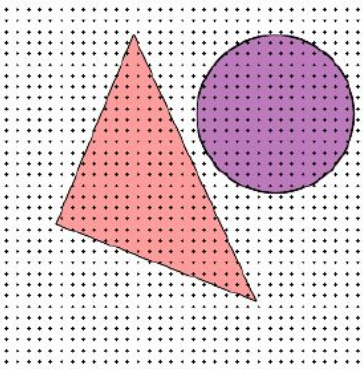
$$\sum_{j=0}^{h-1} \sum_{i=0}^{w-1} \delta(u-i, v-j)$$

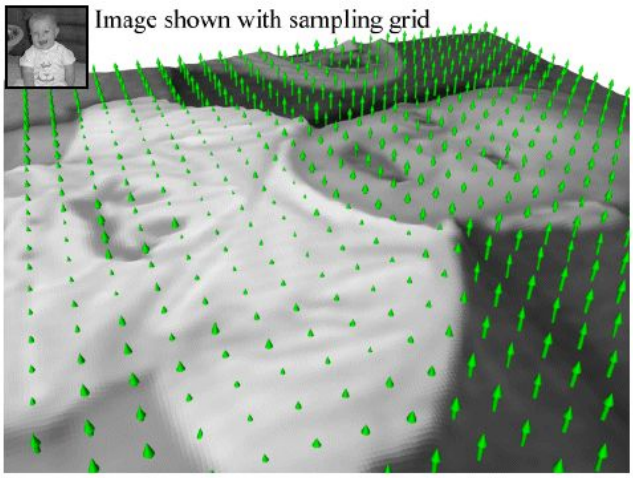


Sampling an Image

- The result is a set of point samples, or pixels.

The same analysis can be applied to geometric objects:



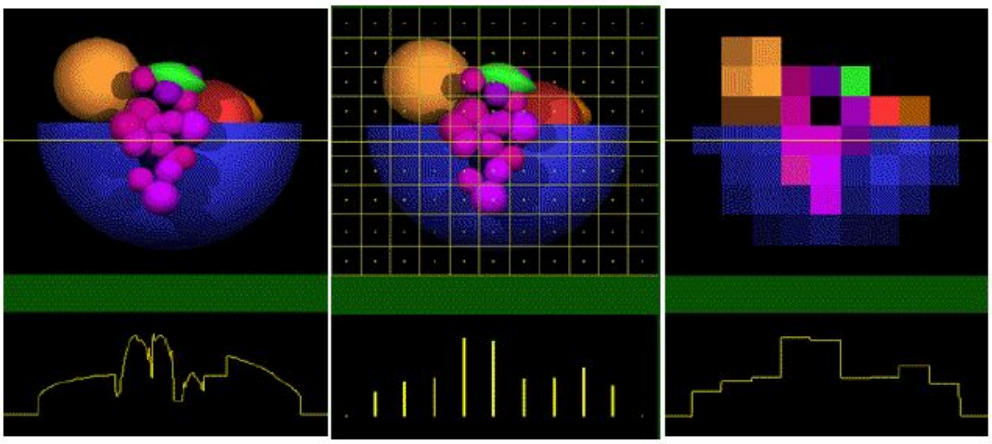


Leftover from Last Time...

- What is a Pixel?
- Examples of Aliasing
- Sampling & Reconstruction
- Filters in Computer Graphics
- Anti-Aliasing for Texture Maps

Examples of Aliasing

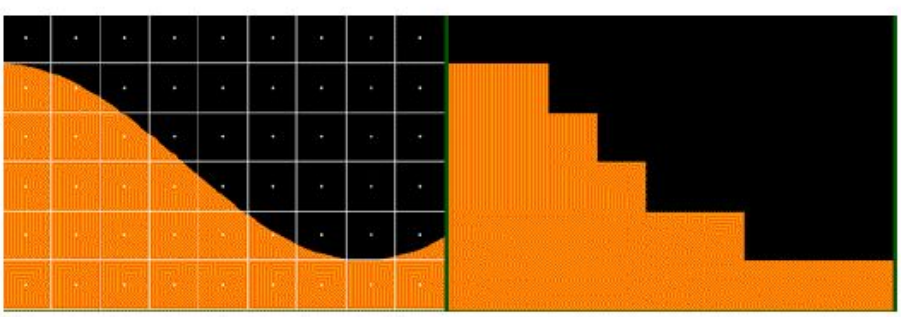
- Aliasing occurs because of *sampling* and *reconstruction*



Original Image Samples Reconstruction

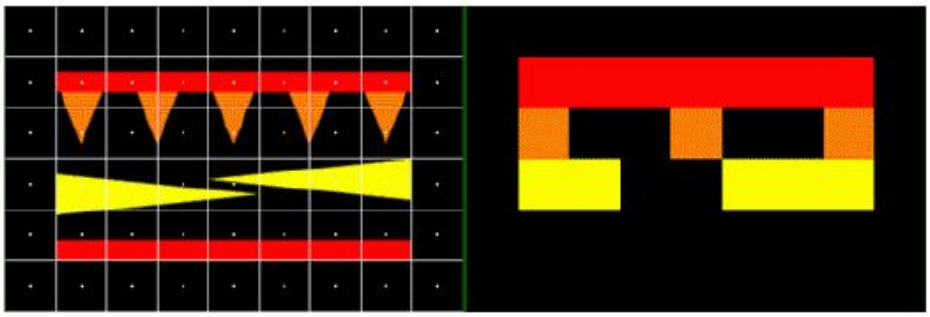
Examples of Aliasing

Jagged boundaries



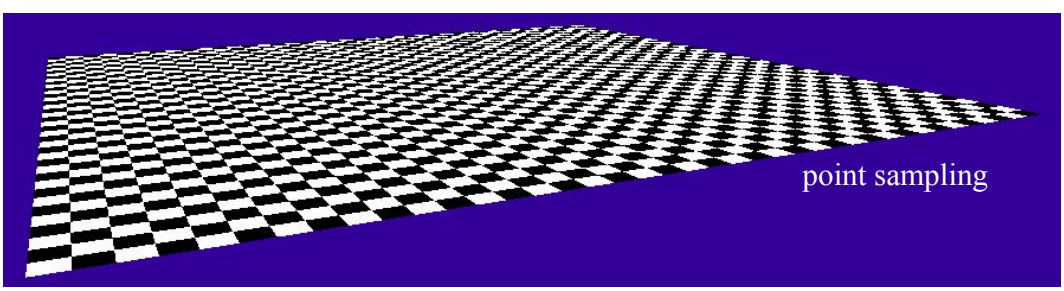
Examples of Aliasing

Improperly rendered detail



Examples of Aliasing

Texture Errors

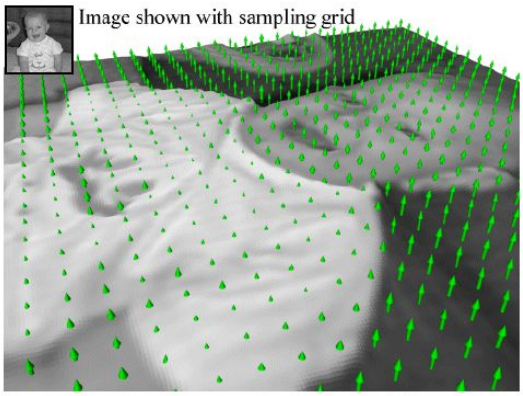


Leftover from Last Time...

- What is a Pixel?
- Examples of Aliasing
- Sampling & Reconstruction
- Filters in Computer Graphics
- Anti-Aliasing for Texture Maps

Sampling Density

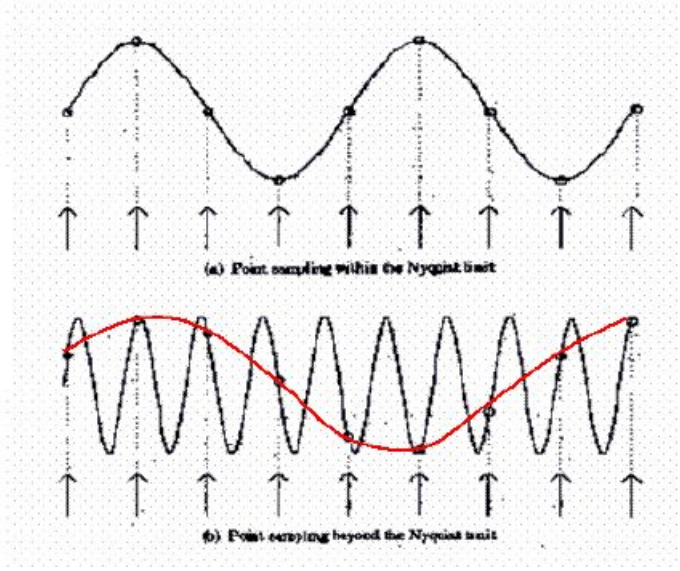
- How densely must we sample an image in order to capture its essence?
- If we under-sample the signal, we won't be able to accurately reconstruct it...



Sampling Density

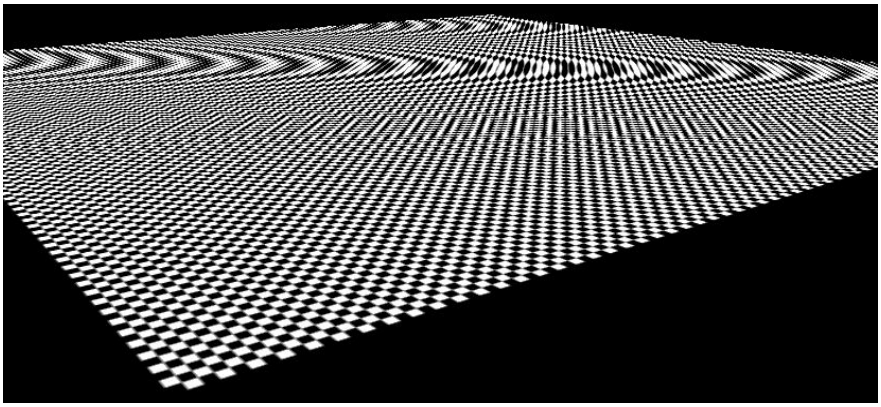
- If we insufficiently sample the signal, it may be mistaken for something simpler during reconstruction (that's aliasing!)

