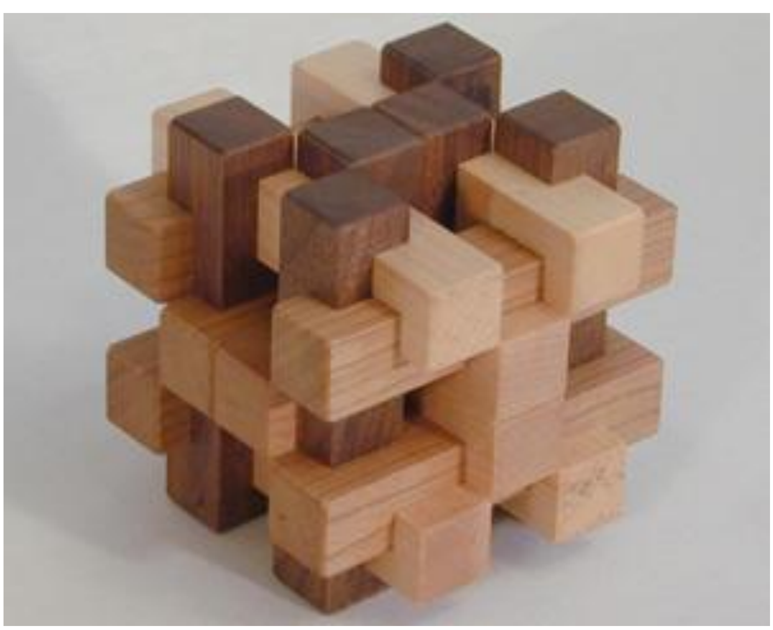
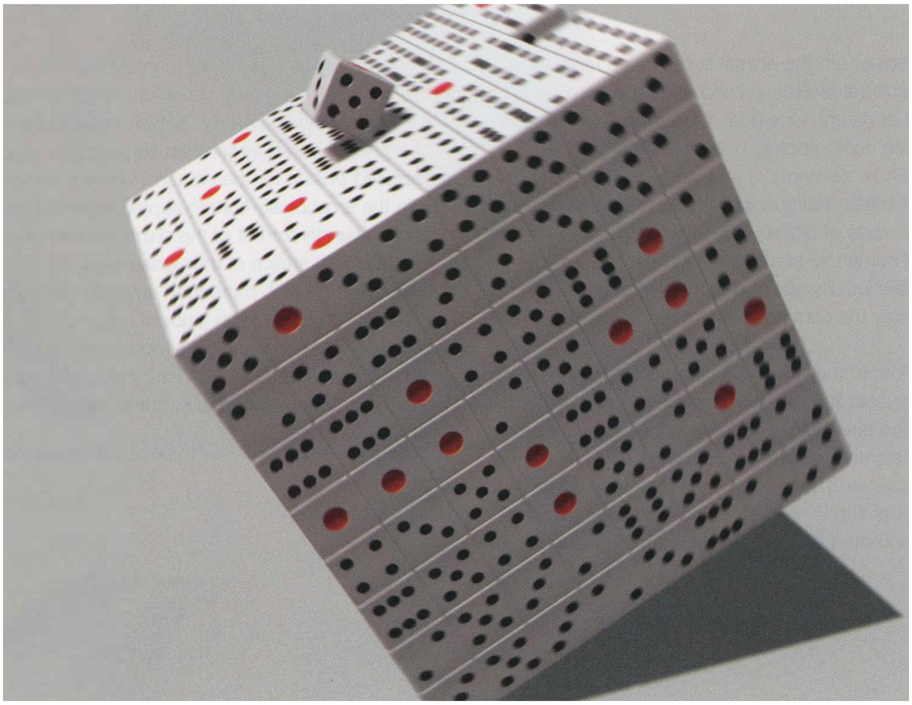
Rigid Body Dynamics, Collision Response, & Deformation

Burr, Justin Legakis ~1999



18 Piece Burr, Bill Cutler Puzzles

Dice, Hitoshi Akayama, SIGGRAPH 2005



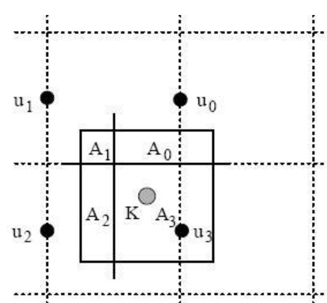
Last Time?

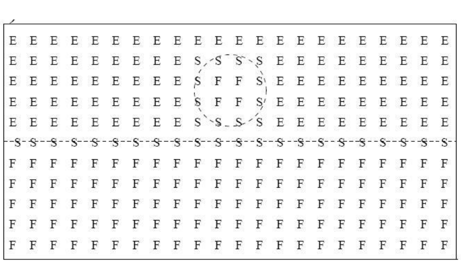
- Navier-Stokes Equations
- Conservation of Momentum & Mass
- Incompressible Flow

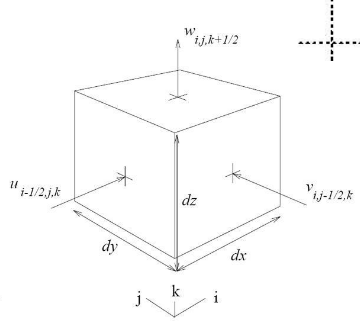
$$\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} = -\frac{\partial p}{\partial x} + g_x + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

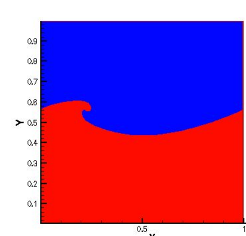
$$\frac{\partial v}{\partial t} + \frac{\partial vu}{\partial x} + \frac{\partial v^2}{\partial y} + \frac{\partial vw}{\partial z} = -\frac{\partial p}{\partial y} + g_y + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$\frac{\partial w}{\partial t} + \frac{\partial wu}{\partial x} + \frac{\partial wv}{\partial y} + \frac{\partial w^2}{\partial z} = -\frac{\partial p}{\partial z} + g_z + \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$









