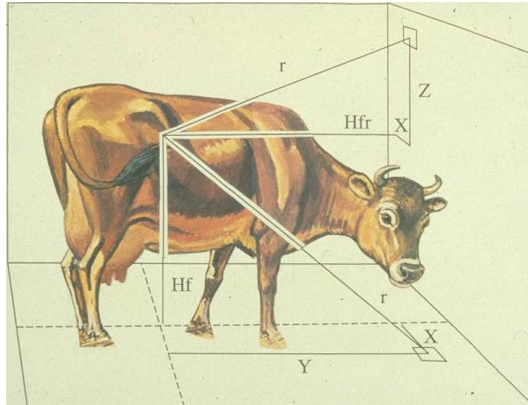
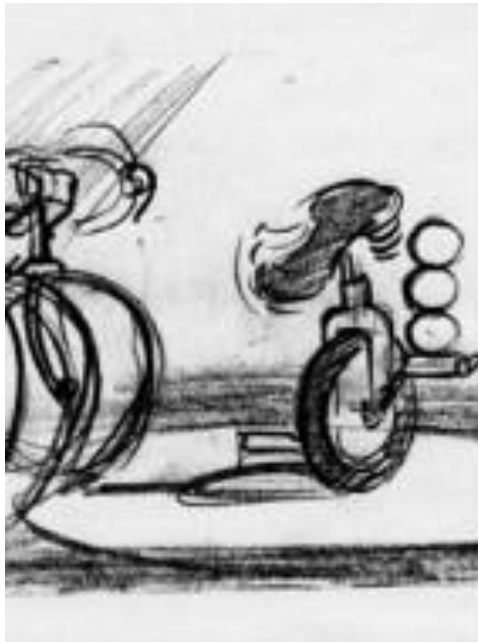


# Local vs. Global Illumination & Radiosity



An early application of radiative heat transfer in stables.

## *Red's Dream*, Pixar, 1987



# Announcement: Quiz 1

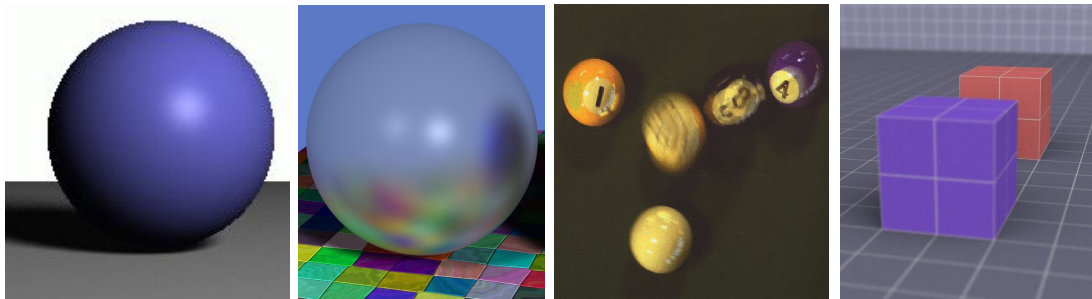
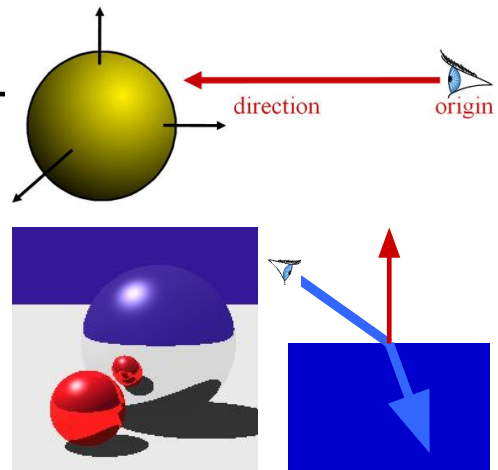
---