Image from Robert L. Cook, "Stochastic Sampling and Distributed Ray Tracing", An Introduction to Ray Tracing, Andrew Glassner, ed., Academic Press Limited, 1989.



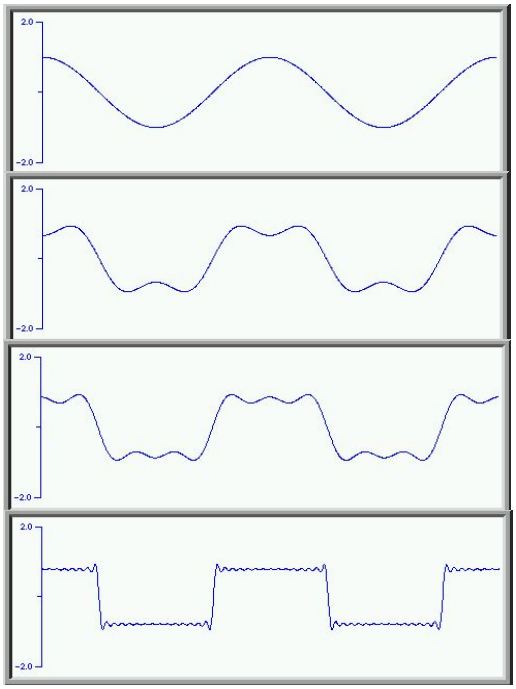
Sampling Density

- Aliasing in 2D because of insufficient sampling density



Remember Fourier Analysis?

- All periodic signals can be represented as a summation of sinusoidal waves.

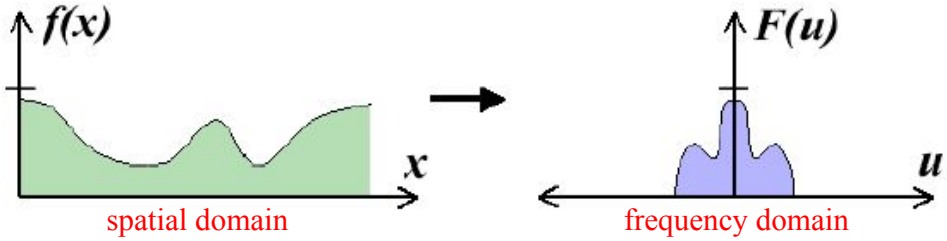


It's a shame that Signals & Systems is not required for CSCI majors...

Images from
http://axion.physics.ubc.ca/341-02/fourier/fourier.html

Remember Fourier Analysis?

- Every periodic signal in the *spatial domain* has a dual in the *frequency domain*.



- This particular signal is *band-limited*, meaning it has no frequencies above some threshold

Remember Fourier Analysis?

- We can transform from one domain to the other using the Fourier Transform.

frequency domain spatial domain

Fourier Transform

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, e^{-i2\pi(ux+vy)}\, dxdy$$

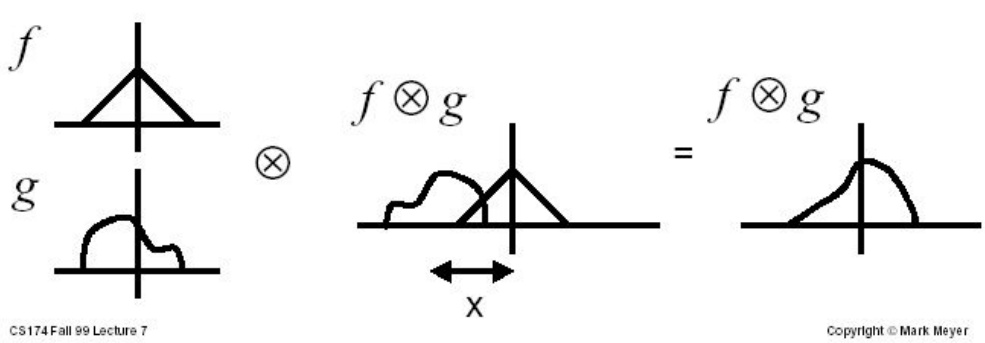
Inverse Fourier Transform

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)\, e^{\,i2\pi(ux+vy)}\, dudv$$

Remember Convolution?

Convolution describes how a system with impulse response, *h(x)*, reacts to a signal, *f(x)*.

$$f(x) * h(x) = \int_{-\infty}^{\infty} f(\lambda) h(x-\lambda)\, d\lambda$$



Images from Mark Meyer
http://www.gg.caltech.edu/~cs174ta/

Remember Convolution?

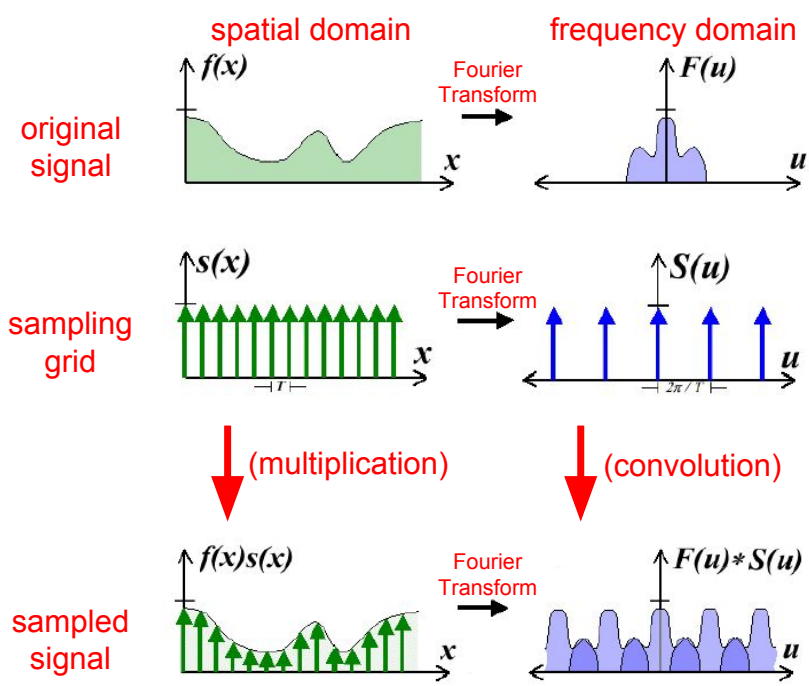
- Some operations that are difficult to compute in the spatial domain can be simplified by transforming to its dual representation in the frequency domain.
- For example, convolution in the spatial domain is the same as multiplication in the frequency domain.

$$f(x) * h(x) \rightarrow F(u)H(u)$$

- And, convolution in the frequency domain is the same as multiplication in the spatial domain

$$F(u) * H(u) \rightarrow f(x)h(x)$$

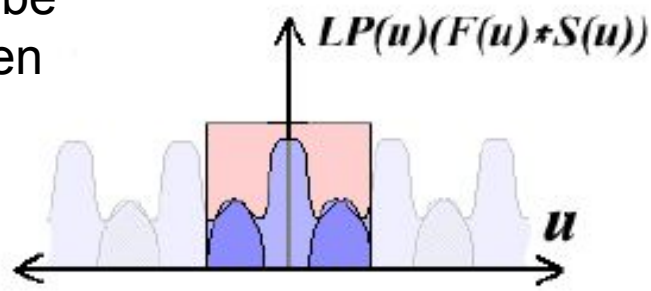
Sampling in the Frequency Domain



Reconstruction

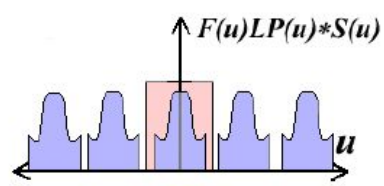
- If we can extract a copy of the original signal from the frequency domain of the sampled signal, we can reconstruct the original signal!

- But there may be overlap between the copies.

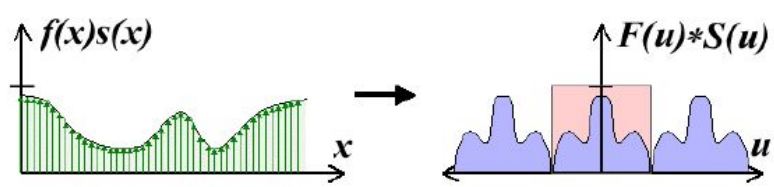


Guaranteeing Proper Reconstruction

- Separate by removing high frequencies from the original signal (low pass pre-filtering)



- Separate by increasing the sampling density



- If we can't separate the copies, we will have overlapping frequency spectrum during reconstruction → *aliasing*.

Sampling Theorem

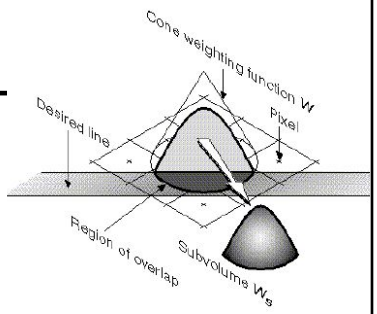
- When sampling a signal at discrete intervals, the sampling frequency must be *greater than twice* the highest frequency of the input signal in order to be able to reconstruct the original perfectly from the sampled version (Shannon, Nyquist)

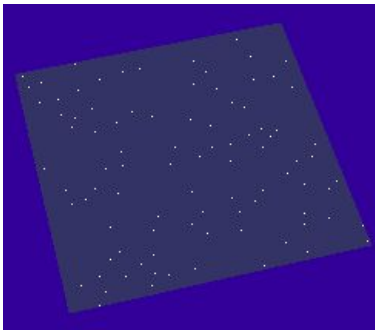
Leftover from Last Time...

