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# Implicit Surfaces, Collision Detection, & Volumetric Data Structures

## Traveler's Insurance, *Snowball*



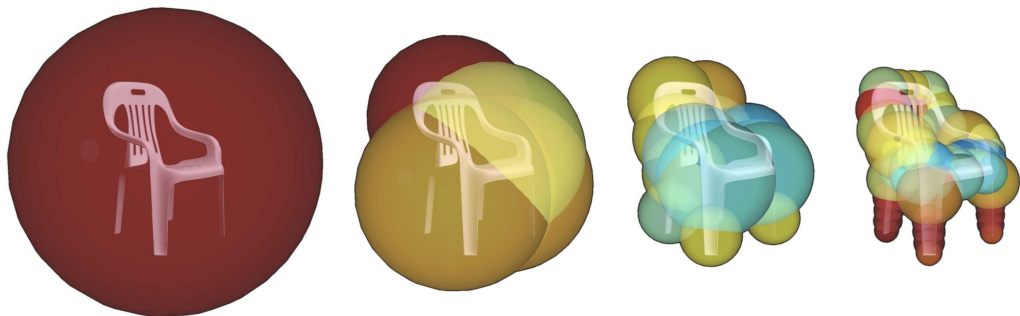
Weta Digital, 2007

# Katamari



BANDAI NAMCO Entertainment Inc., 2004-2018

## Output-Sensitive Collision Processing for Reduced-Coordinate Deformable Models



Level 0



Level 2



Level 4

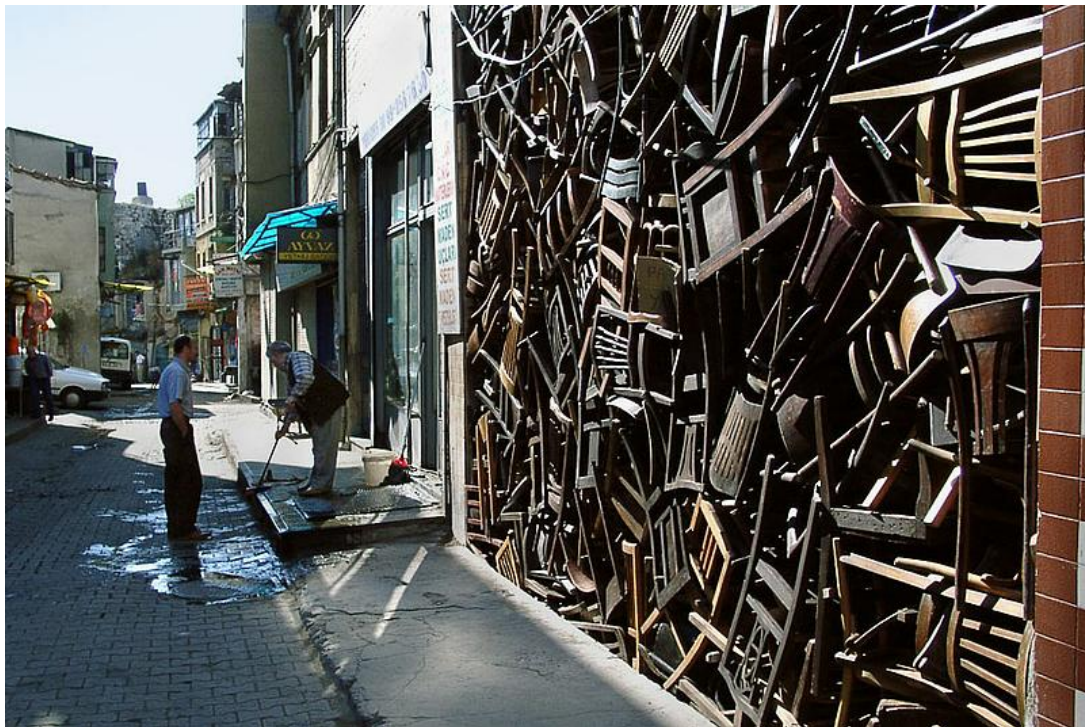


Level 6

James, Pai, and Twigg, SIGGRAPH 2004



*Untitled, 1550 chairs stacked, Doris Salcedo, 2003*

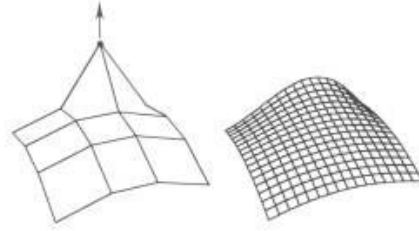


*Untitled, 1550 chairs stacked, Doris Salcedo, 2003*

# Last Time?

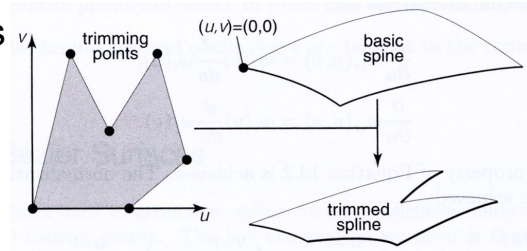
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- Spline Surfaces
  - complex topology is challenging, requires trimming curves

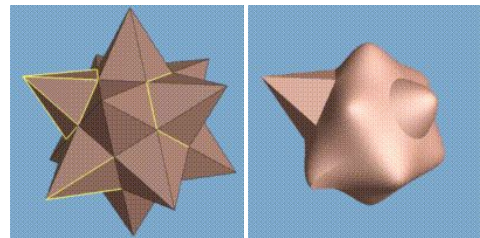
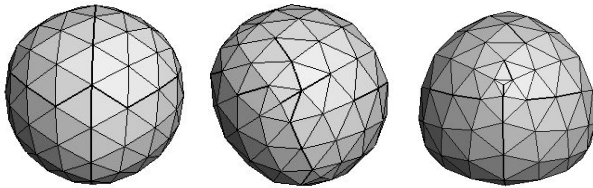