- On Submittity
  - Tuesday (March 9th), during class (2:30-4:20pm)
  - Short answer & pencil/digital sketching/upload
  - Let me know ASAP if you have a network outage or other personal situation
- Closed book / no internet / no collaboration
  - one double-sided 8.5"x11" sheet of notes allowed
- Practice Problems (2017 quiz) on the calendar
- Coverage:
  - Lecture and assigned readings thru Lecture 10
  - When there was a choice of papers: you are responsible for having read one paper per lecture,
  - Worksheets thru Lecture 10
  - Homeworks 0, 1, & 2

## Last Time?

---

- Ray Casting & Ray-Object Intersection
- Recursive Ray Tracing
- Distributed Ray Tracing



# Today

---

- Paper for Today: Distributed Ray Tracing
- Local Illumination
- Why is Global Illumination Important?
- Radiosity Matrix
- Calculating the Form Factors
- Advanced Radiosity
- Worksheet

# Reading for Today

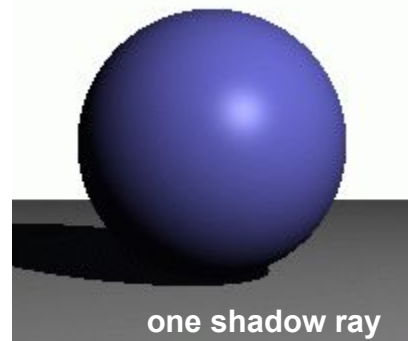
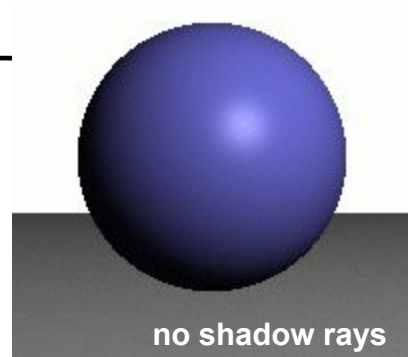
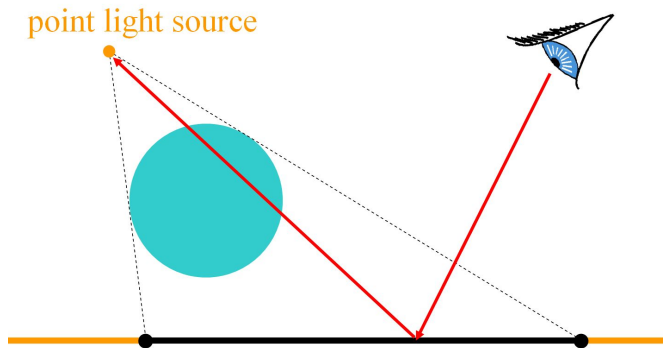
---

- "Distributed Ray Tracing", Cook, Porter, & Carpenter, SIGGRAPH 1984.



# Shadows

- one shadow ray per intersection per point light source



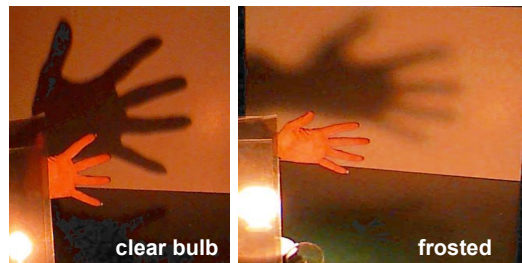
# Shadows & Light Sources



[http://3media.initialized.org/photos/2000-10-18/index\\_gall.htm](http://3media.initialized.org/photos/2000-10-18/index_gall.htm)