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- Flow Simulation Discussion
- Worksheet
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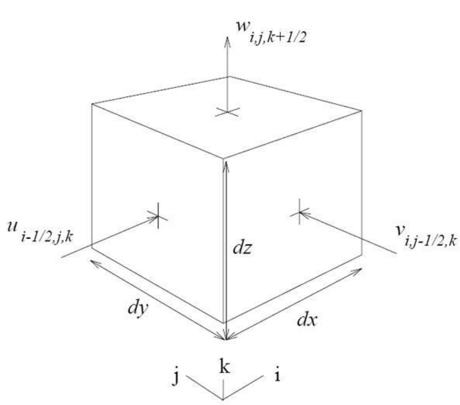
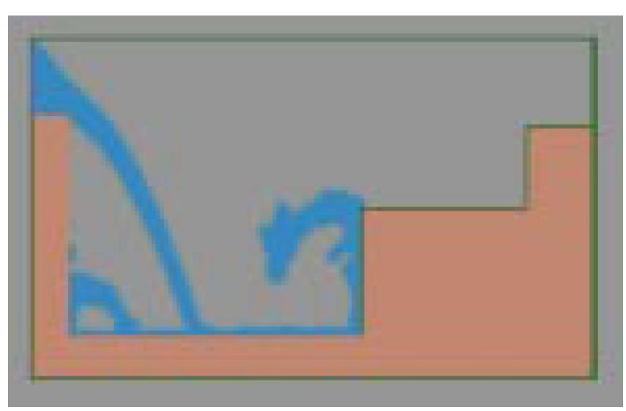
Reading for Today

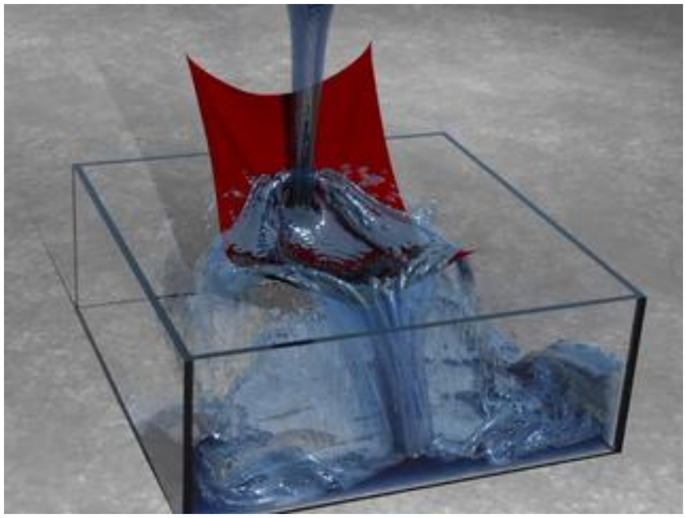
Everyone should read this
(simple fluid model used in HW2)

- "Realistic Animation of Liquids",
  Foster & Metaxas, 1996



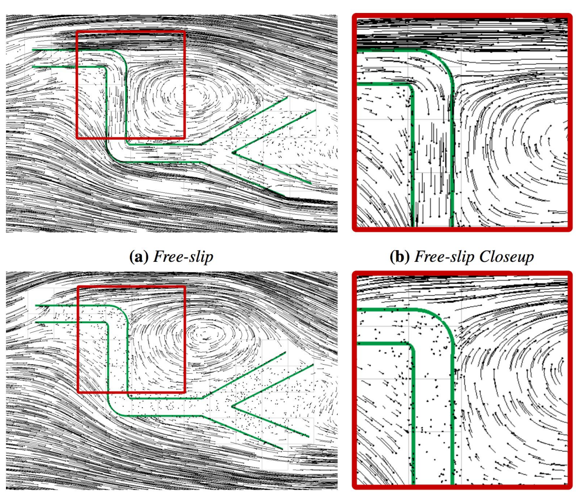


Optional Reading



"Coupling Water and Smoke to Thin Deformable and Rigid Shells", Guendelman, Selle, Losasso, & Fedkiw, SIGGRAPH 2005.

Optional Reading



(a) *Free-slip* **(b)** *Free-slip Closeup*

(c) *No-slip* **(d)** *No-slip Closeup*

"Preserving Geometry and Topology for Fluid Flows with Thin Obstacles and Narrow Gaps"
Azevedo, Batty, & Oliveira, SIGGRAPH 2016

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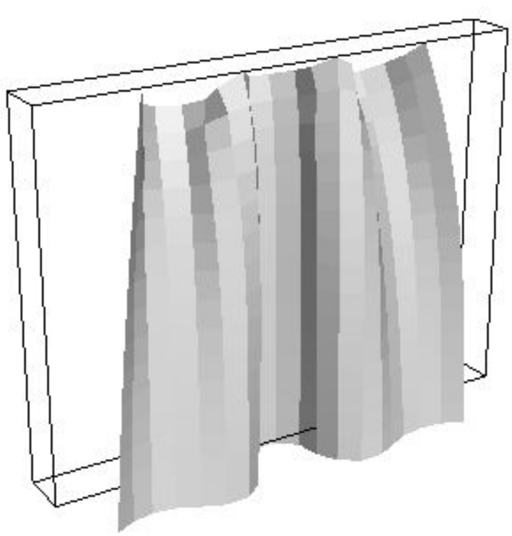
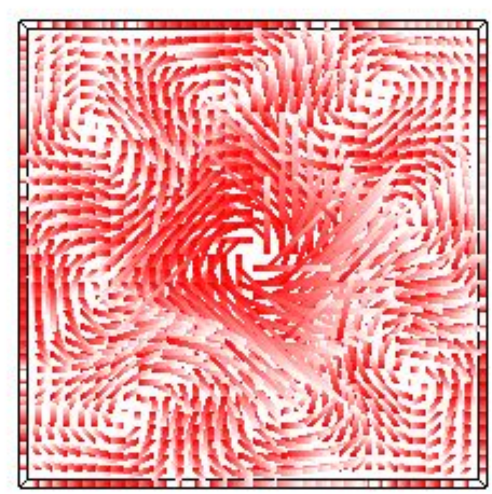
HW2: Cloth & Fluid Simulation

FYI: Deadlines in this course *are on the calendar*

(please report mismatched dates on Submitty)





At each Timestep:

- Identify which cells are Empty, Full, or on the Surface
- Compute new velocities
- Adjust the velocities to maintain an incompressible flow
- Move the particles
  - Interpolate the velocities at the faces
- Render the geometry and repeat!

