Subdivision Surfaces

Geri’s Game

Pixar Animation Studios, 1986
Questions on Homework 1?

• What’s an illegal edge collapse?

![Diagram of a graph with vertices 1 to 7 and edges between them.]

What if vertex 1 is the same as vertex 4?

• To be legal, the ring of neighboring vertices must be unique (have no duplicates)!

Notes about HW Autograding

• HW is run on a Linux desktop machine
• Automated:
  – Keyboard & mouse commands
  – Reasonable pauses (sleep)
  – Screenshots
• Will have longer wait times
  – not parallelized (one student at a time)
  – … now two desktops
• Due to COVID
  – Your submission is received & stored at RPI
  – Shipped to Barb’s house for grading
  – w/ Spectrum router… Networking is suspect

Don't panic if autograding takes a while or gets stuck. Post on the forum if you experience problems.
Last Time?

- Curves & Surfaces
- Continuity Definitions
  - $C^0$, $G^1$, $C^1$, … $C^\infty$
- Interpolation vs. Approximation Splines
- Cubic Bezier & BSpline

Today

- Papers for Today
  - “Subdivision Surfaces in Character Animation”
  - “Piecewise Smooth Surface Reconstruction”
- Misc. Mesh/Surface Vocabulary
- Subdivision Surface “Zoo”
- Interpolating Subdivision
- Papers for Next Time
- Worksheet: Bezier Spline vs. BSpline
Reading for Today

- DeRose, Kass, & Truong, "Subdivision Surfaces in Character Animation", SIGGRAPH 1998

Quad Meshes
two common in artistic practice (e.g. Pixar’s Geri’s Game)

![Image of a hand with a piecewise smooth Catmull-Clark surface.](image)

Figure 5: Geri’s hand as a piecewise smooth Catmull-Clark surface. Infinitely sharp creases are used between the skin and the finger nails.

Subdivision Surfaces in Character Animation

- Catmull Clark Subdivision Rules
- Semi-sharp vs. Infinitely-sharp creases
- Mass-Spring Cloth (next week)
- Hierarchical Mesh for Collision
- Texturing Subdivision Surfaces

![Images of subdivision surfaces](images)

Figure 11: (a) A texture mapped regular pentagon comprised of 5 triangles; (b) the pentagonal model with its vertices moved; (c) A subdivision surface whose control mesh is the same 5 triangles in (a), and where boundary edges are marked as creases; (d) the subdivision surface with its vertices positioned as in (b).
Reading for Today

• Hoppe et al., “Piecewise Smooth Surface Reconstruction” SIGGRAPH 1994

Piecewise Smooth Surface Reconstruction

• From input: scanned mesh points
  – Estimate topological type (genus)
  – Mesh optimization (a.k.a. simplification)
  – Smooth surface optimization
Piecewise Smooth Surface Reconstruction

- Optimization
- Remeshing

- Crease subdivision masks *decouple* behavior of surface on either side of crease
- Crease rules cannot model a cone
- Optimization can be done locally
  - subdivision control points have only local influence
- Results
  - Noise?
  - Applicability?
  - Limitations?
  - Running Time
Spline-Based Modeling Headaches

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Misc. Mesh/Surface Vocabulary

- **Genus**: The maximum number of disjoint simple closed curves which can be cut from an orientable surface of genus $g$ without disconnecting it is $g$.

Misc. Mesh/Surface Vocabulary

- **Homeomorphic/Topological equivalence**: a continuous stretching and bending of the object into a new shape

• **Dihedral Angle:**
  - the angle between the planes of two triangular faces
  - “looking down the edge” between two faces, the angle between the faces.

• **Valence (a.k.a. degree):**
  the number of edges incident to the vertex.

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**Misc. Mesh/Surface Vocabulary**

• **Warp & weft:** Yarns used in weaving. Because the weft does not have to be stretched in the way that the warp is, it can generally be less strong.

http://en.wikipedia.org/wiki/Weft
Misc. Mesh/Surface Vocabulary

• Extraordinary Vertex
  – Quad mesh: vertices w/ valence ≠ 4
  – Hex mesh: vertices w/ valence ≠ 3
  – Tri mesh: vertices w/ valence ≠ 6
Today

- Papers for Today
- Misc. Mesh/Surface Vocabulary
- Subdivision Surface “Zoo”
  - Doo Sabin (anything!)
  - Loop, Butterfly, $\sqrt{3}$ (triangles only)
  - Catmull Clark (turns everything into quads)
  - … many others!
- Interpolating Subdivision
- Papers for Next Time
- Worksheet: Bezier Spline vs. BSpline

Chaikin's Algorithm

![Chaikin's Algorithm Diagram]
Doo-Sabin Subdivision

Idea: introduce a new vertex for each face
At the midpoint of old vertex, face centroid

Original Cube  The 1st subdivision  The 2nd subdivision

The 3rd subdivision  The 5th subdivision

http://www.ke.ics.saitama-u.ac.jp/xuz/pic/doo-sabin.gif
Loop Subdivision

Subdivision Rules. The masks for the Loop scheme are shown in Figure 4.3. For boundaries and edges tagged as crease edges, special rules are used. These rules produce a cubic spline curve along the boundary/crease. The curve only depends on control points on the boundary/crease.