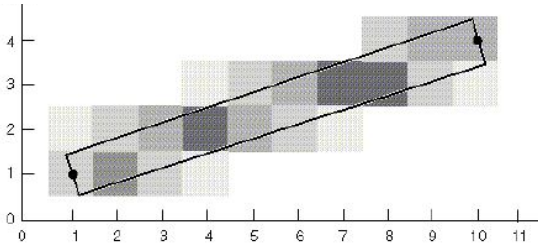
- What is a Pixel?
- Examples of Aliasing
- Sampling & Reconstruction
- Filters in Computer Graphics
  - Ideal, Gaussian, Box, Bilinear, Bicubic
- Anti-Aliasing for Texture Maps

Filters

- Weighting function (convolution kernel)
- Area of influence often bigger than "pixel"
- Sum of weights = 1
  - Each sample contributes the same total to image
  - Constant brightness as object moves across the screen.
- No negative weights/colors (optional)







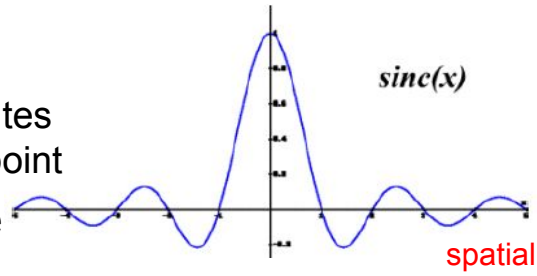
Source: Foley, VanDam, Feiner, Hughes - Computer Graphics, Second Edition,

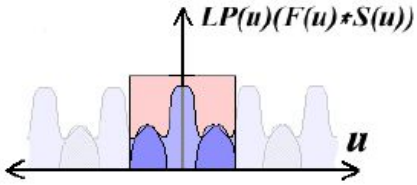
Filters

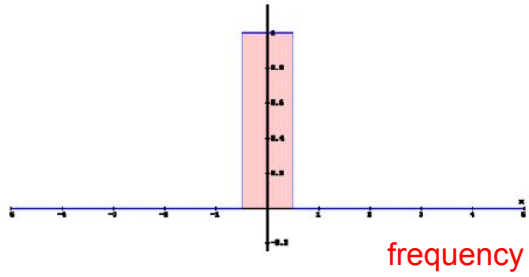
- Filters are used to
  - reconstruct a continuous signal from a sampled signal (reconstruction filters)
  - band-limit continuous signals to avoid aliasing during sampling (low-pass filters)
- Desired frequency domain properties are the same for both types of filters
- Often, the same filters are used as reconstruction and low-pass filters

The Ideal Filter

- Unfortunately it has *infinite* spatial extent
  - Every sample contributes to every interpolated point
- Expensive/impossible to compute

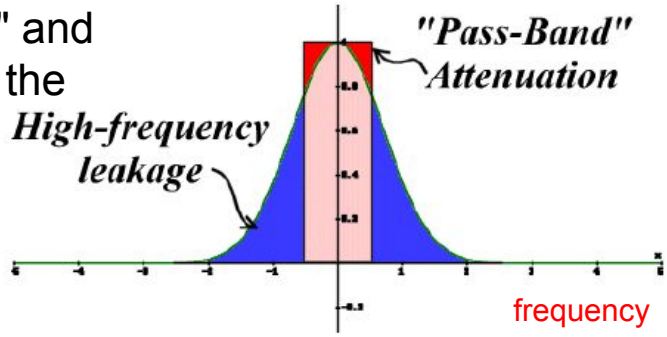






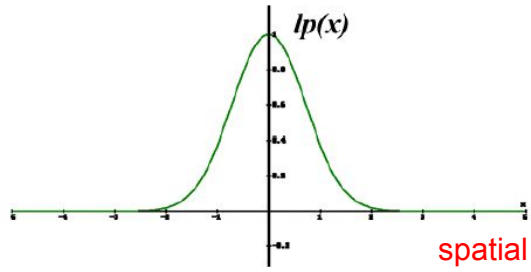
Problems with Practical Filters

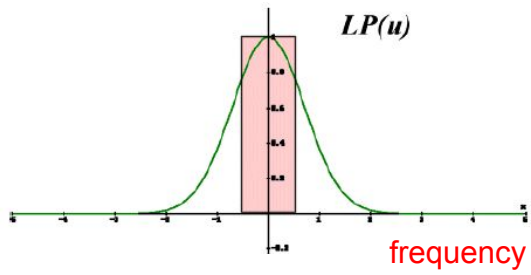
- Many visible artifacts in re-sampled images are caused by poor reconstruction filters
- Excessive pass-band attenuation results in blurry images
- Excessive high-frequency leakage causes "ringing" and can accentuate the sampling grid (anisotropy)



Gaussian Filter

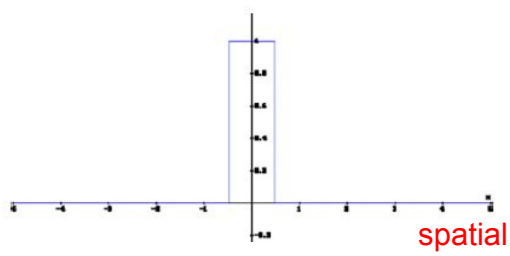
- This is what a CRT does for free!

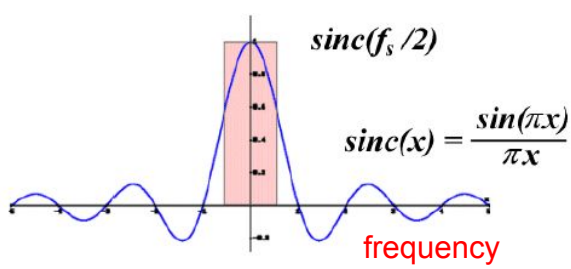




Box Filter / Nearest Neighbor

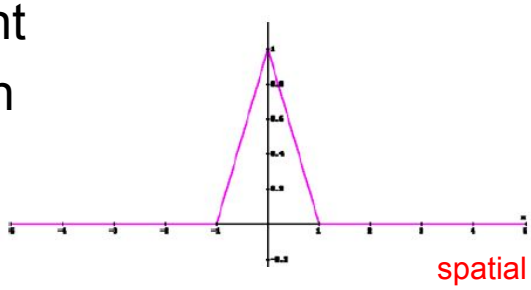
- Pretending pixels are little squares.

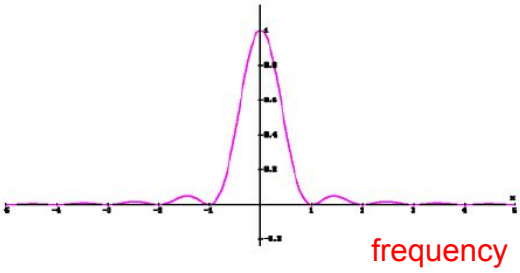




Tent Filter / Bi-Linear Interpolation

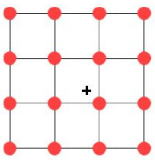
- Simple to implement
- Reasonably smooth

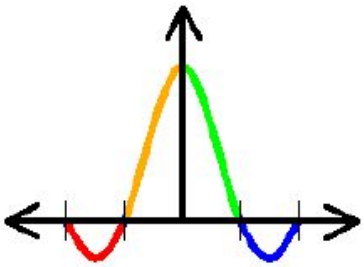


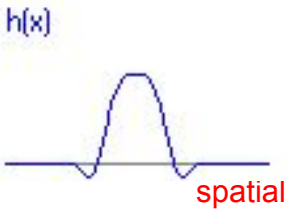


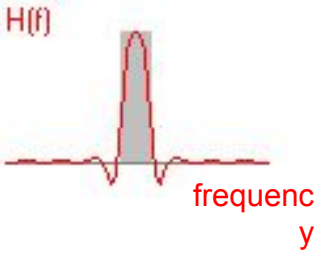
Bi-Cubic Interpolation

- Begins to approximate the ideal spatial filter, the sinc function







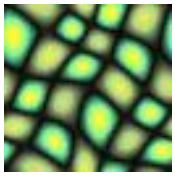


Leftover from Last Time...

- What is a Pixel?
- Examples of Aliasing
- Sampling & Reconstruction
- Filters in Computer Graphics
- Anti-Aliasing for Texture Maps
  - Magnification & Minification, Mipmaps

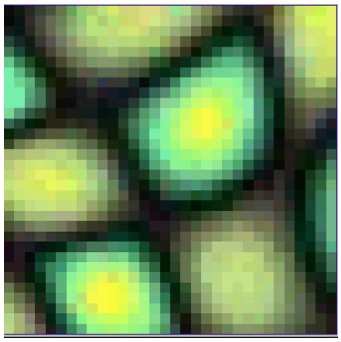
Sampling Texture Maps

- When texture mapping it is rare that the screen-space sampling density matches the sampling density of the texture.