Adjusting the Velocities

- Calculate the *divergence* of the cell (the extra in/out flow)
- The divergence is used to update the *pressure* within the cell
- Adjust each face velocity uniformly to bring the divergence to zero
- Iterate across the entire grid until divergence is < ε

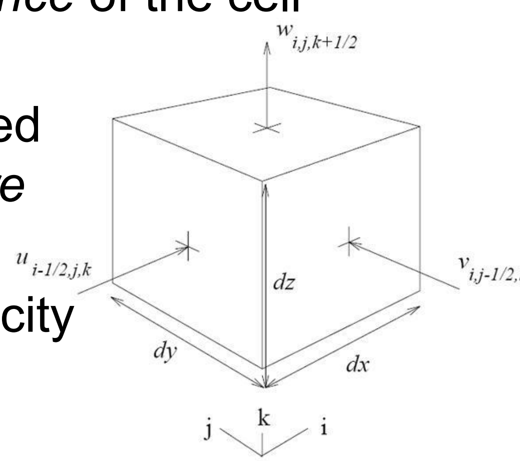
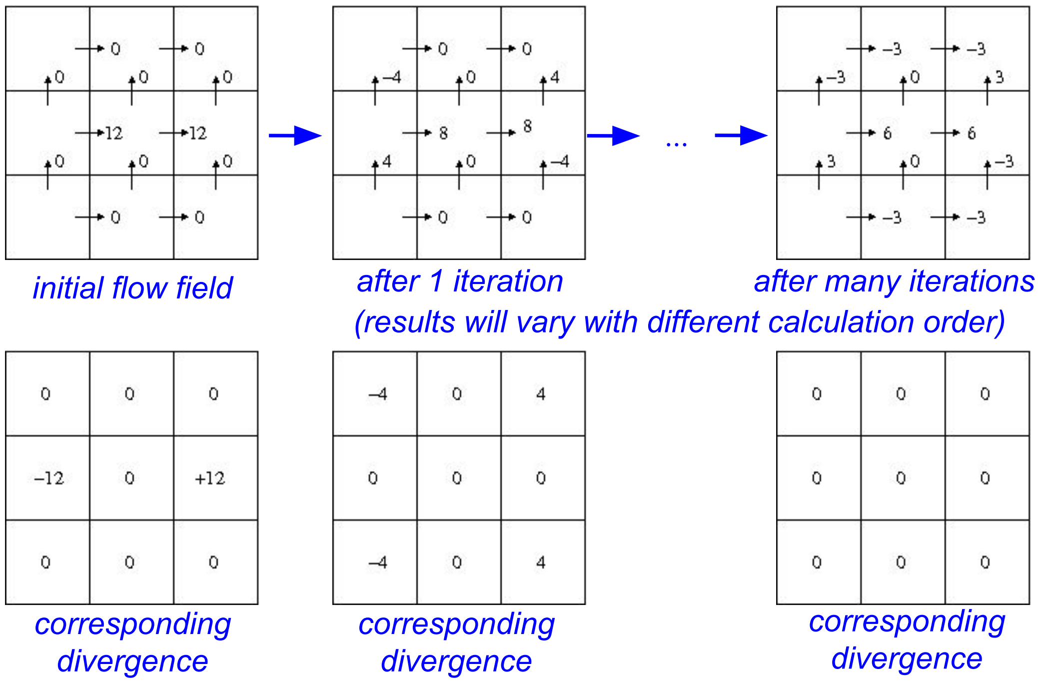


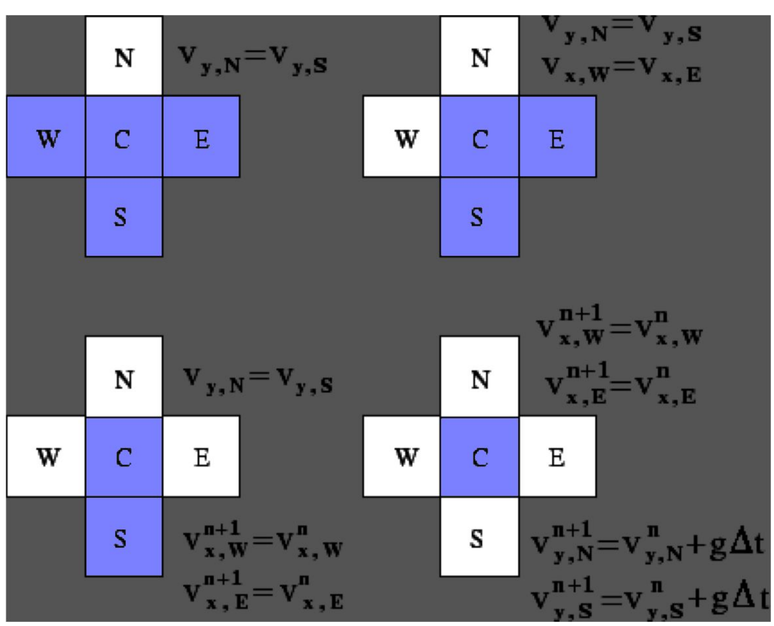
Image from Foster & Metaxas, 1996

Calculating/Eliminating Divergence



Handing Free Surface with MAC

- Divergence in surface cells:
  - Is divided equally amongst neighboring empty cells
  - Or other similar strategies?
- Zero out the divergence & pressure in empty cells