- Subdivision Zoo

- Doo-Sabin
- Loop
- Catmull-Clark



- Subdivision w/ Creases



# Today

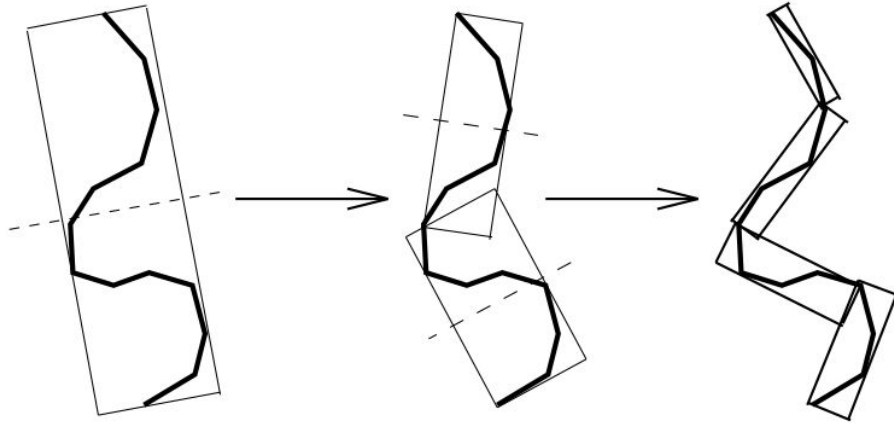
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- **Readings for Today**
- Motivation: Collision Detection is Expensive
- Conservative Bounding Region
- Spatial Acceleration Data Structures
- Papers for Friday
- Questions about Homework 1
- Worksheet on Subdivision Surfaces

# Reading for Today

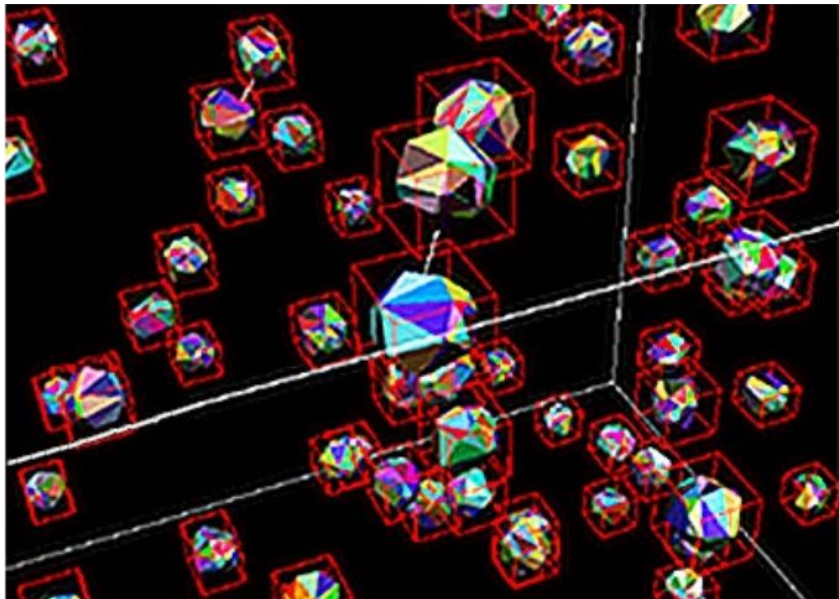
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- Oriented Bounding Box (OBB):  
generalization of the (axis-aligned) BVH



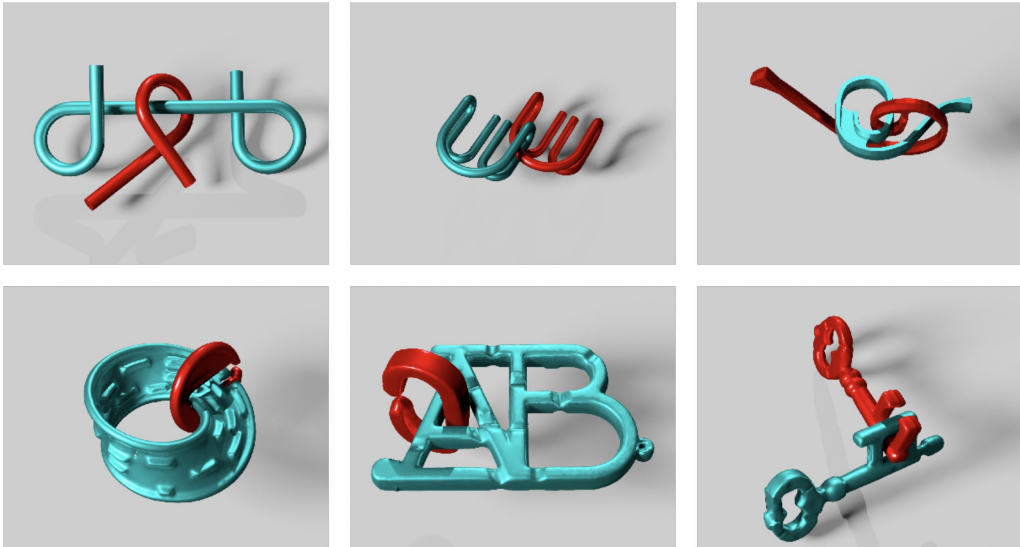
OBB-Tree: A Hierarchical Structure for Rapid Interference Detection,  
Gottschalk, Lin, & Manocha, SIGGRAPH 1996.

"I-COLLIDE: An Interactive and Exact Collision  
Detection System for Large-scaled Environments",  
Cohen, Lin, Manocha, & Ponamgi, I3D 1995.