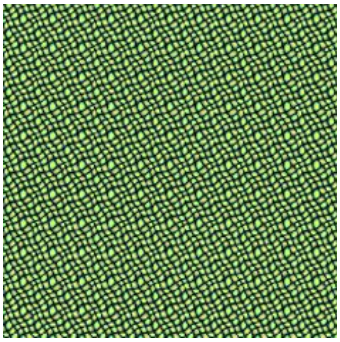


64x64 pixels

Original Texture



Magnification for Display

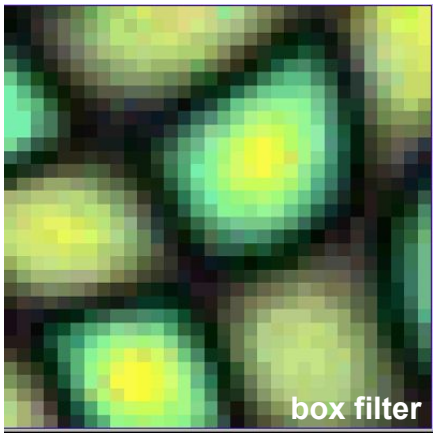


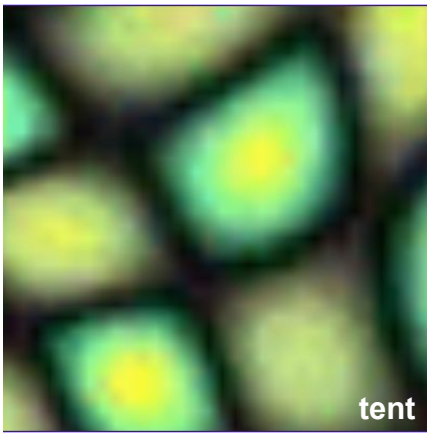
Minification for Display

for which we must use a reconstruction filter

Linear Interpolation

- Tell OpenGL to use a tent filter instead of a box filter.
- Magnification looks better, but blurry
  - (texture is under-sampled for this resolution)

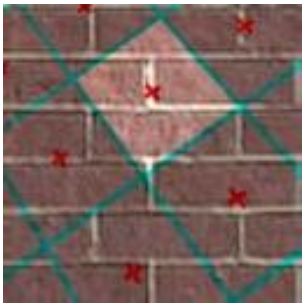




Spatial Filtering

- Remove the high frequencies which cause artifacts in texture minification.
- Compute a spatial integration over the extent of the pixel
- This is equivalent to convolving the texture with a filter kernel centered at the sample (i.e., pixel center)!
- Expensive to do during rasterization, but an approximation it can be precomputed

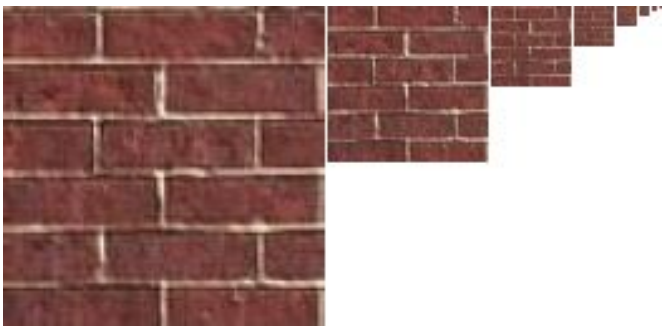
projected texture in image plane



box filter in texture plane

MIP Mapping

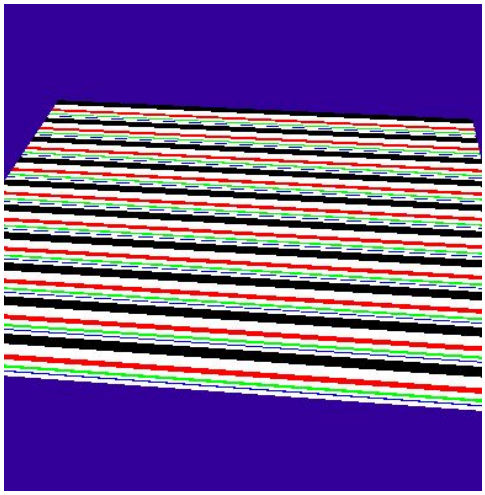
- Construct a pyramid of images that are pre-filtered and re-sampled at 1/2, 1/4, 1/8, etc., of the original image's sampling



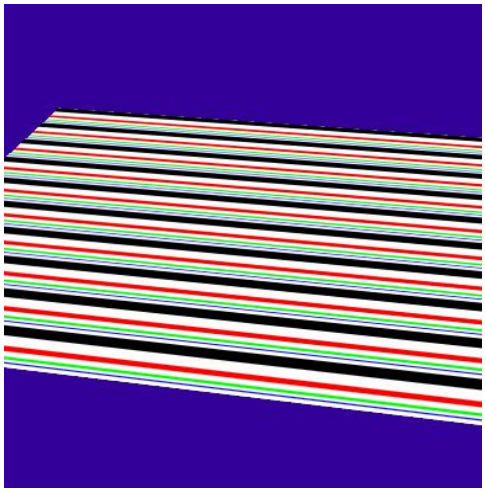
- During rasterization we compute the index of the decimated image that is sampled at a rate closest to the density of our desired sampling rate
- MIP stands for *multum in parvo* which means *many in a small place*

MIP Mapping Example

- Thin lines may become disconnected / disappear



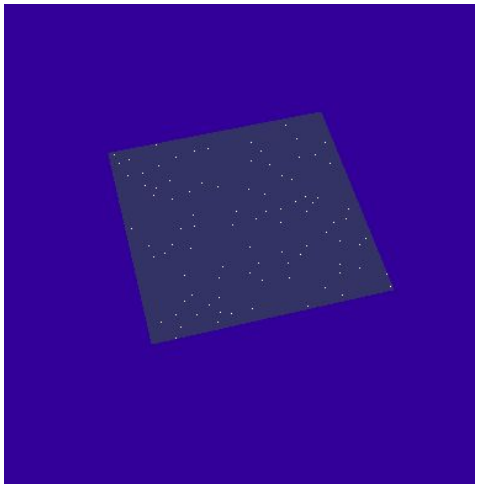
Nearest Neighbor



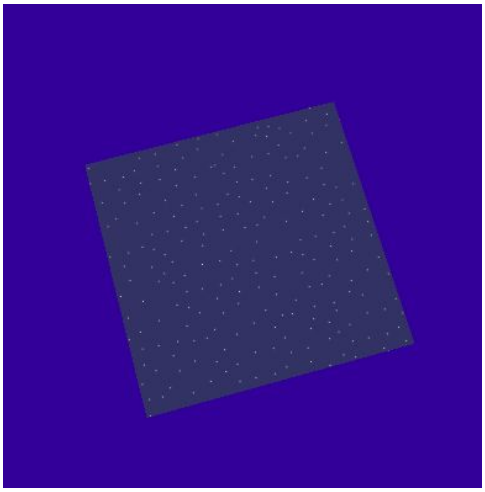
MIP Mapped (Bi-Linear)

MIP Mapping Example

- Small details may "pop" in and out of view



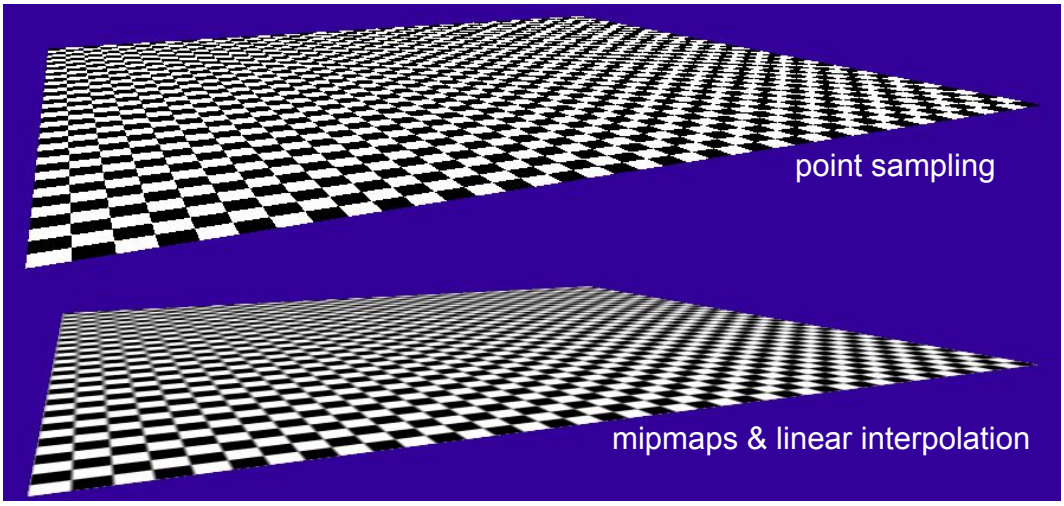
Nearest Neighbor



MIP Mapped (Bi-Linear)

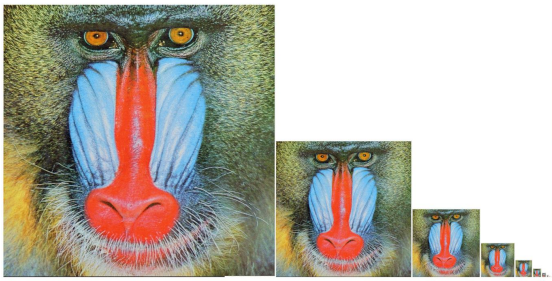
Examples of Aliasing

Texture Errors

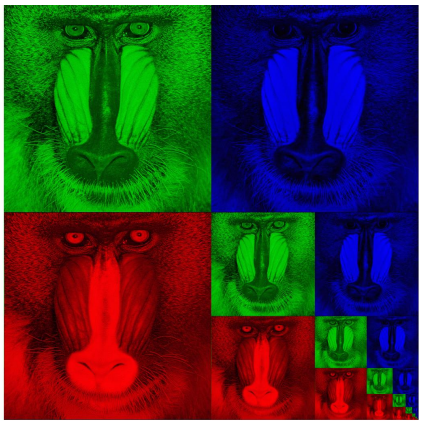


Storing MIP Maps

- Can be stored compactly
- Illustrates the 1/3 overhead of maintaining the MIP map