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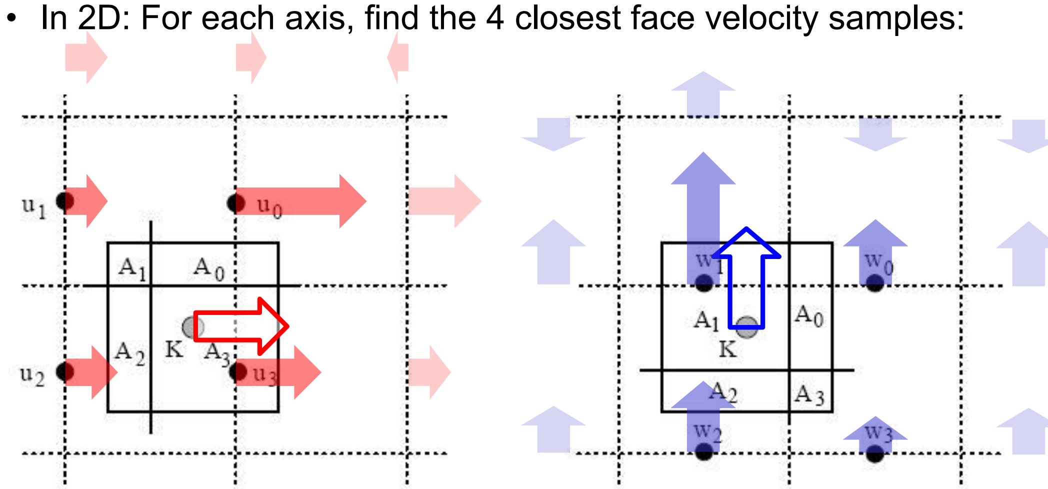
Velocity Interpolation

Original image from Foster & Metaxas, 1996

- In 2D: For each axis, find the 4 closest face velocity samples:

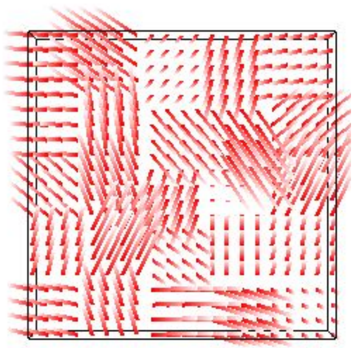


$u_k = A_0u_0 + A_1u_1 + A_2u_2 + A_3u_3$ $w_k = A_0w_0 + A_1w_1 + A_2w_2 + A_3u_3$

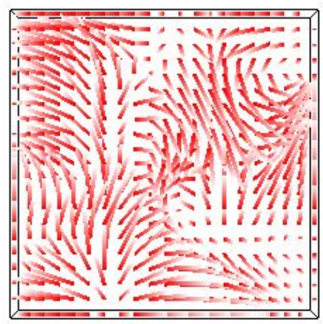
- (In 3D… Find 8 closest face velocities in each dimension)

Correct Velocity Interpolation

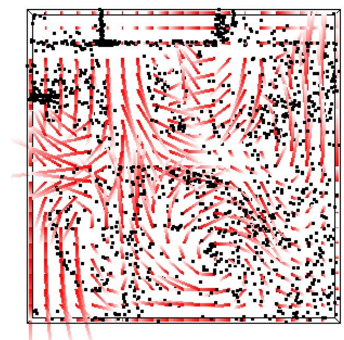
- NOTE: The complete implementation isn't particularly elegant… Storing velocities at face midpoints (req'd for conservation of mass) makes the index math messy!



No interpolation (just use the left/bottom face velocity)
Note the discontinuities in velocity at cell boundaries



Correct Interpolation
Note that the velocity perpendicular to the outer box is zero



Buggy Interpolation
Note the clumping particles, and the discontinuities at some of the cell borders (& particles might escape the box!)

Pop Worksheet!

Det
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2D
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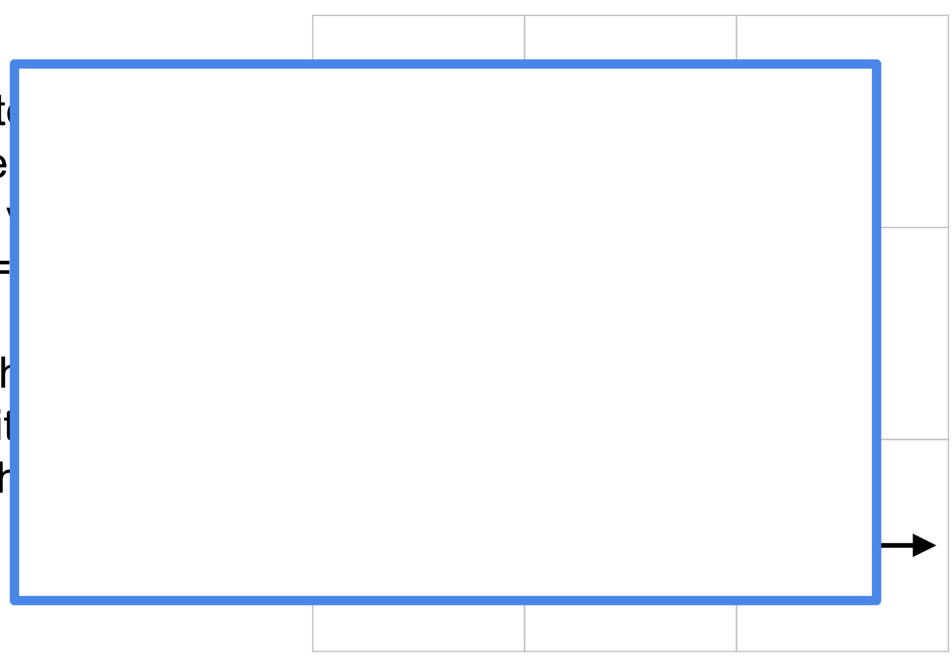

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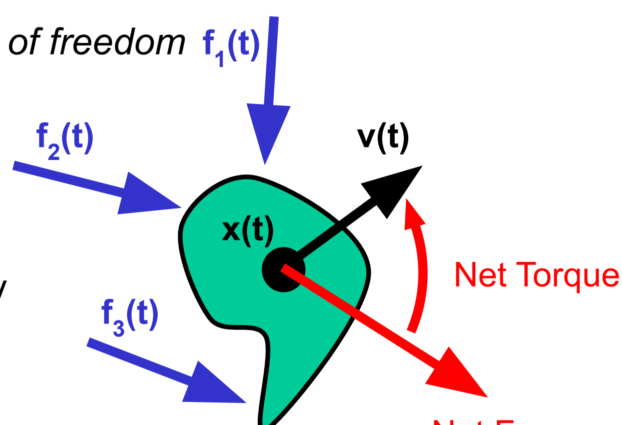
Rigid Body Dynamics