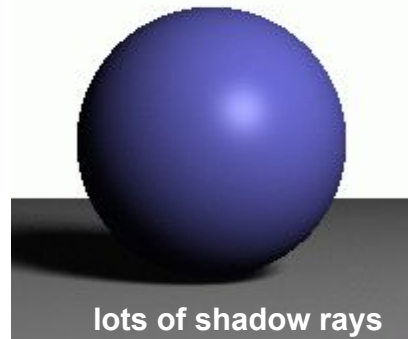
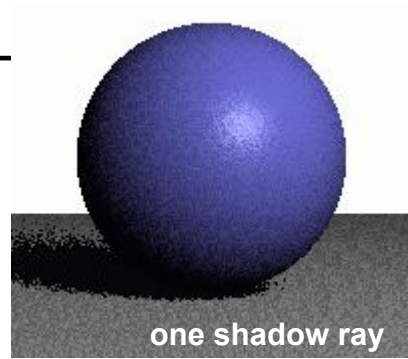
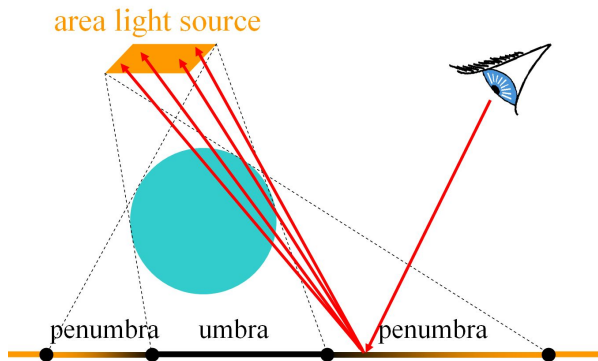
<http://www.davidfay.com/index.php>



<http://www.pa.uky.edu/~sciworks/light/preview/bull2.1.ttc1>

# Soft Shadows

- multiple shadow rays to sample area light source



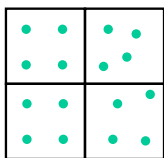
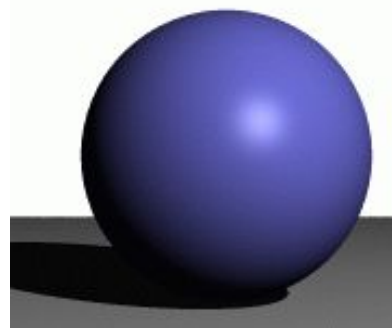
# Antialiasing – Supersampling

- multiple rays per pixel

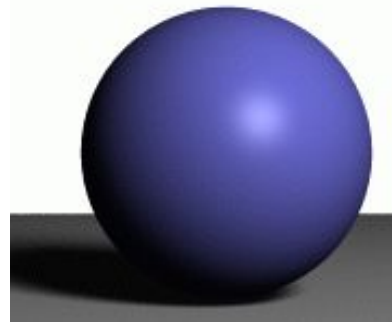
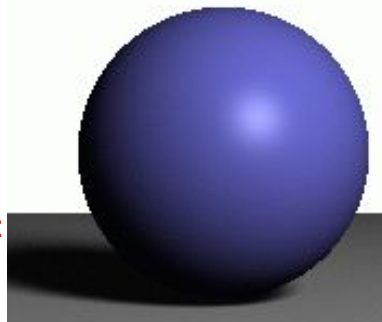
point light