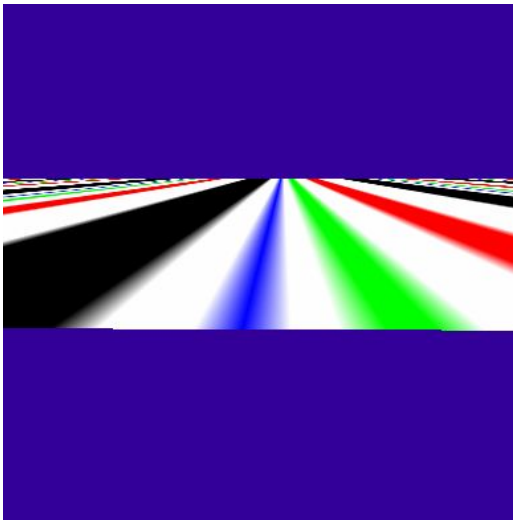
10-level mip map



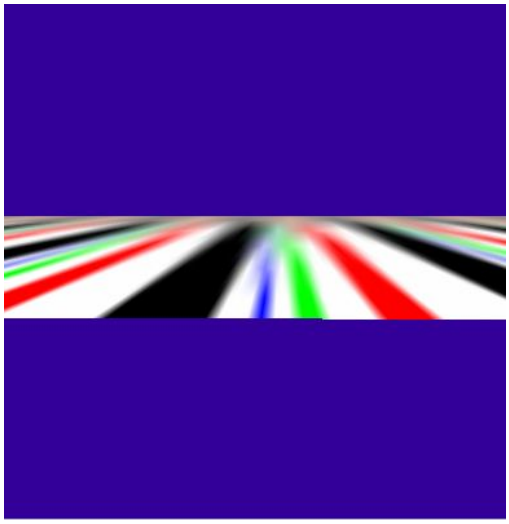
Memory format of a mip map

Anisotropic MIP-Mapping

- What happens when the surface is tilted?

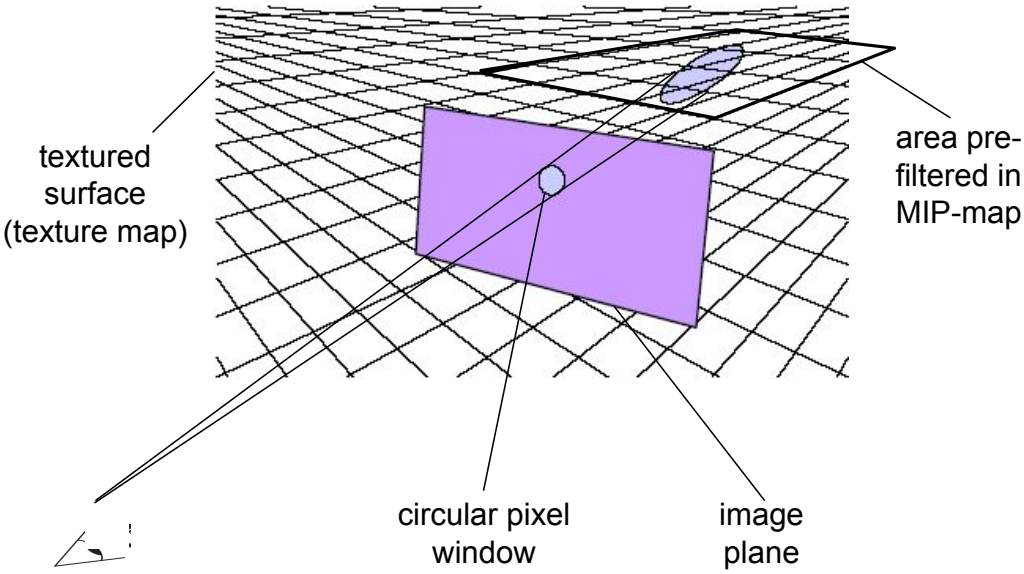


Nearest Neighbor



MIP Mapped (Bi-Linear)

Anisotropic MIP-Mapping



- Square MIP-map area is a bad approximation

Anisotropic MIP-Mapping

- We can use different mipmaps for the 2 directions
- Additional extensions can handle non axis-aligned views

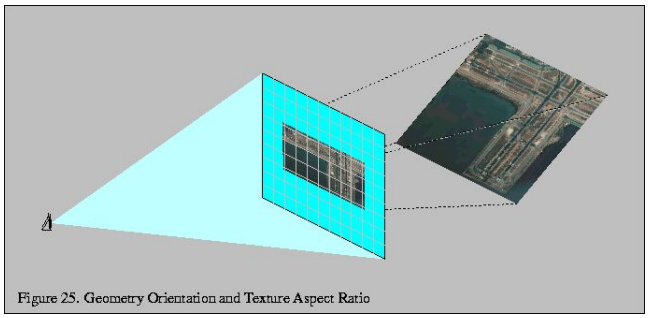


Figure 25. Geometry Orientation and Texture Aspect Ratio

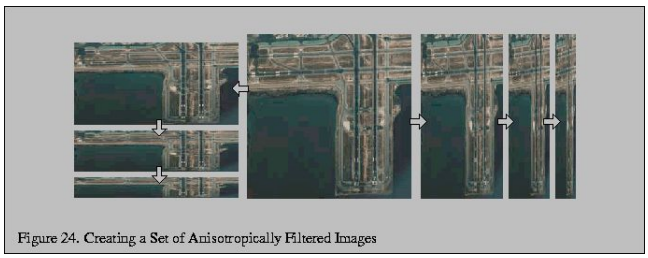


Figure 24. Creating a Set of Anisotropically Filtered Images

Images from http://www.sgi.com/software/opengl/advanced98/notes/node37.html

Today

- Readings for Today
- Measuring BRDFs
- 3D Digitizing & Scattering
- Complex Material Properties
- Importance of Participating Media
- BSSRDFs
- Other Complex Materials

Reading for Today:

- "Correlated Multi-Jittered Sampling", Andrew Kensler, Pixar Technical Memo, 2013

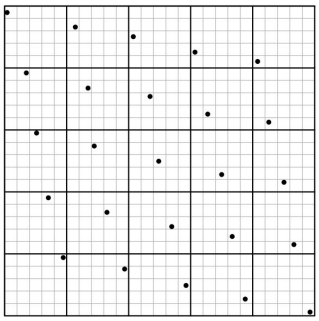


Figure 1: The canonical arrangement. Heavy lines show the boundaries of the 2D jitter cells. Light lines show the horizontal and vertical substrata of N-rooks sampling. Samples are jittered within the subcells.

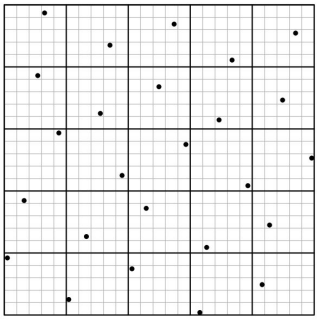


Figure 3: With correlated shuffling.

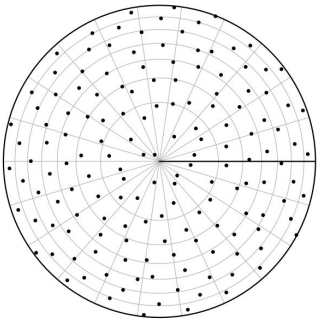
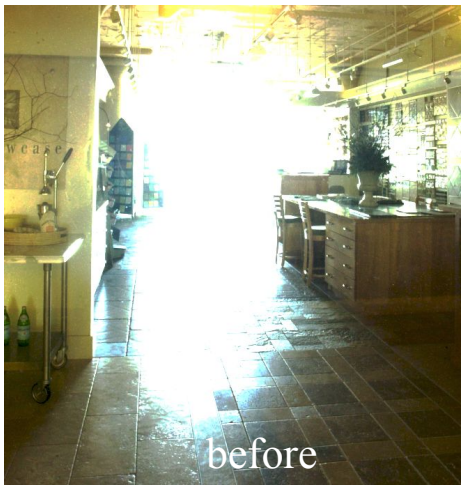


Figure 9: Polar warp with $m = 22$, $n = 7$.

[9] G. J. Ward and P. S. Heckbert. Irradiance gradients. In *Third Eurographics Rendering Workshop*, pages 85–98, May 1992.

Reading for Today:

"Fast Bilateral Filtering for the Display of High-Dynamic Range Images",
Durand & Dorsey, SIGGRAPH 2002

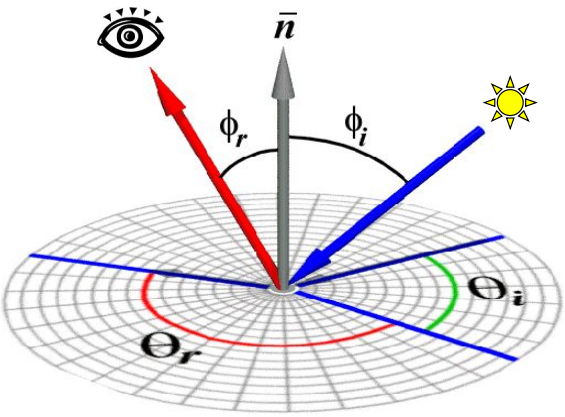


Today

- Readings for Today
- Measuring BRDFs
- 3D Digitizing & Scattering
- Complex Material Properties
- Importance of Participating Media
- BSSRDFs
- Other Complex Materials

BRDF

- Ratio of light coming from one direction that gets reflected in another direction
- Bidirectional Reflectance Distribution Function
  - 4D
  - $R(\theta_i, \varphi_i ; \theta_r, \varphi_r)$
  - Note: BRDF for *isotropic* materials is 3D