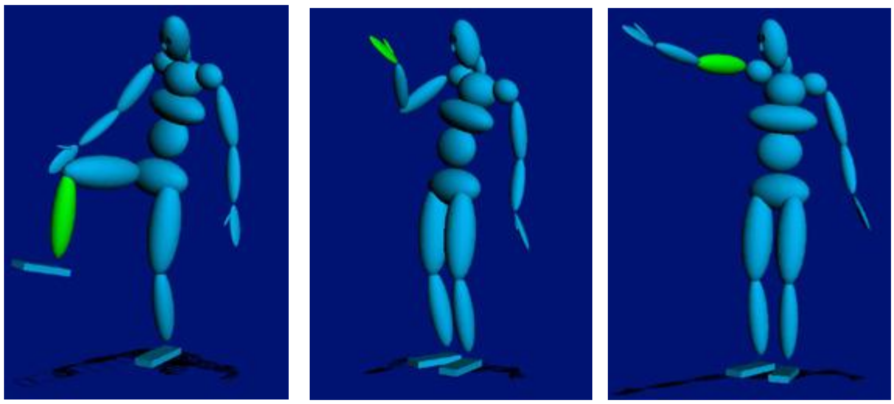
- How do we simulate this object's motion over time?
- We could discretize the object into many particles…
  - But a rigid body does *not* deform
  - Only a few *degrees of freedom*
- Instead, we use only one particle at the center of mass
  - Body has velocity *and* angular velocity
  - Compute net force & net torque



Nice Reference Material: http://www.pixar.com/companyinfo/research/pbm2001/

Degree of Freedom (DOF)

- Rotations:

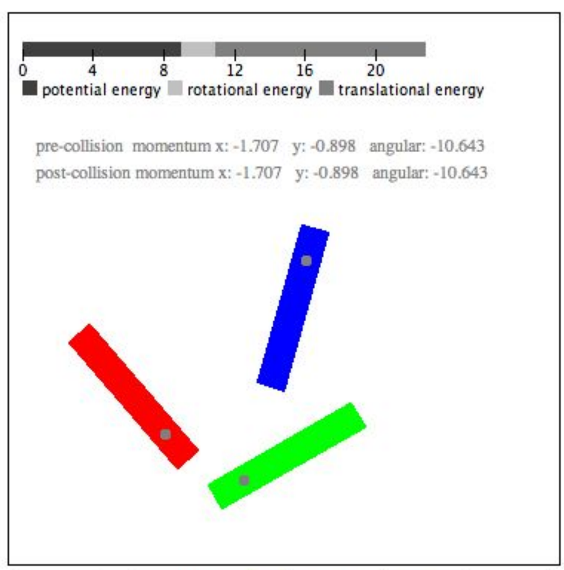


1 DOF: knee　　2 DOF: wrist　　3 DOF: arm

- Translations count too… ➔ 6 DOF

Energy & Rigid Body Collisions

- Total Energy = Kinetic Energy + Potential Energy + Rotational Energy
- Total Energy stays constant if there is no damping and no friction
- Rotational Energy is constant between collisions



http://www.myphysicslab.com/collision.html

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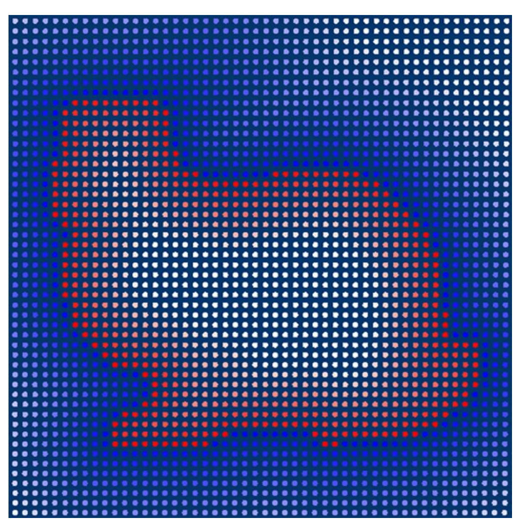
Implicit Surfaces

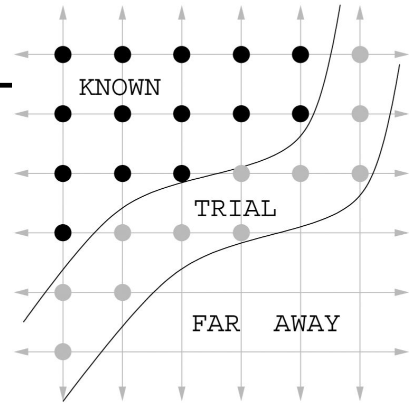
- For a sphere:
  $H(x,y,z) = x^2 + y^2 + z^2 - r^2$



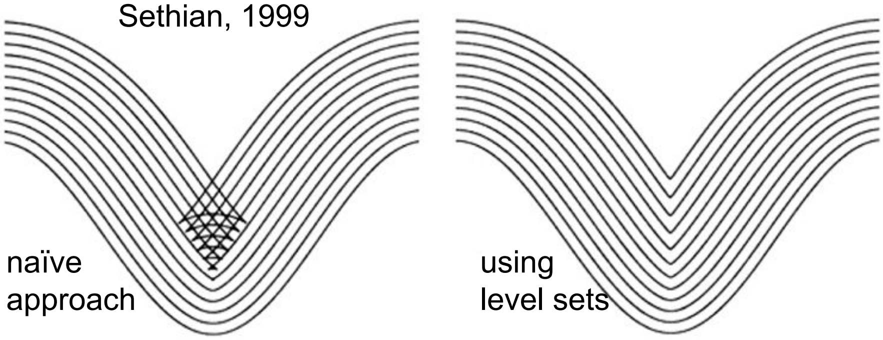
- If $H(x,y,z) = 0$, on surface
- If $H(x,y,z) > 0$, outside surface
- If $H(x,y,z) < 0$, inside surface

Level Sets

- Efficient method for computing signed distance field



Level Set Methods and Fast Marching Methods, Sethian, 1999



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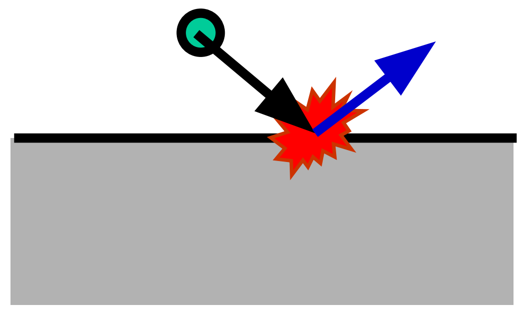
Collisions