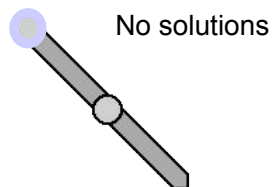


# Reading for Today

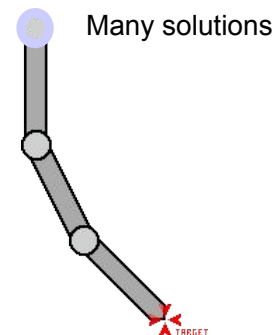
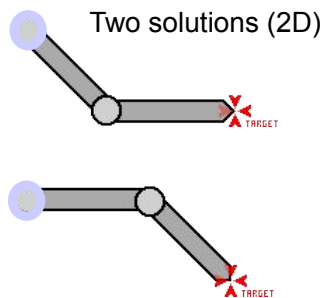
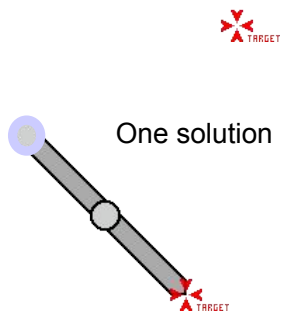
- "C-Space Tunnel Discovery for Puzzle Path Planning", Zhang, Belfer, Kry, & Voucha, SIGGRAPH 2020.



# Searching Configuration Space

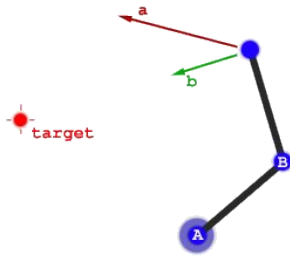


- Application: Robot Motion Planning



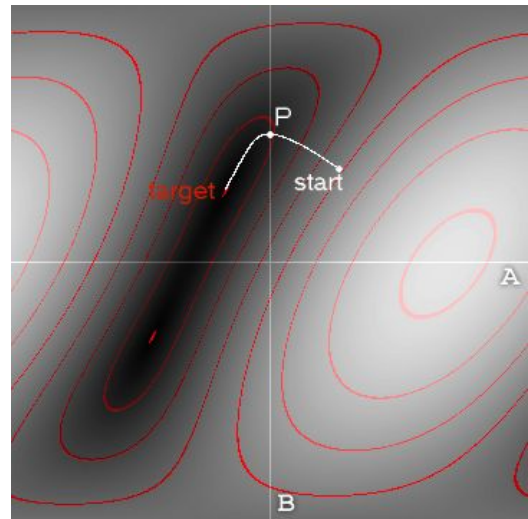
“The good-looking textured light-sourced bouncy fun smart and stretchy page”  
Hugo Elias, [http://freespace.virgin.net/hugo.elias/models/m\\_ik.htm](http://freespace.virgin.net/hugo.elias/models/m_ik.htm)

# Searching Configuration Space



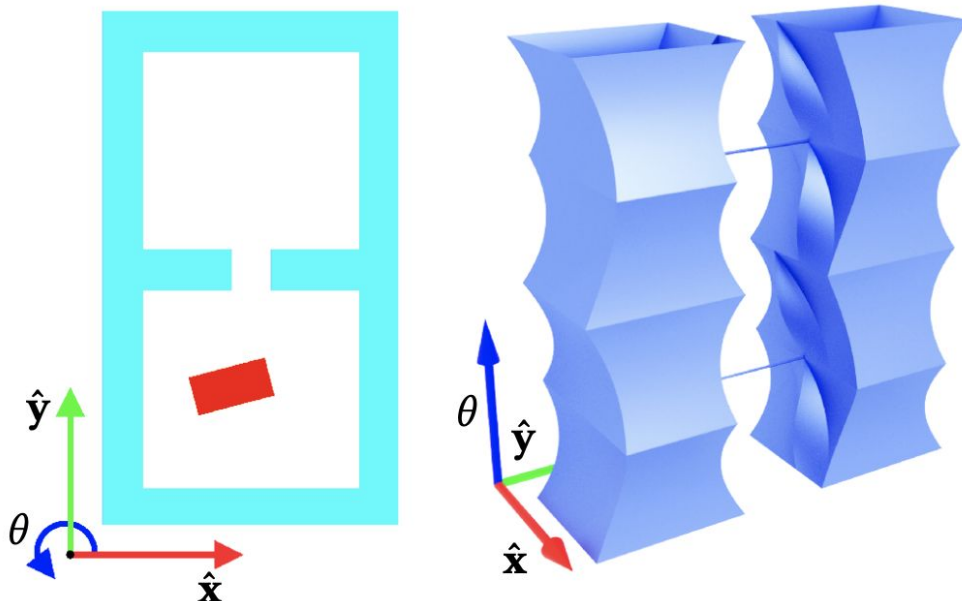
- What are the unknowns? What are the “degrees of freedom” of our robot arm?
- More degrees of freedom = higher dimensional configuration space

pose space shaded by distance to target



“The good-looking textured light-sourced bouncy fun smart and stretchy page”  
Hugo Elias, [http://freespace.virgin.net/hugo.elias/models/m\\_ik2.htm](http://freespace.virgin.net/hugo.elias/models/m_ik2.htm)

# Searching Configuration Space



“C-Space Tunnel Discovery for Puzzle Path Planning”,  
Zhang, Belfer, Kry, & Voucha, SIGGRAPH 2020.

# Today

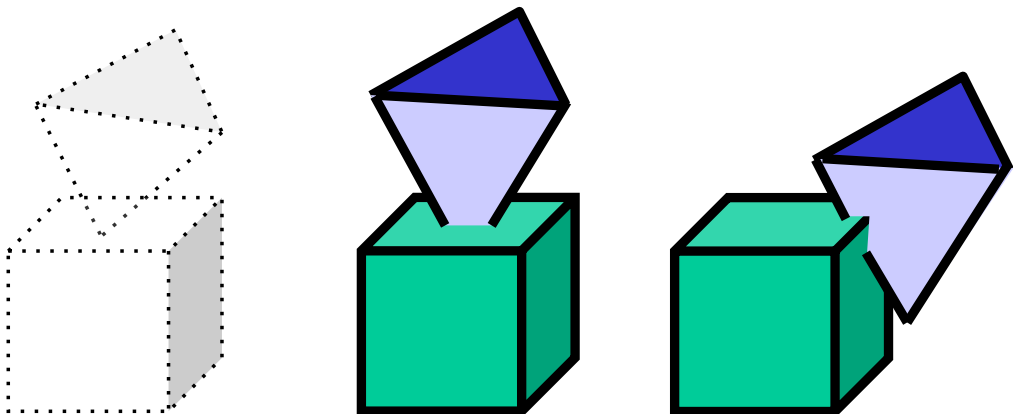
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- **Motivation: Collision Detection is Expensive**
- Conservative Bounding Region
- Spatial Acceleration Data Structures
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## Collision Detection for Solids