w/ antialiasing

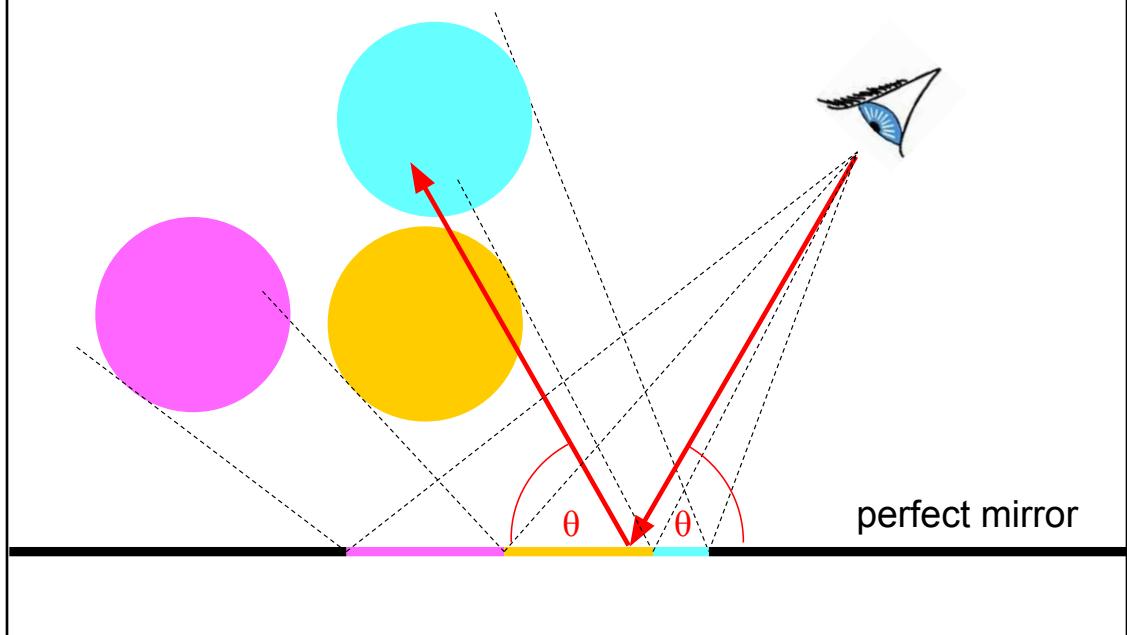


area light



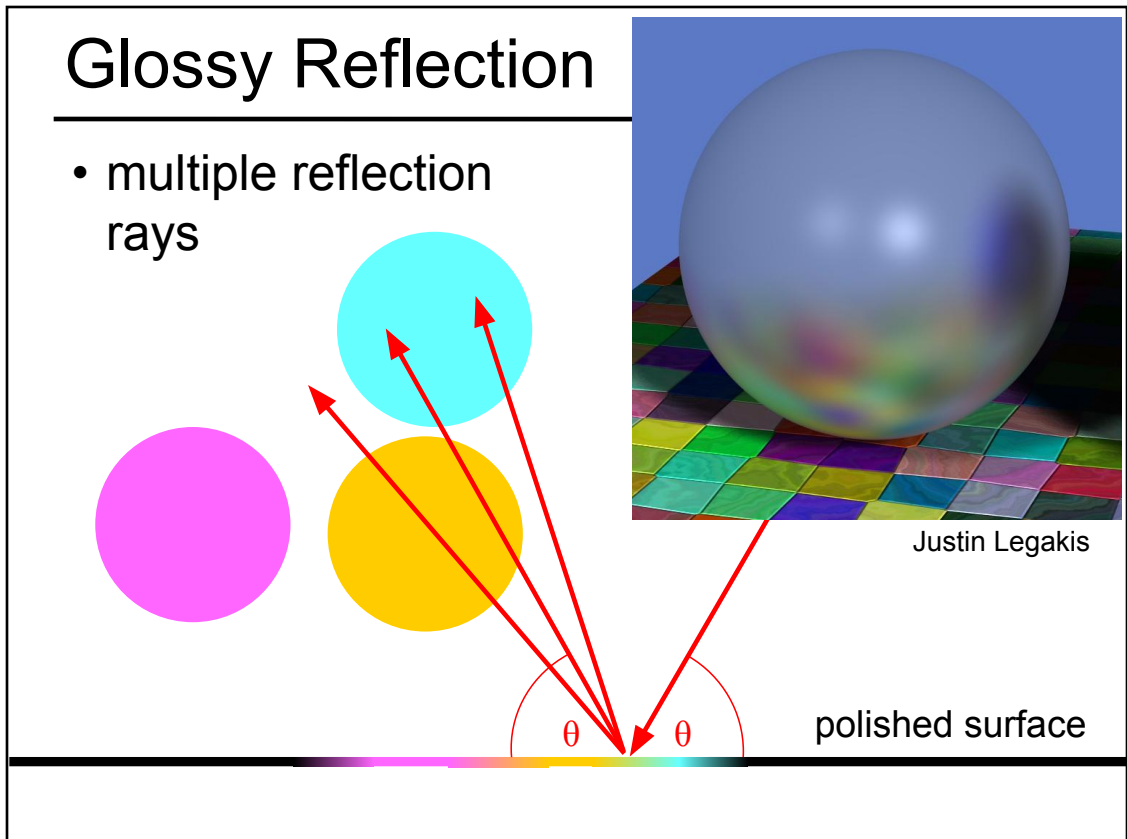
# Reflection

- one reflection ray per intersection



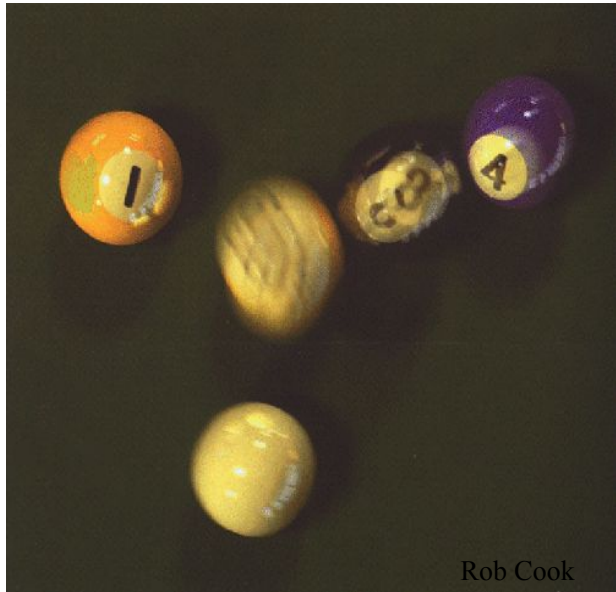