- Detection
- Response
- Overshooting problem
  (when we enter the solid)

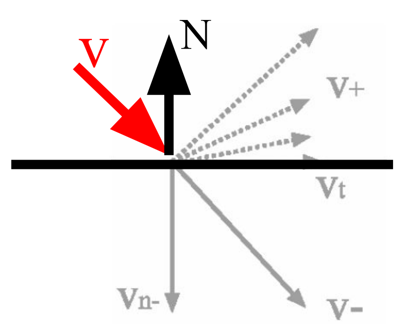


Collision Response

- tangential velocity v_t unchanged
- normal velocity v_n reflects:

$$v = v_t + v_n$$

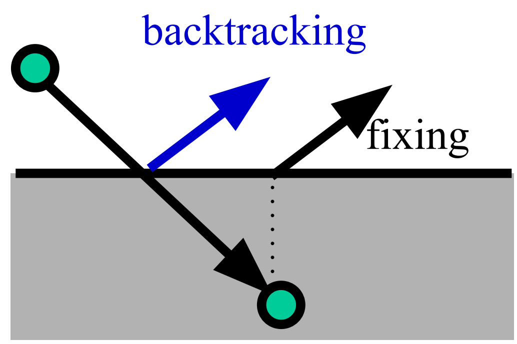
$$v \leftarrow v_t - \varepsilon v_n$$



- coefficient of restitution
  - 1 for elastic
  - 0 for plastic
- change of velocity = *-(1+є)v*
- change of momentum *Impulse = -m(1+є)v*

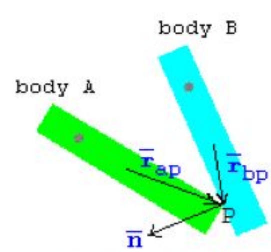
Collisions - Overshooting

- Usually, we detect collision when it's too late: we're already inside
- Solutions: back up
  - Compute intersection point
  - Compute response there
  - Advance for remaining fractional time step
- Other solution: Quick and dirty fixup
  - Just project back to object closest point



Collision Between Two Objects

- Suppose a vertex on body A is colliding into an edge of body B at point P. Define the following variables:



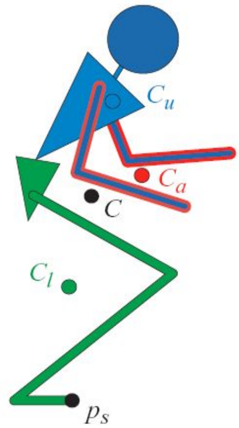
vectors involved in collision

m_a, m_b = mass of bodies A, B
$\bar{r}_{ap}$ = distance vector from center of mass of body A to point P
$\bar{r}_{bp}$ = distance vector from center of mass of body B to point P
ω_{a1}, ω_{b1} = initial pre-collision angular velocity of bodies A, B
ω_{a2}, ω_{b2} = final post-collision angular velocity of bodies A, B
$\bar{v}_{a1}, \bar{v}_{b1}$ = initial pre-collision velocities of center of mass bodies A, B
$\bar{v}_{a2}, \bar{v}_{b2}$ = final post-collision velocities of center of mass bodies A, B
$\bar{v}_{ap1}$ = initial pre-collision velocity of impact point on body A
$\bar{v}_{bp1}$ = initial pre-collision velocity of impact point on body B
$\bar{n}$ = normal (perpendicular) vector to edge of body B
e = elasticity (0 = inelastic, 1 = perfectly elastic)

http://www.myphysicslab.com/collision.html

Center of Mass & Moment of Inertia

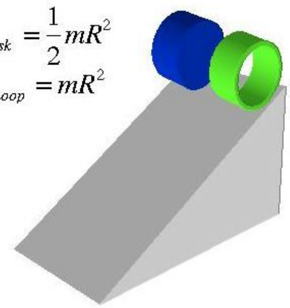
- Center of Mass: mean location of all mass in the system
- Moment of Inertia: a measure of an object's resistance to changes to its rotation
- If a solid cylinder & a hollow tube have the ame radius & the same mass, which will reach the bottom of the ramp first?



Liu & Popović

http://en.wikipedia.org/wiki/Fosbury_Flop

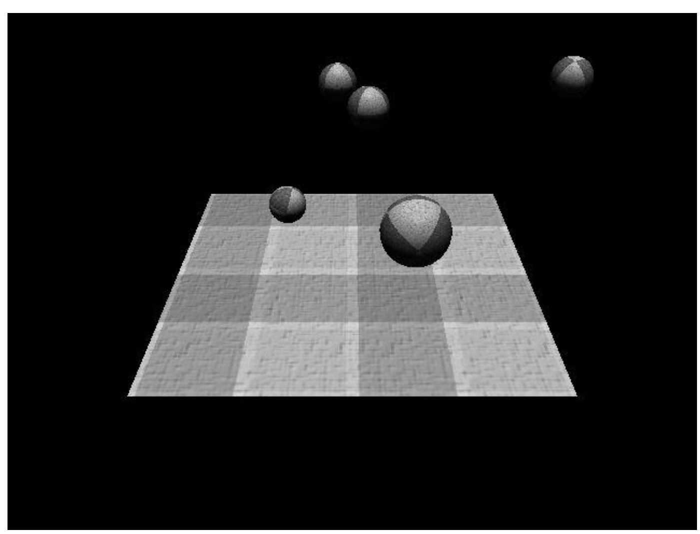
$$I_{disk} = \frac{1}{2} mR^2$$
$$I_{hoop} = mR^2$$



http://solomon.physics.sc.edu/~tedeschi/demo/demo12.html
http://hyperphysics.phy-astr.gsu.edu/hbase/hoocyl2.html

Rigid Body Dynamics

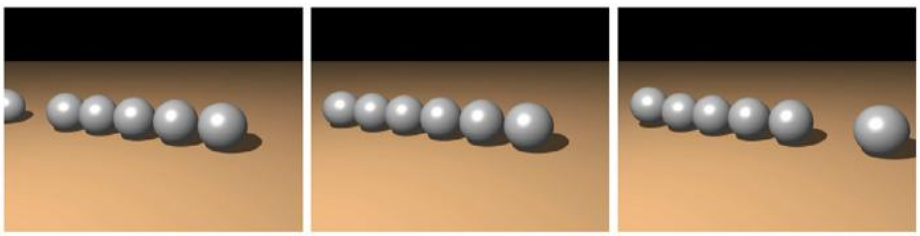
- Physics
  - Velocity
  - Acceleration
  - Angular Momentum
- Collisions
- Friction



from: Darren Lewis
http://www-cs-students.stanford.edu/~dalewis/cs448a/rigidbody.html

Advanced Collisions

- What about friction?
- What if the contact between two objects is not a single point?
- What if more than two objects collide simultaneously?