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- How to detect collision between 2 polyhedra?
- Need an inside/outside test
- Test if a vertex is inside the other polyhedron
- But treat also edge-edge intersection





# Cost of Detection?

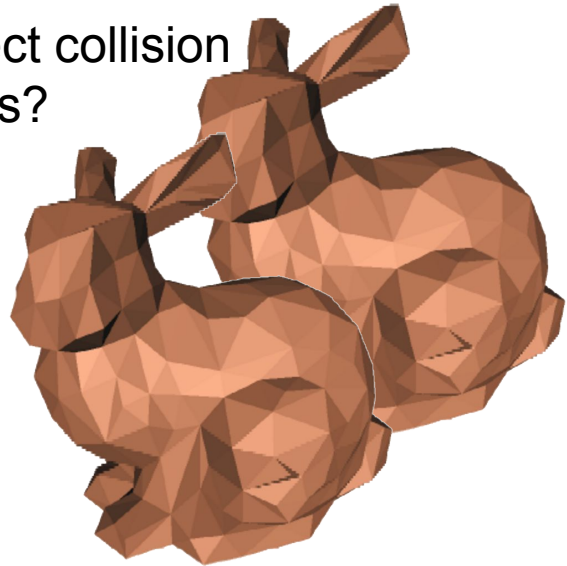
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- Test each edge with each face?

$O(N^2)$

- How would you detect collision between two bunnies?

- $O(N^2)$  is too expensive!
- Let's use a spatial data structure



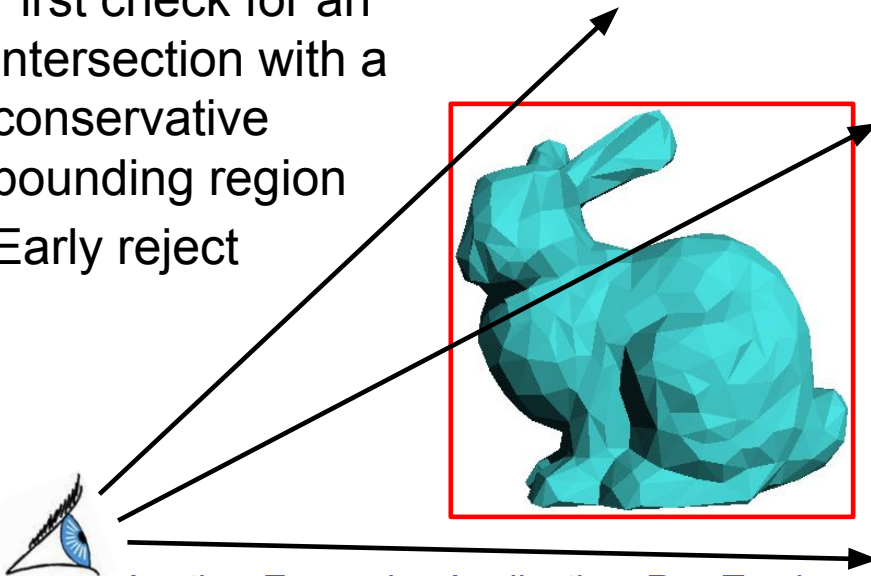
# Today

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# Conservative Bounding Region

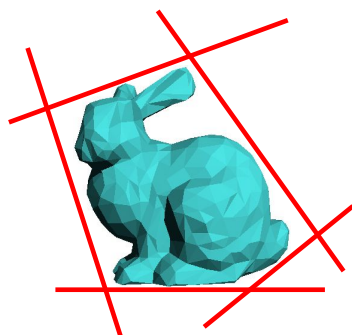
- First check for an intersection with a conservative bounding region
- Early reject



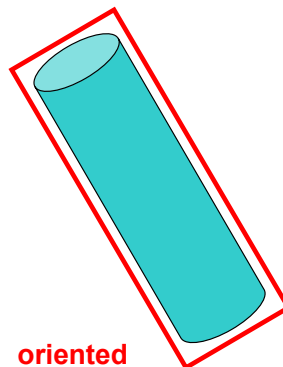
Another Expensive Application: Ray Tracing  
Intersect object & ray... *more later this semester!!*

# Conservative Bounding Regions

- tight  $\rightarrow$  avoid false positives
- fast to intersect
- easy/fast/perfect construction (*less important*)



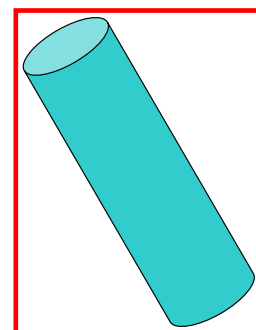
arbitrary convex region  
(bounding half-spaces)



oriented  
bounding box



bounding  
sphere

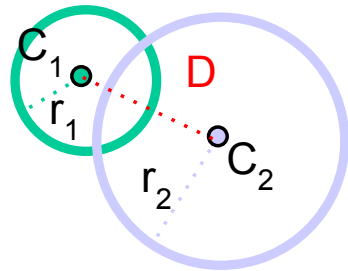
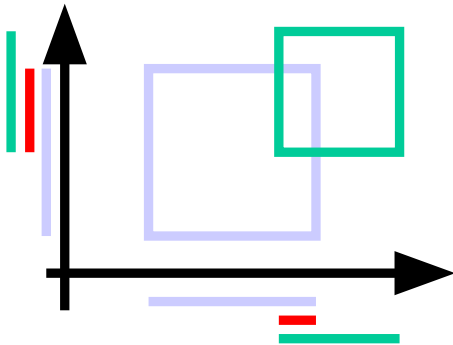


axis-aligned  
bounding  
box

# Overlap Test

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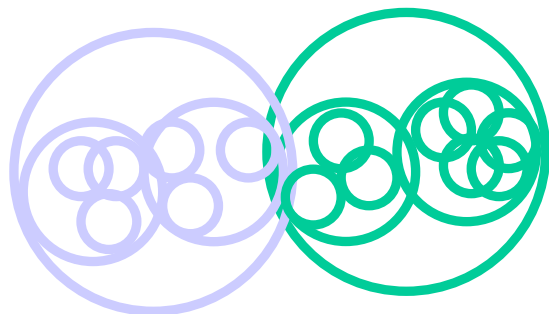
- Overlap between two axis-aligned boxes?
  - Check if the intervals along the 3 dimensions overlap
- Overlap test between two spheres?
  - $D(\text{center}_1, \text{center}_2) < r_1 + r_2$