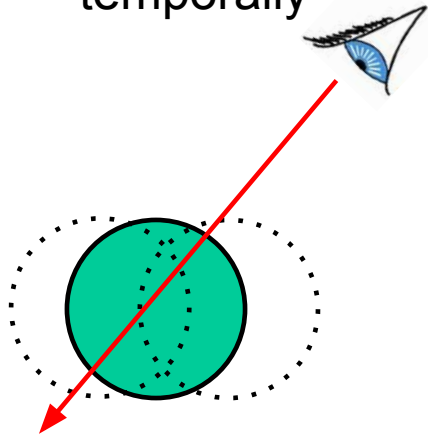
# Glossy Reflection

- multiple reflection rays



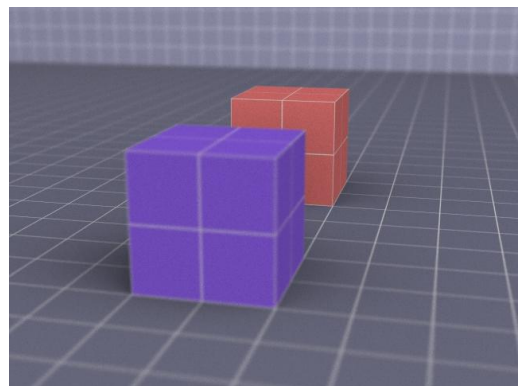
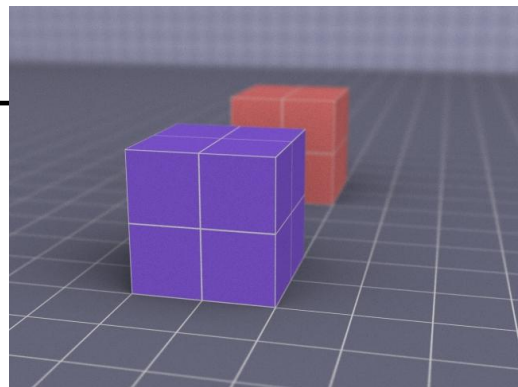
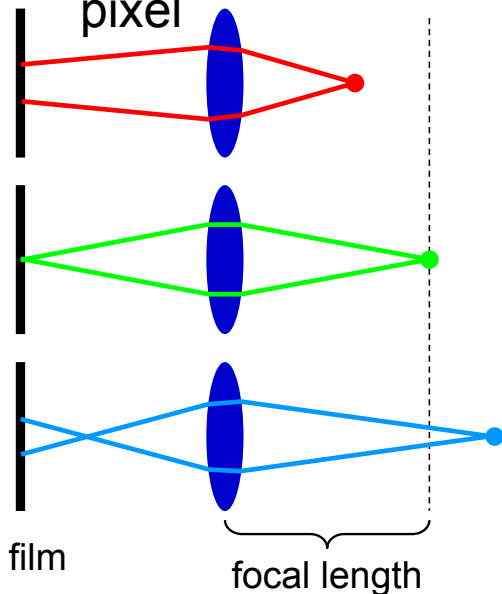
# Motion Blur