Figure 4.3: Loop subdivision: in the picture above, $\beta$ can be chosen to be either $\frac{1}{3}(5/8 - (\frac{1}{5} + \frac{1}{4}\cos \frac{2\pi}{n})^2)$ (original choice of Loop [16]), or for $n > 3$, $\beta = \frac{1}{4n}$ as proposed by Warren [33]. For $n = 3$, $\beta = 3/16$ can be used.
Adding creases to Loop Subdivision

- Vertex & edge masks
- Limit masks
  - Position
  - Tangent

Catmull Clark Subdivision

\[
e_{ij}^{t+1} = \frac{e_{ij} + e_{ij}^{t} + e_{ij}^{t+1} + e_{ij}^{t+1}}{4},
\]

where subscripts are taken modulo the valence of the central vertex \( v^0 \). (The valence of a vertex is the number of edges incident to it.) Finally, a vertex point \( v' \) is computed as

\[
v^{t+1} = \frac{n-2}{n} v + \frac{1}{n} \sum_{j} e_{ij} + \frac{1}{n} \sum_{j} e_{ij}^{t+1}
\]

Vertices of valence 4 are called ordinary; others are called extraordinary.

Figure 3: Recursive subdivision of a topologically complicated mesh: (a) the control mesh; (b) after one subdivision step; (c) after two subdivision steps; (d) the limit surface.

Figure 4: The situation around a vertex \( v^0 \) of valence \( n \).

"Subdivision Surfaces in Character Animation",
DeRose, Kass & Truong, SIGGRAPH 1998
Catmull-Clark Subdivision

https://team.inria.fr/virtualplants/teaching/informatique-graphique-2016/tp4-instructions/

Catmull-Clark preferred by Artists

- Catmull-Clark is based on quadrilaterals
  - Like NURBS, specifically cubic bsplines
  - Implicit adjacency in subdivided microgeometry
  - Better than triangles for symmetric objects
Butterfly Subdivision

- Triangle-based subdivision
- Alternate scheme to Loop

$\sqrt{3}$ Subdivision Kobbelt, SIGGRAPH 2000

Adaptive Subdivision (Loop): Need to close gaps between different levels of refinement

Loop: less localized refinement

$\sqrt{3}$: more localized refinement
Questions?

Justin Legakis

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Interpolation vs. Approximation Curves

- Interpolation Curve – over constrained → lots of (undesirable?) oscillations

- Approximation Curve – more reasonable?

Interpolating Subdivision

- Chaikin:

- Doo-Sabin:

  of the centroids of each edge/face
Interpolating Subdivision

- **Interpolation vs. Approximation** of control points
- Handle arbitrary topological type
- Reduce the “extraneous bumps & wiggles”

"Efficient, fair interpolation using Catmull-Clark surfaces", Halstead, Kass & DeRose, SIGGRAPH 1993

Interpolation of Catmull-Clark Surfaces

- Solve for a new control mesh (generally “bigger”) such that when Catmull-Clark subdivision is applied it interpolates the original control mesh
Vertex Position in Limit

- $V_n$ stores the center vertex & surrounding edge & face vertices as a big column vector:

$$V_{n}^{i+1} = S_n V_n^i$$

- When $n = 4$: (n = valence)

$$V_n^\infty := \lim_{i \to \infty} S_n^i V_n^1$$

Solve for New Positions

- Goal: Find the control mesh vertex positions, $x$ (a column vector of 3D points), such that the position of the vertices in the limit match the input vertices, $b$ (also a column vector of points)

- Use Least Squares to solve

$$A x = b$$

where $A$ is a square matrix with the interpolation rules and connectivity of the mesh

- See paper for extension to match limit normals
Fairing

- Fairing: an additional part or structure added to an aircraft, tractor-trailer, etc. to smooth the outline and thus reduce drag

- Subdivide initial resolution twice so that all constrained vertex positions are independent

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Reading for Next Time:  *(pick one)*

- Oriented Bounding Box (OBB): generalization of the (axis-aligned) BVH


Reading for Next Time:  *(pick one)*

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Connecting Cubic Bézier Curves

- Where is this curve
  - $C^0$ continuous?
  - $G^1$ continuous?
  - $C^1$ continuous?
- What’s the relationship between:
  - the # of control points, and
  - the # of cubic Bézier subcurves?
BSpline Curve Control Points

Default BSpline  BSpline with Discontinuity  BSpline which passes through end points
Repeat interior control point  Repeat end points

Pop Worksheet!

- What is the minimum number of cubic Bezier curve segments needed to approximately reproduce the two curves below? Sketch the positions of the control vertices.

- Repeat for cubic BSplines curve segments.