# General Collision Detection

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- Put a hierarchy around your objects
- Use the fast overlap test recursively
- Handle exact case at the leaves (when necessary)
- More difficult for self-collision (e.g. cloth)
  - Because there is more overlap



# Today

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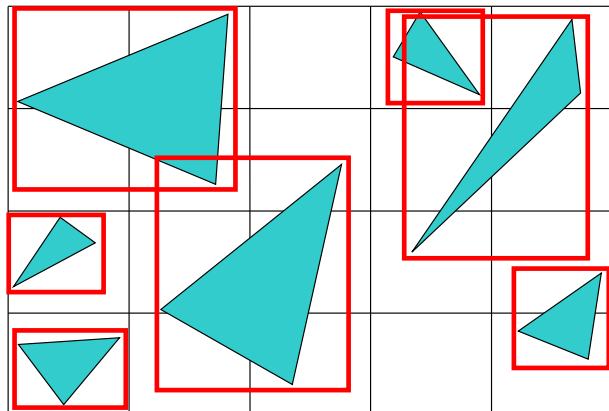
- Readings for Today
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- **Spatial Acceleration Data Structures**
  - Fixed/Uniform/Regular Grid
  - Nested Grid
  - Octree
  - Binary Space Partition
  - K-d tree
  - Bounding Volume Hierarchy
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## Fixed/Uniform/Regular Grid

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- Separate geometry into regions
- Reduces pairwise comparisons
- Primitives that overlap multiple cells?

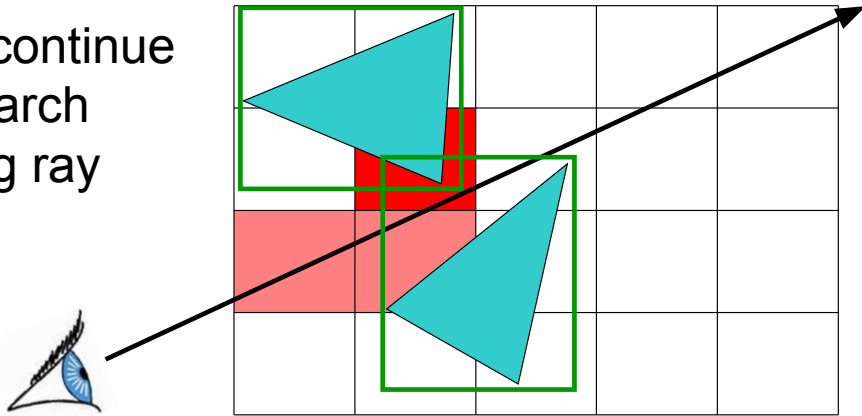
**Insert into multiple cells  
(use pointers)**



## For Each Cell Along a Ray

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- Does the cell contain an intersection?
- Yes: return closest intersection
- No: continue to march along ray



## Fixed/Uniform Grid Discussion

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- Advantages?
  - easy to construct
  - easy to traverse
- Disadvantages?
  - may be only sparsely filled
  - geometry may still be clumped

# Today

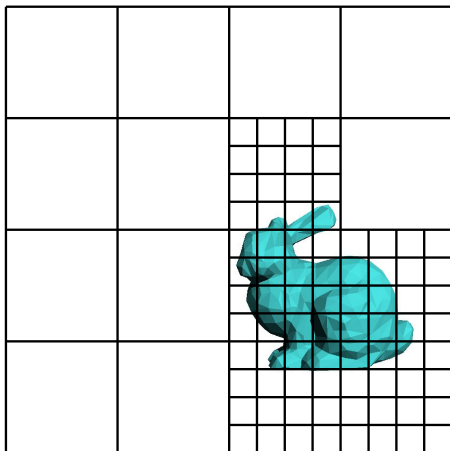
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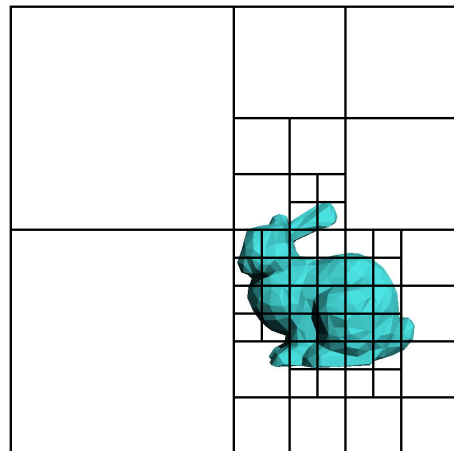
## Adaptive Grids

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- Subdivide until each cell contains no more than  $n$  elements, or maximum depth  $d$  is reached



Nested Grids

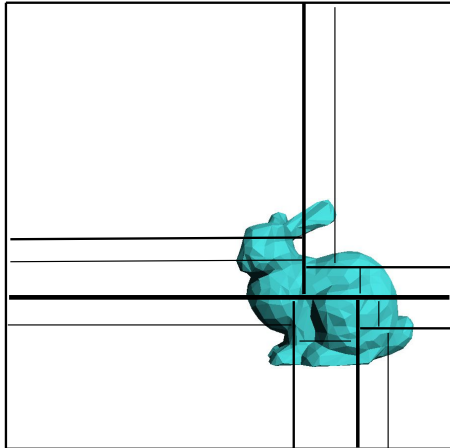


Octree/(Quadtree)