- Sample objects temporally



# Depth of Field

- multiple rays per pixel



Justin Legakis

# Ray Tracing Algorithm Analysis

---

- Ray casting
- Lots of primitives
- Recursive
- Distributed Ray Tracing Effects
  - Soft shadows
  - Anti-aliasing
  - Glossy reflection
  - Motion blur
  - Depth of field

$$\text{cost} \approx \text{height} * \text{width} * \text{num primitives} * \text{intersection cost} * \text{size of recursive ray tree} * \left\{ \begin{array}{l} \text{num shadow rays} * \\ \text{num supersamples} * \\ \text{num glossy rays} * \\ \text{num temporal samples} * \\ \text{num focal samples} * \\ \dots \end{array} \right.$$

**can we reduce this?**

**these can serve double duty**

## Today

---

- Paper for Today: Distributed Ray Tracing
- **Local Illumination**
  - BRDF
  - **Ideal Diffuse Reflectance**
  - **Ideal Specular Reflectance**
  - **The Phong Model**
- Why is Global Illumination Important?
- Radiosity Matrix
- Calculating the Form Factors
- Advanced Radiosity
- Worksheet



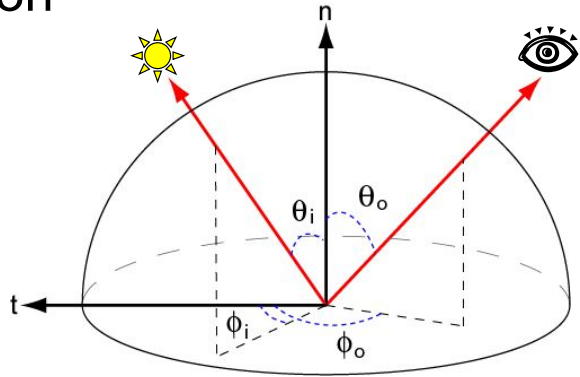
# BRDF

- Ratio of light coming from one direction that gets reflected in another direction
- Bidirectional Reflectance Distribution Function

– 4D

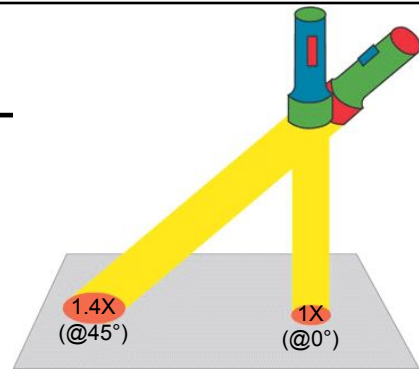
–  $R(\theta_i, \phi_i; \theta_o, \phi_o)$

– Note: BRDF for *isotropic* materials is 3D

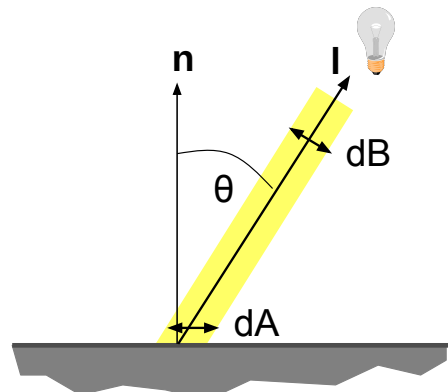


# Incoming Radiance

- The amount of light received by a surface depends on incoming angle
  - Bigger at normal incidence (Winter/Summer difference)
- By how much?
  - $dB = dA \cos \theta$
  - Same as:  $I \cdot n$  (dot product with normal)