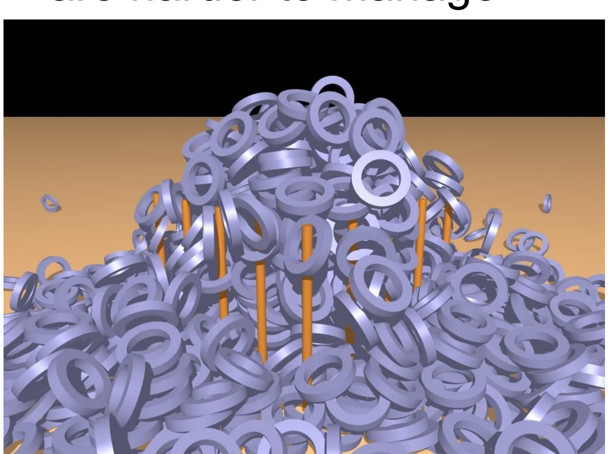


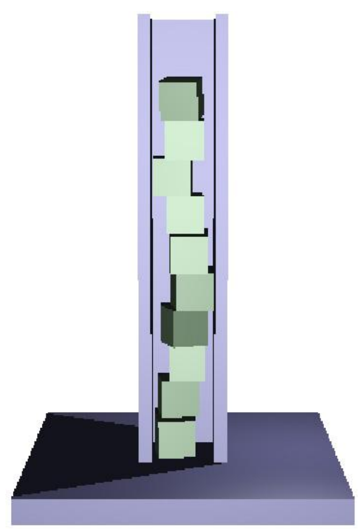
Guendelman, Bridson & Fedkiw
Nonconvex Rigid Bodies with Stacking
SIGGRAPH 2003

Resting Collisions

Victor J. Milenkovic & Harald Schmidl
Optimization-Based Animation
SIGGRAPH 2001

- We know how to simulate bouncing really well
- But resting collisions are harder to manage





Guendelman, Bridson & Fedkiw
Nonconvex Rigid Bodies with Stacking, SIGGRAPH 2003

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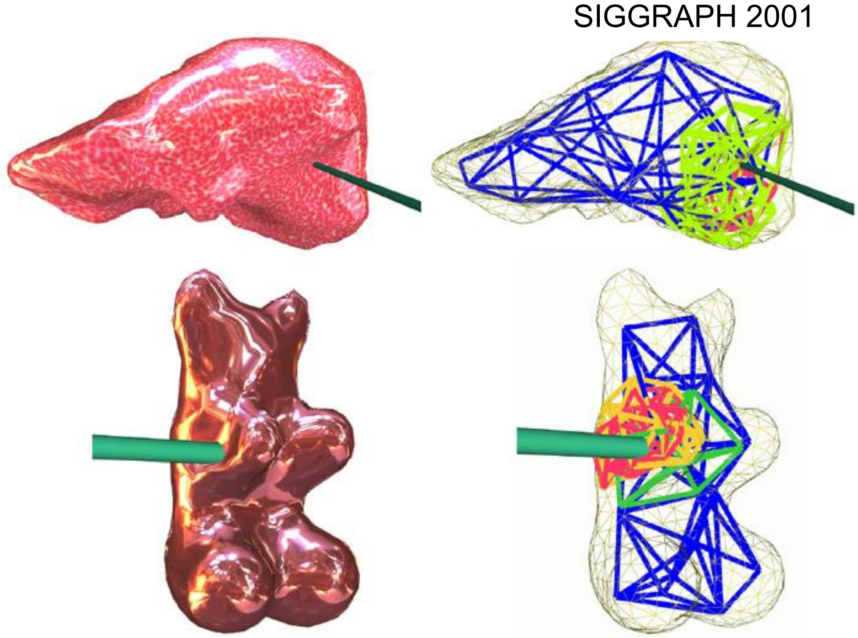
Deformation & Level of Detail

Gilles Debunne , Mathieu Desbrun, Marie-Paule Cani, & Alan H. Barr
Dynamic Real-Time Deformations using Space & Time Adaptive Sampling
SIGGRAPH 2001



Simulation of Non-Rigid Objects

- We modeled string & cloth using mass-spring systems. Can we do the same?
- Yes…
- But a more physically accurate model uses *volumetric elements:*

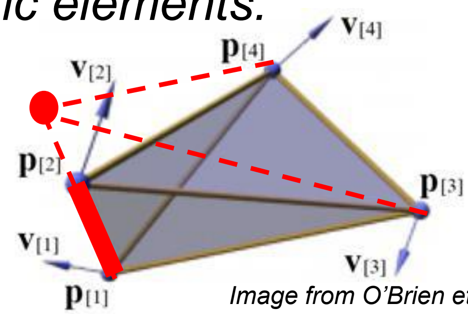
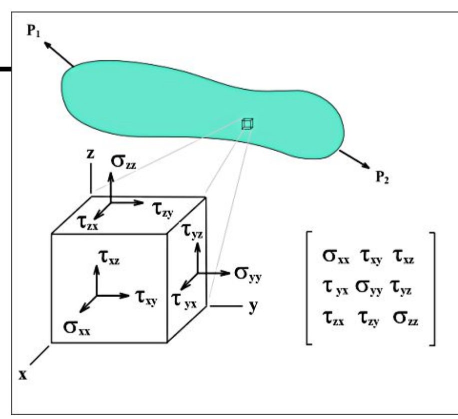