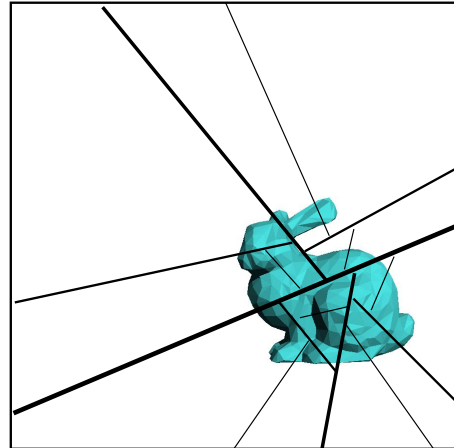
# Adaptive Grids

---

- Subdivide until each cell contains no more than  $n$  elements, or maximum depth  $d$  is reached



K-D Tree

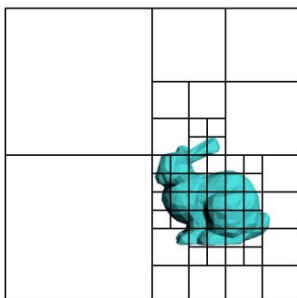


Binary Space Partition (BSP)

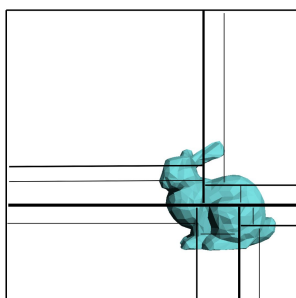
# Variations of Adaptive Grids

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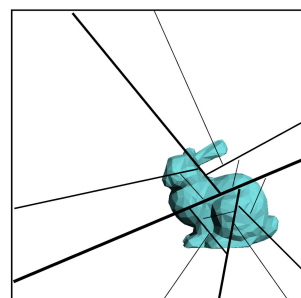
- **When to split?** When a cell contains “lots” of geometry, but has not yet reached the max tree depth
- **Where to split?**
  - Quadtree/Octree: split *every* dimension in half, always axis aligned
  - kd-tree: choose *one* dimension (often the largest dimension) and split it axis aligned (but not necessarily at the midpoint)
  - Binary Space Partition (BSP): choose an *arbitrary* cut plane
- **Which one is best?** *It depends.... Often they are all equally good!*



Quadtree/Octree



kd tree

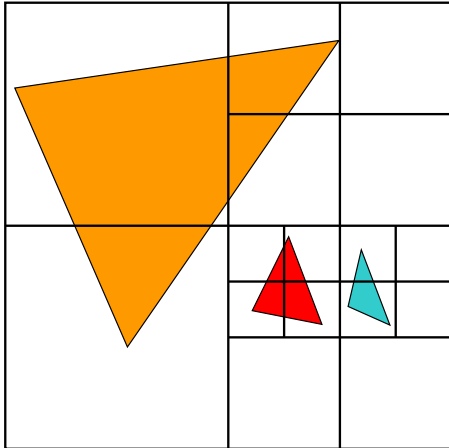


BSP Tree

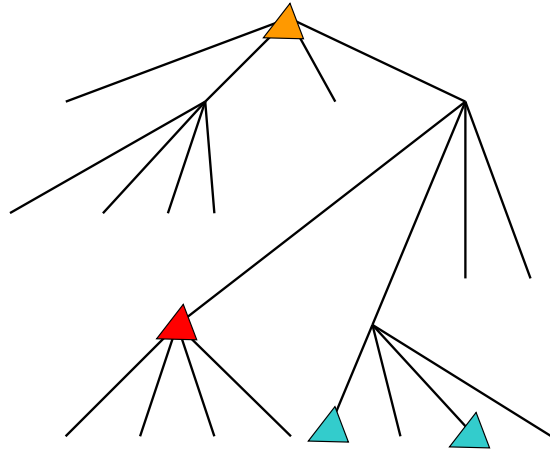
# Primitives in an Adaptive Grid

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- Can live at intermediate levels, or be pushed to lowest level of grid



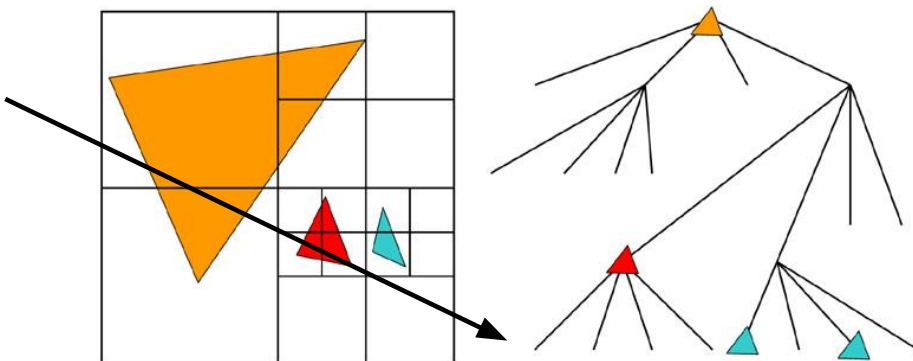
Octree/(Quadtree)



# Adaptive Grid Discussion

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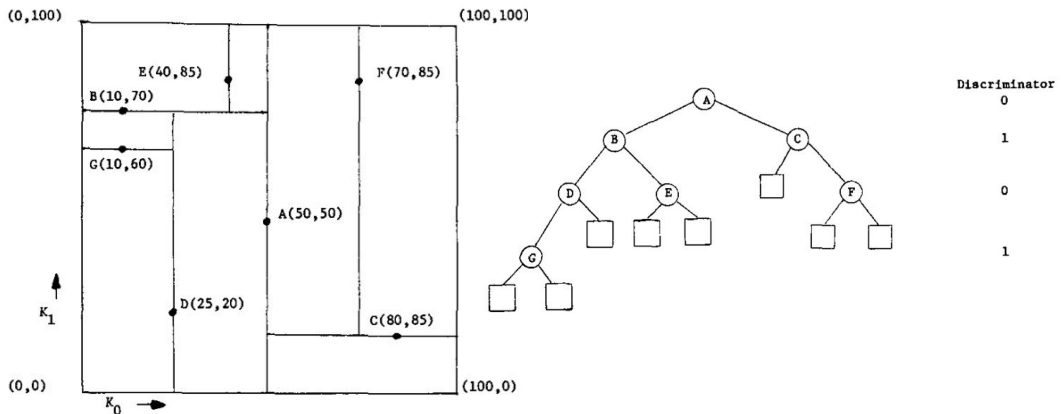
- Advantages?
  - grid complexity matches geometric density
- Disadvantages?
  - more expensive to traverse (binary tree, lots of pointers)