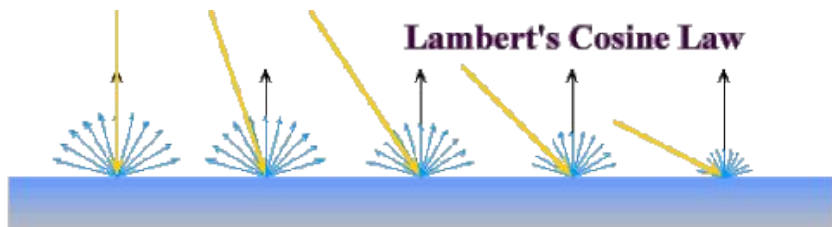
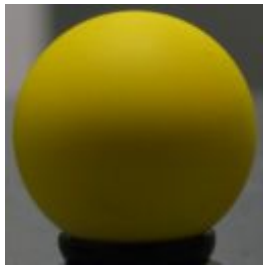
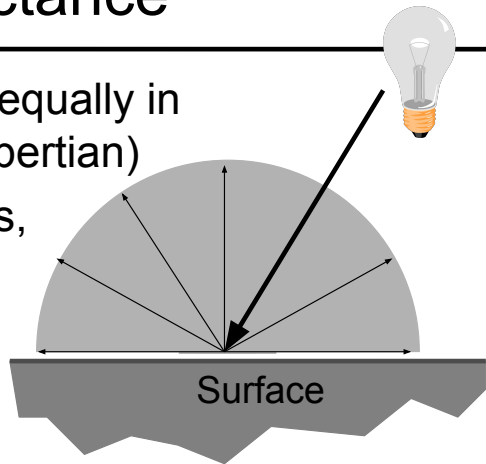


[http://www.shsu.edu/~dl\\_www/bkonline/131online/f02latitude/02index.htm](http://www.shsu.edu/~dl_www/bkonline/131online/f02latitude/02index.htm)



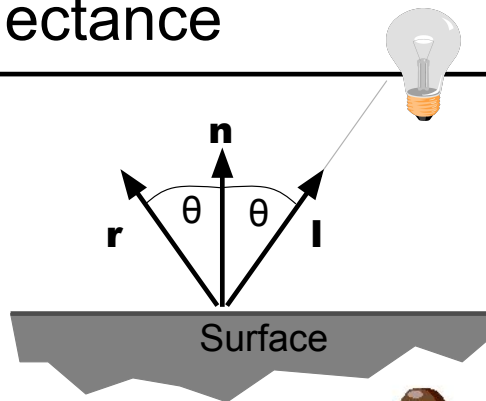
# Ideal Diffuse Reflectance

- Assume surface reflects equally in all directions (a.k.a. Lambertian)
- An ideal diffuse surface is, at the microscopic level, a very rough surface
- Examples: chalk, clay, some paints



# Ideal Specular Reflectance

- Assume surface reflects only in mirror direction
  - View dependent
- Microscopic surface elements are oriented in the same direction as the surface
- Examples: mirrors, highly polished metals



# Non-Ideal Reflectors

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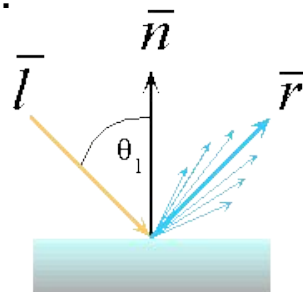
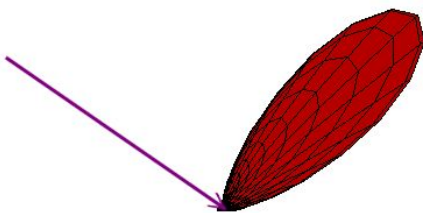
- Real materials tend to be *neither* ideal diffuse *nor* ideal reflective
- Highlight is blurry, looks glossy