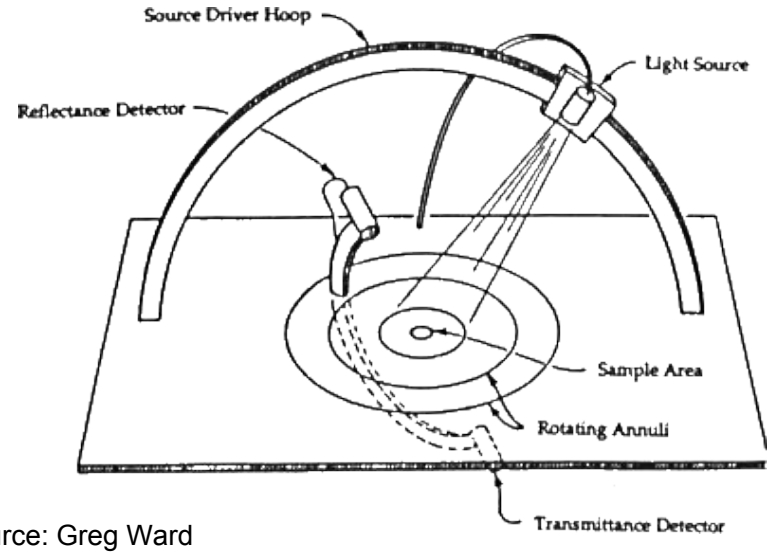
BRDFs in the Movie Industry

- Agent Smith's clothes are CG, with measured BRDF

Measured BRDF in film production: realistic cloth appearance for "The Matrix Reloaded"
Borshukov, SIGGRAPH 2003 Sketches & Applications

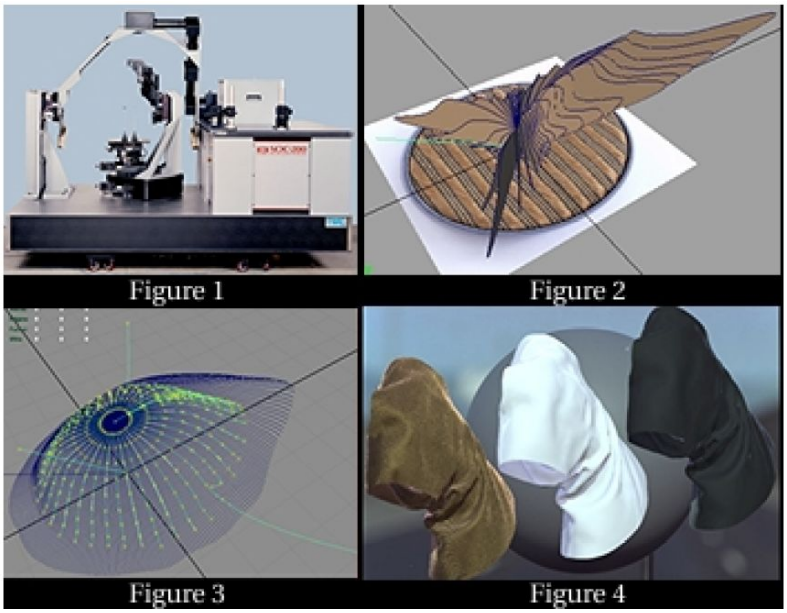
How Do We Obtain BRDFs?

- Gonioreflectometer
  - 4 degrees of freedom



Source: Greg Ward

BRDFs in the Movie Industry



Measured BRDF in film production: realistic cloth appearance for "The Matrix Reloaded"
Borshukov, SIGGRAPH 2003 Sketches & Applications

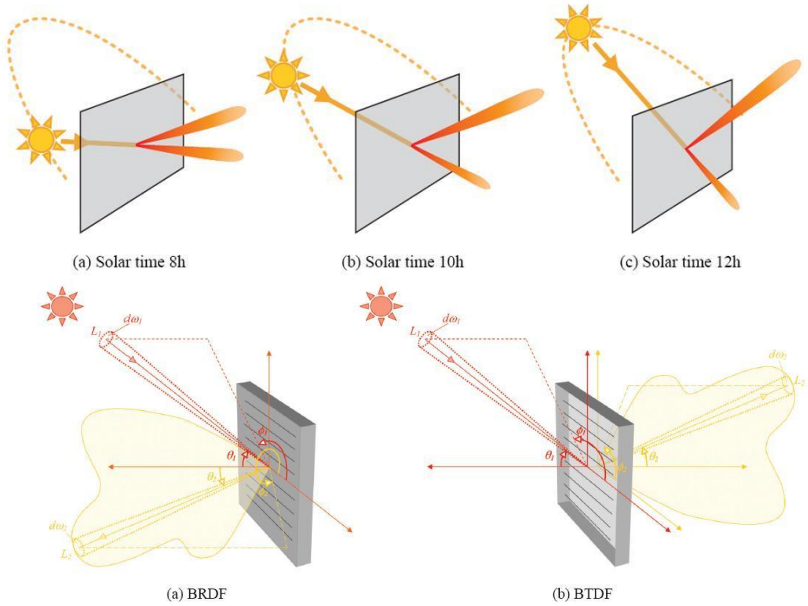
BRDFs in the Movie Industry

Photo CG Photo CG

Not just a BRDF…

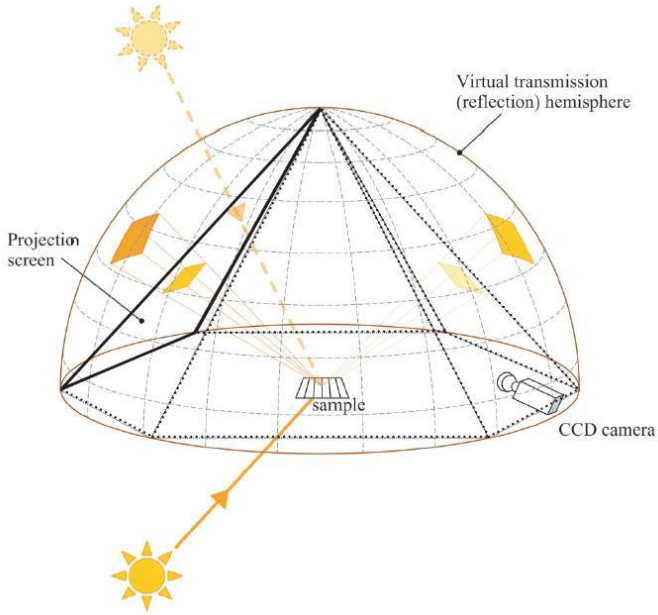
Realistic human face rendering for "The Matrix Reloaded"
Borshukov & Lewis, SIGGRAPH 2003 Sketches & Applications

Materials – BRDF & BTDF



M. Andersen, “Innovative bi-directional video-goniophotometer for advanced fenestration systems”, 2004.

Measuring Materials



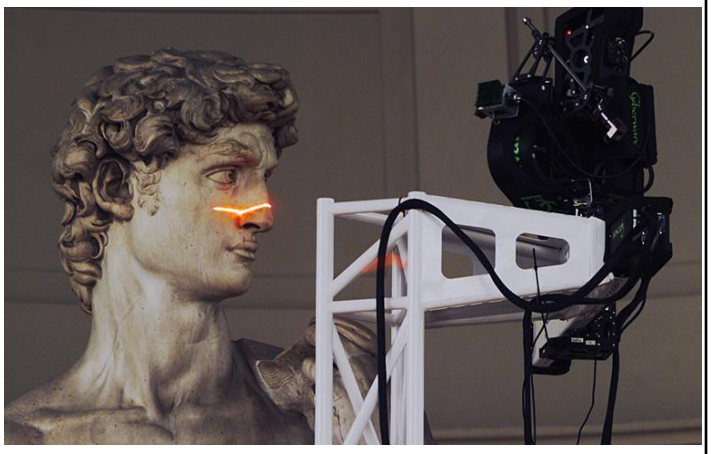
M. Andersen, “Innovative bi-directional video-goniophotometer for advanced fenestration systems”, 2004.

Today

- Readings for Today
- Measuring BRDFs
- 3D Digitizing & Scattering
- Complex Material Properties
- Importance of Participating Media
- BSSRDFs
- Other Complex Materials

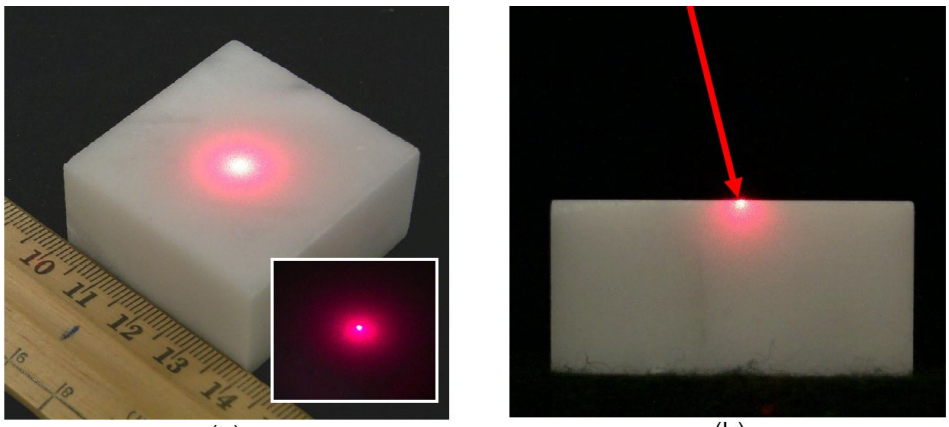
3D Digitizing

Cyberware



The Digital Michelangelo Project: 3D Scanning of Large Statues,
Levoy et al., *SIGGRAPH 2000*

Scattering & Scanning



(a) (b)

Figure 1: Diffusion in a sample of Carrara Statuario marble.