Image from O'Brien et al. 1999

Strain & Stress



http://en.wikipedia.org/wiki/Image:Stress_tensor.png

- Stress
  - the internal distribution of forces within a body that balance and react to the loads applied to it
  - *normal stress* & *shear stress*
- Strain
  - material deformation caused by stress.
  - measured by the change in length of a line or by the change in angle between two lines

$$\varepsilon = \frac{\Delta_l}{l_0}$$

Finite Element Method

- To solve the continuous problem (deformation of all points of the object)
  - Discretize the problem
  - Express the interrelationship
  - Solve a big linear system
- More principled than Mass-Spring

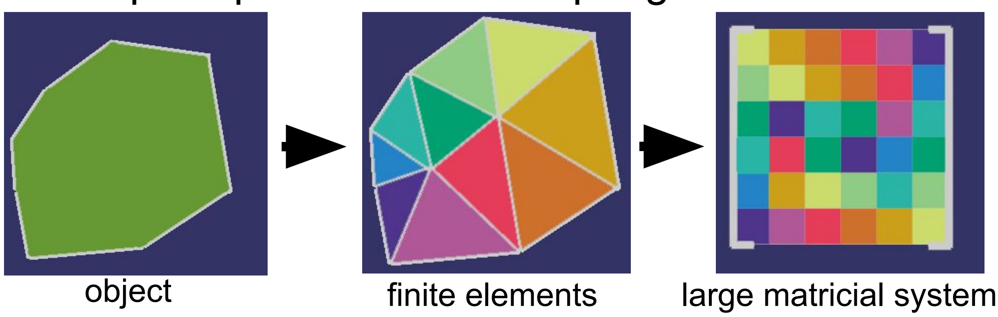


Diagram from Debunne et al. 2001

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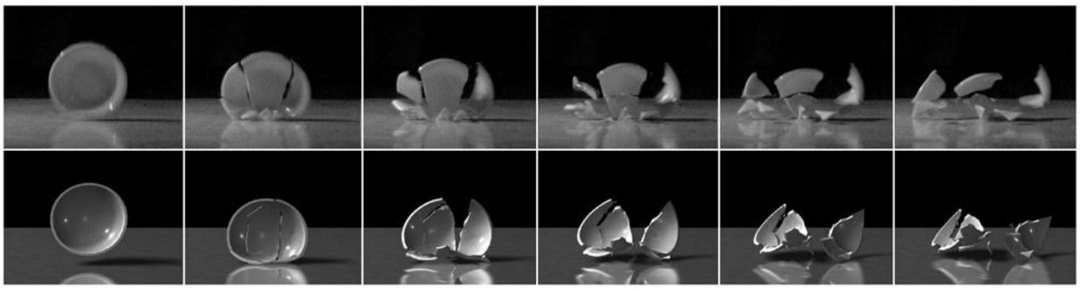
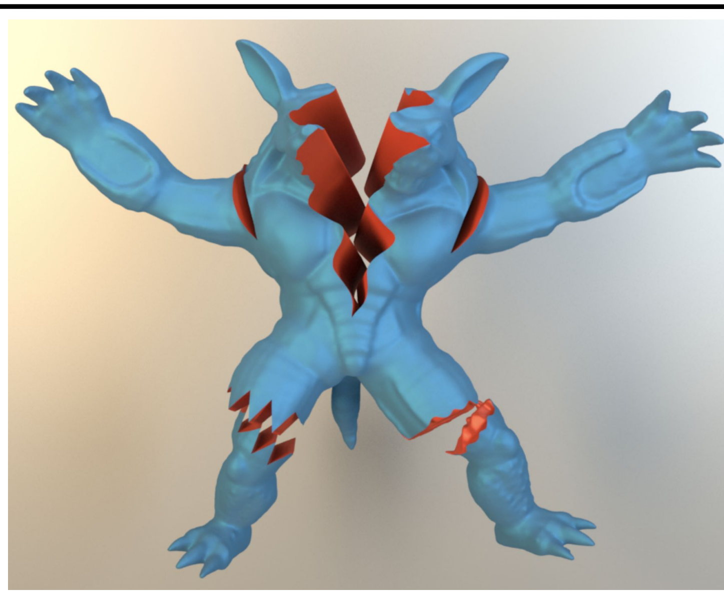
Readings for Tuesday… *(pick one)*

- James O'Brien & Jessica Hodgins "*Graphical Modeling and Animation of Brittle Fracture*" SIGGRAPH 1999.



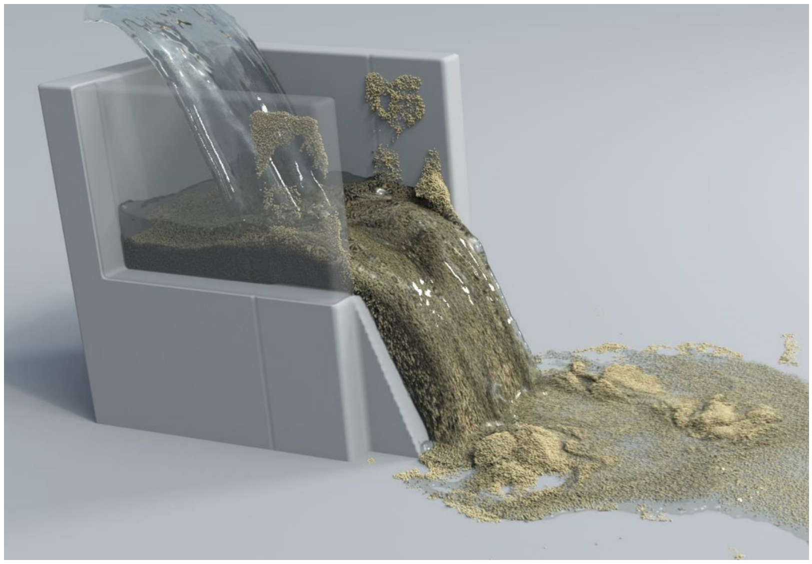
- Fracture threshhold
- Remeshing
  - need connectivity info!
- Material properties
- Parameter tuning

Readings for Tuesday… *(pick one)*



"Robust eXtended Finite Elements for Complex Cutting of Deformables", Koschier, Bender, & Thuerey, SIGGRAPH 2017

Readings for Tuesday… *(pick one)*



"Multi-species simulation of porous sand and water mixtures", Pradhana, Gast, Klar, Fu, Teran, Jiang, and Museth, SIGGRAPH 2017.