# Early k-d tree paper

- "Multidimensional Binary Search Trees Used for Associative Searching", Bentley, Communications of the ACM, 1975



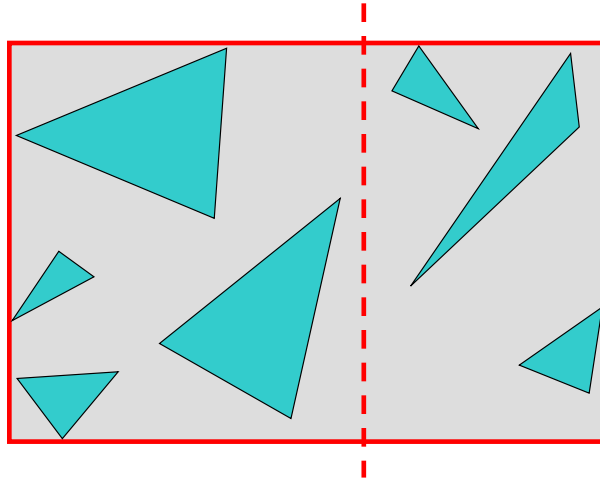
# Today

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# Bounding Volume Hierarchy

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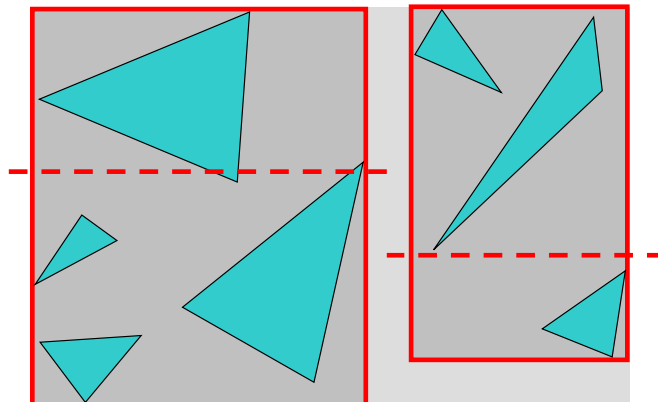
- Find bounding box of objects
- Split objects into two groups
- Recurse



# Bounding Volume Hierarchy

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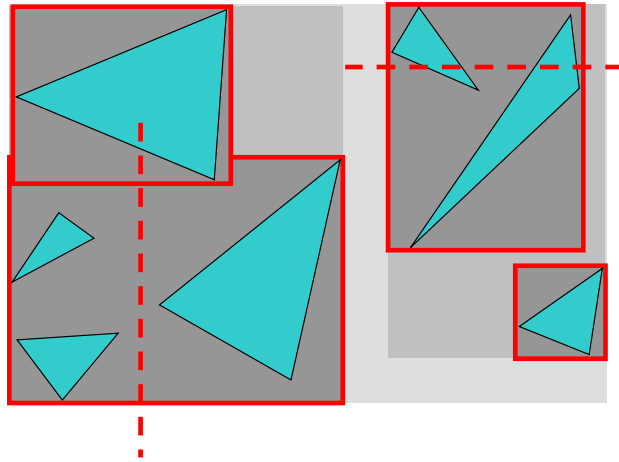
- Find bounding box of objects
- Split objects into two groups
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# Where to split objects?

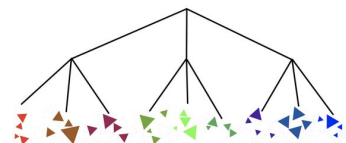
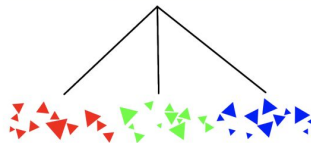
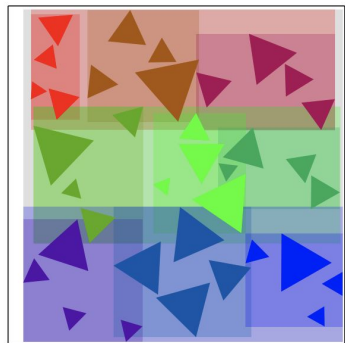
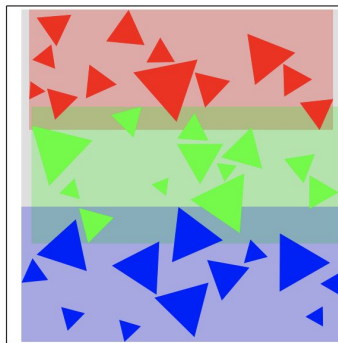
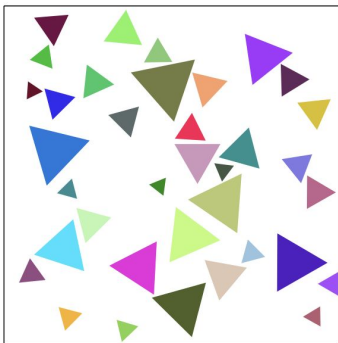
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- At midpoint OR
- Sort, and put half of the objects on each side OR
- Use modeling hierarchy