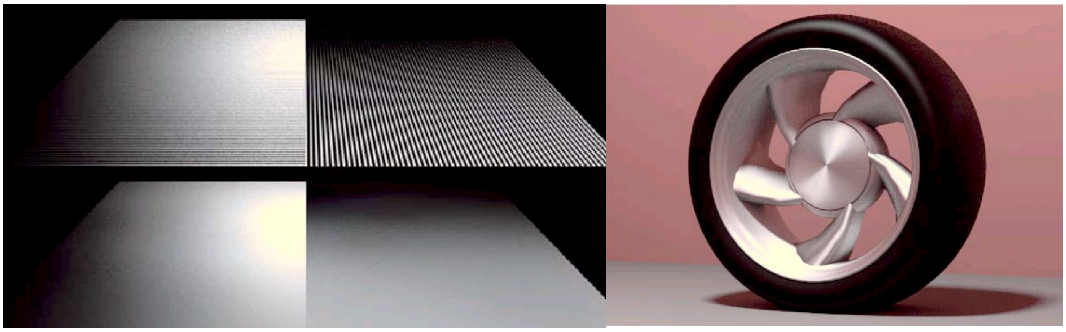
An Assessment of Laser Range Measurement of Marble Surfaces, Godin et al, 2001.

Today

- Readings for Today
- Measuring BRDFs
- 3D Digitizing & Scattering
- Complex Material Properties
- Importance of Participating Media
- BSSRDFs
- Other Complex Materials

Anisotropic BRDFs

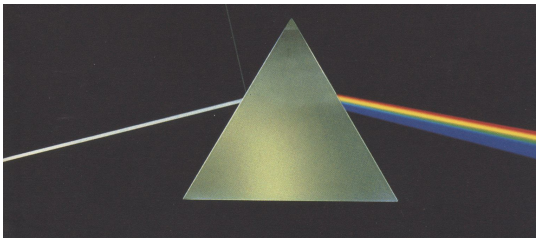
- Surfaces with strongly oriented microgeometry
- Examples:
  - brushed metals, hair, fur, cloth, velvet



Source: Westin et.al 92

What makes a Rainbow?

- Refraction is wavelength-dependent
  - Refraction increases as the wavelength of light decreases
  - violet and blue experience more bending than orange and red
- Usually ignored in graphics
- Rainbow is caused by refraction + internal reflection + refraction

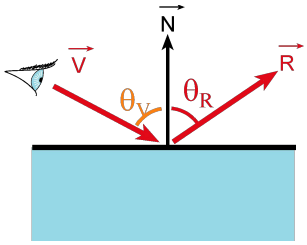


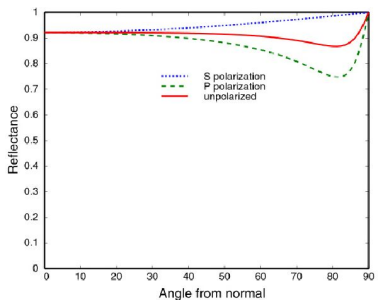
Pink Floyd, *The Dark Side of the Moon*

From “Color and Light in Nature” by Lynch and Livingstone

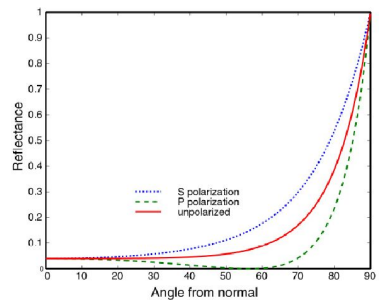
Amount of Reflection

- Traditional ray tracing (hack)
  - Constant **reflectionColor**
- More realistic:
  - Fresnel reflection term (more reflection at grazing angle)
  - Schlick's approximation: $R(\theta)=R_0+(1-R_0)(1-\cos\theta)^5$





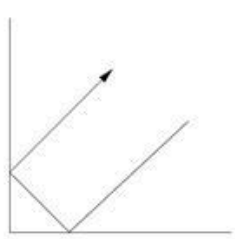
metal

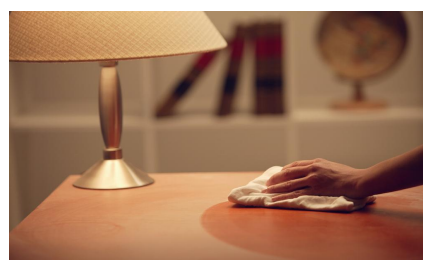


Dielectric (glass)

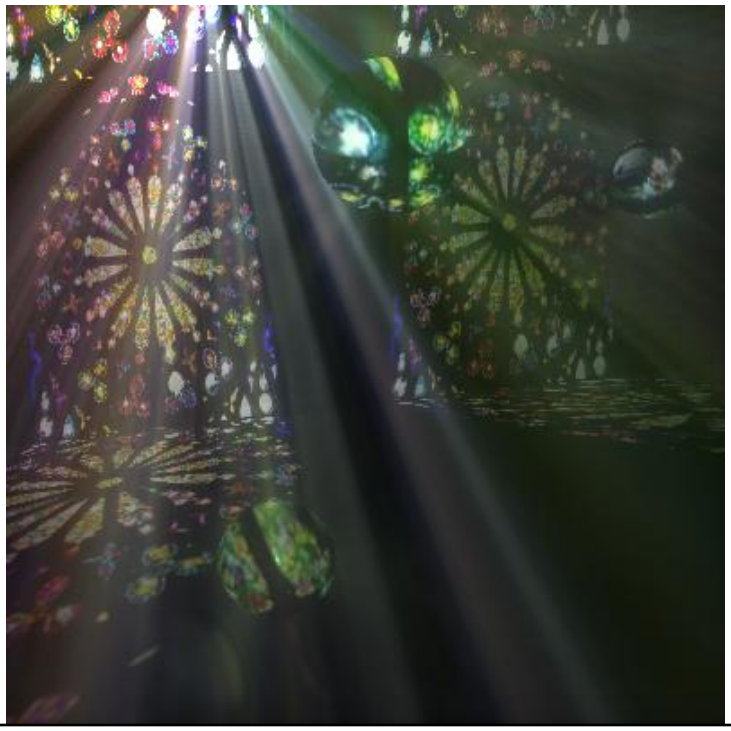
Dusty Surfaces & Retro-Reflection

- Viewed perpendicular to the surface, there is little scattering off dust
- At grazing angles, there is increased scattering with the dust making the surface appear brighter
- Earth viewed from space appears brighter near the edges, due to increased atmospheric scattering
- Road paint is intentionally retro-reflective (so drivers see road markings illuminated by their own headlights)



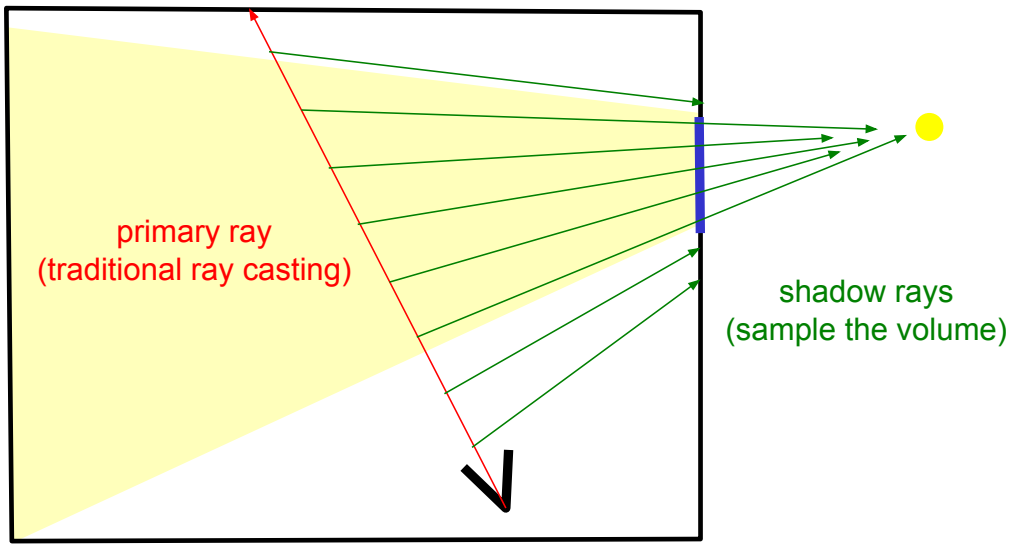


Light Rays in a Dusty Room



Annie Ding, MIT
6.837 Final Project
December, 2004

Ray Tracing Participating Media



Participating Media

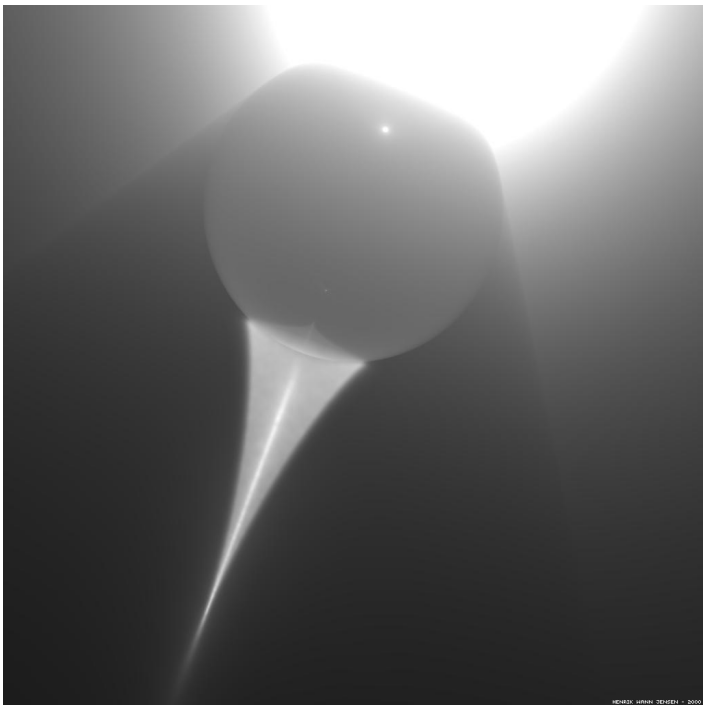
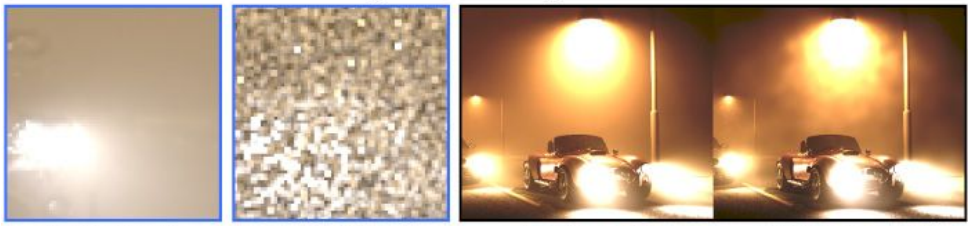


Image by Henrik Wann Jensen

"Radiance Caching for Participating Media", Jarosz, Donner, Zwicker, & Jensen, 2008.