# Data Structures Homework 8

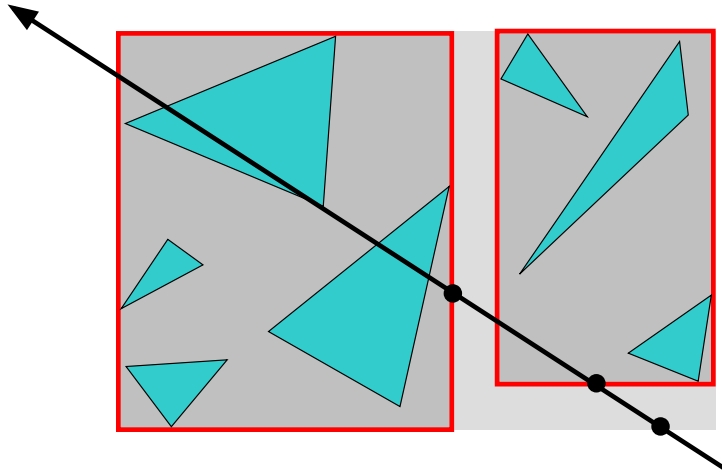
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# Intersection with BVH

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- Check sub-volume with closer intersection first



# Bounding Volume Hierarchy Discussion

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- Advantages
  - easy to construct
  - easy to traverse
  - binary
- Disadvantages
  - may be difficult to choose a good split for a node
  - poor split may result in minimal spatial pruning

# Today

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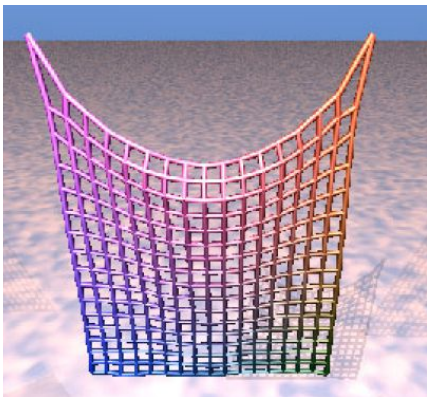
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## Reading for Next Time:

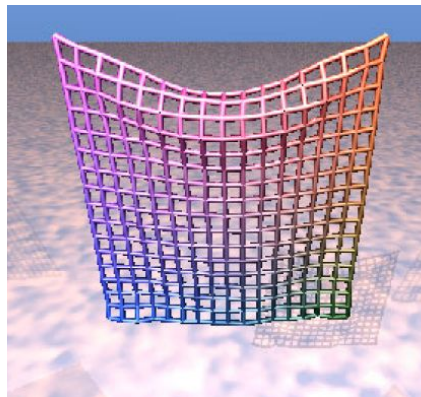
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Everyone should read this  
(simple cloth model used in HW2)

- “Deformation Constraints in a Mass-Spring Model to Describe Rigid Cloth Behavior”, Provot, 1995.



Simple mass-spring system



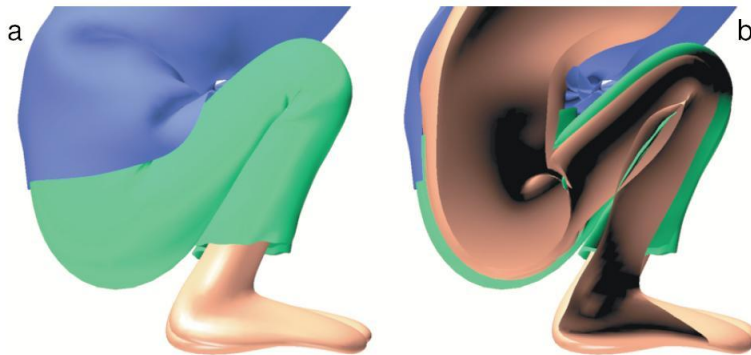
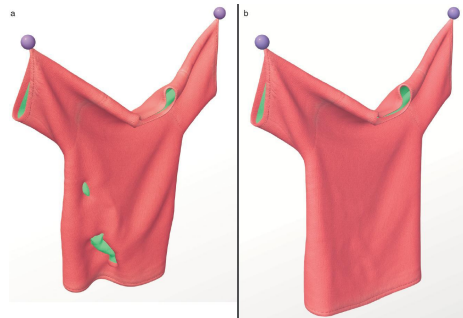
Improved solution

# Cloth in Practice (w/ Animation)

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## OPTIONAL READING FOR NEXT TIME

- Baraff, Witkin & Kass  
*Untangling Cloth*  
SIGGRAPH 2003



## Today

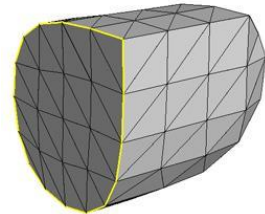
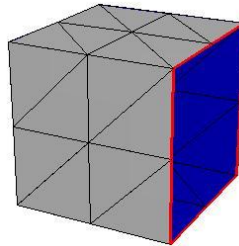
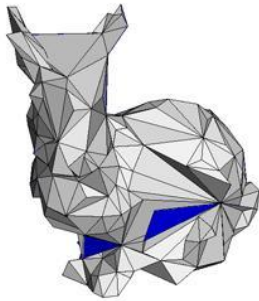
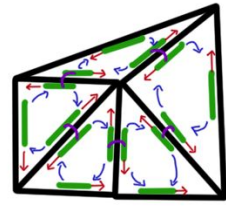
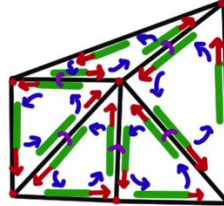
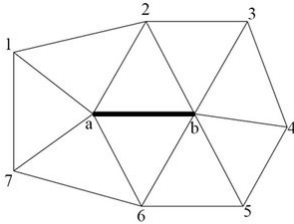
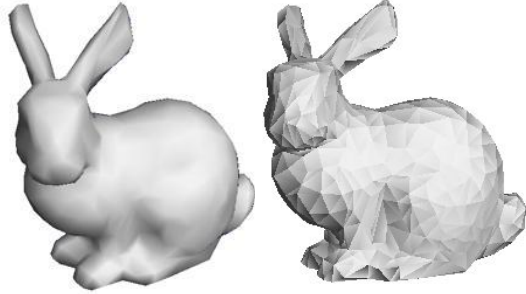
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# Homework 1:

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- Questions/Comments?



# Pop Worksheet!

---

Sketch the polygonal mesh after performing 2 iterations of  
st  
If  
st