Equal-time Comparisons



Our Method | Path Tracing | Our Method | Photon Mapping

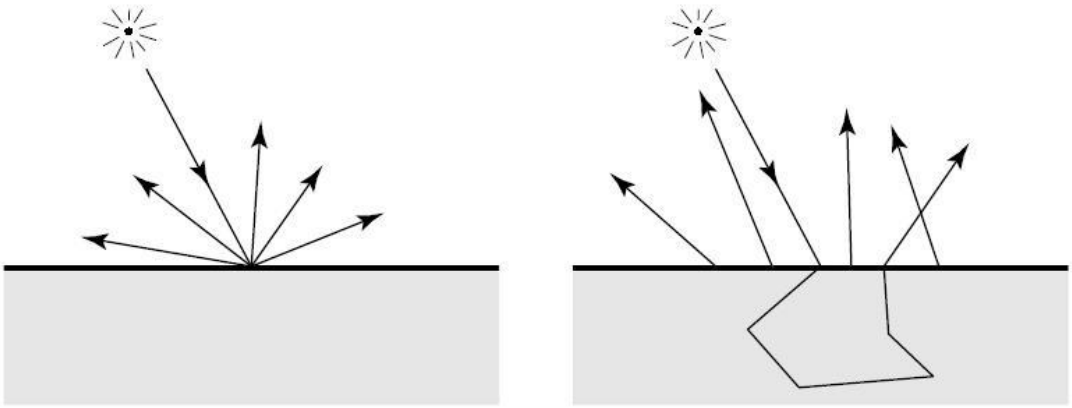
Today

- Readings for Today
- Measuring BRDFs
- 3D Digitizing & Scattering
- Complex Material Properties
- Importance of Participating Media
- BSSRDFs
- Other Complex Materials

Reading for Next Time: *(pick one)*

"A Practical Model for Subsurface Light Transport",
Jensen, Marschner, Levoy, & Hanrahan, SIGGRAPH 2001

BRDF vs. BSSRDF



Images from "A Practical Model for Subsurface Light Transport"
Jensen, Marschner, Levoy, & Hanrahan SIGGRAPH 2001

Single Scattering

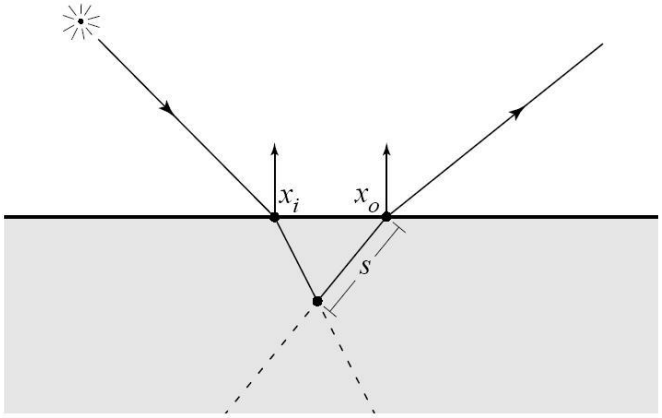


Figure 4: Single scattering occurs only when the refracted incoming and outgoing rays intersect, and is computed as an integral over path length s along the refracted outgoing ray.

Images from "A Practical Model for Subsurface Light Transport"
Jensen, Marschner, Levoy, & Hanrahan SIGGRAPH 2001

Dipole Approx. for Diffuse Scattering

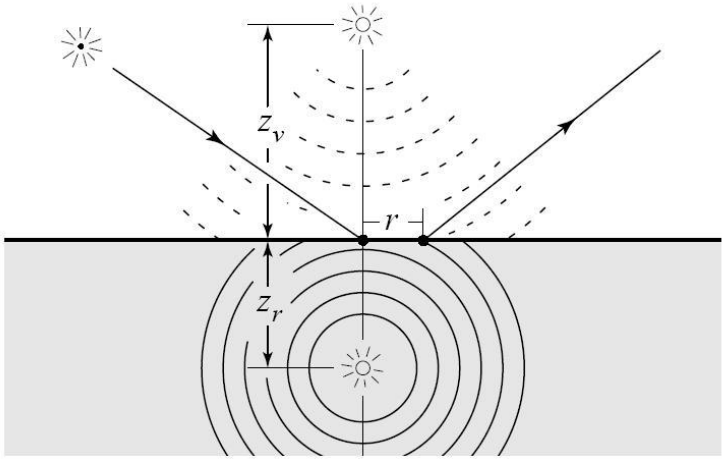


Figure 3: An incoming ray is transformed into a dipole source for the diffusion approximation.

Images from "A Practical Model for Subsurface Light Transport"
Jensen, Marschner, Levoy, & Hanrahan SIGGRAPH 2001

Today

- Readings for Today
- Measuring BRDFs
- 3D Digitizing & Scattering
- Complex Material Properties
- Importance of Participating Media
- BSSRDFs
- Other Complex Materials

(a) 3D mesh (close-up of nostril) (b) Color data (c) Diffuse rendering

(d) Oily layer (e) Subsurface scattering (f) Final result

"Digital Face Cloning", Jensen, SIGGRAPH Sketch 2003

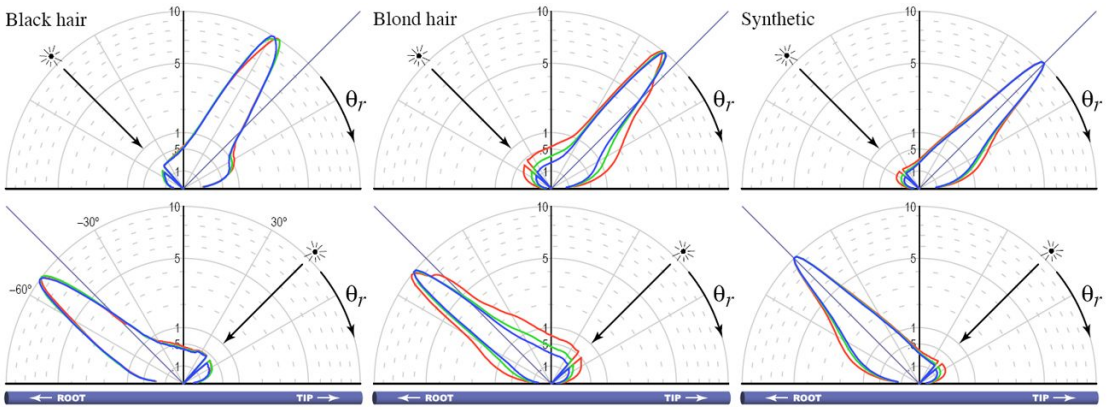
"Light Diffusion in Multi-Layered Translucent Materials" Donner & Jensen, *SIGGRAPH 2005*

Measuring BSSRDF by Dilution

"Acquiring Scattering Properties of Participating Media by Dilution"
Narasimhan et al. SIGGRAPH 2006

(a) Acquired photographs (b) Rendering at low concentrations (c) Rendering at natural concentrations

Reading for Next Time *(pick one)*



Black hair **Blond hair** **Synthetic (wig)**

"Light Scattering from Human Hair Fibers"
Marschner et al., SIGGRAPH 2003

Reading for Next Time *(pick one)*

Old Method **New Method** **Photo**

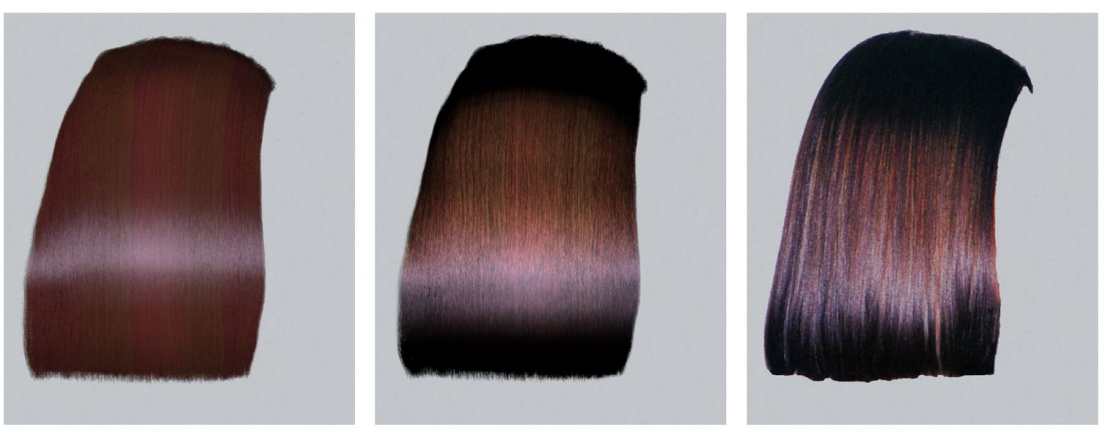


Figure 12: A comparison of Kajiya and Kay's model (left) under a single point source, our proposed model (center) with the same lighting, and the hair from the photograph in Figure 11 (removed from context to simplify the comparison). The Kajiya model's diffuse term results in a flat appearance, while the secondary highlight in our model correctly captures the colored shading of the real hair.

"Light Scattering from Human Hair Fibers"
Marschner et al., SIGGRAPH 2003