# Non-Ideal Reflectors

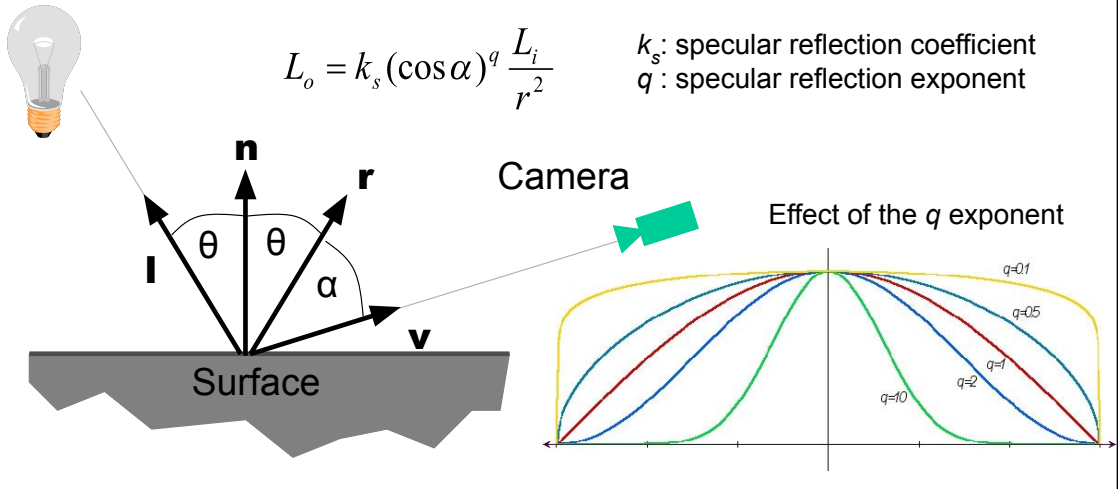
---

- Most light reflects in the ideal reflected direction
- Microscopic surface variations will reflect light just slightly offset
- How much light is reflected?



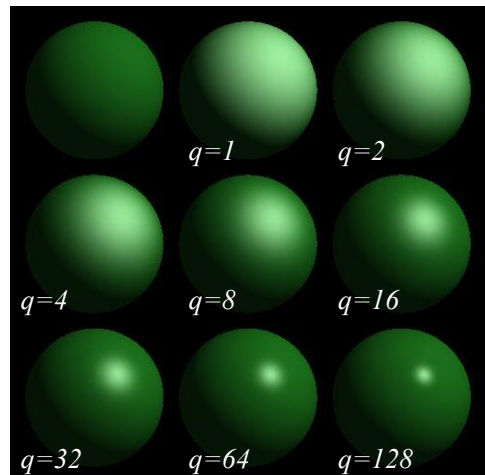
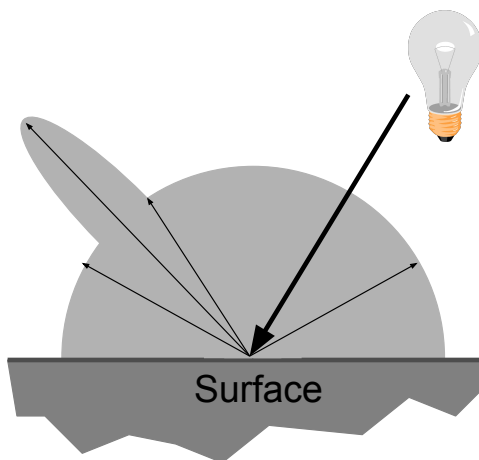
# The Phong Model

- An empirical (observational) model
- How much light is reflected “specularly”?
  - Depends on the angle  $\alpha$ , between the ideal reflection direction  $r$  and the viewer direction  $l$



# The Phong Model

- Sum of three components:
  - diffuse reflection + specular reflection + “ambient”.

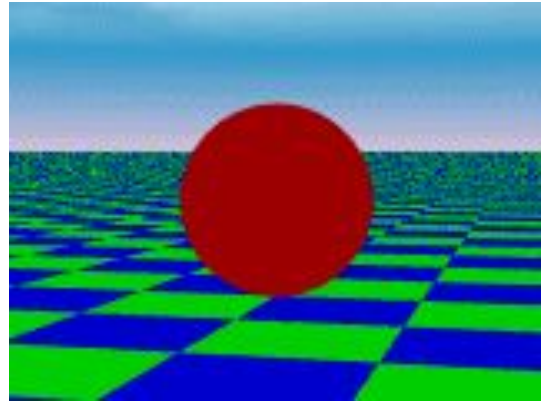


variations in Phong specular exponent

# Ambient Illumination

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- In a typical room, everything receives at least a little bit of light
- Ambient illumination represents the reflection of all indirect illumination



$$L(\omega_r) = k_a$$

- This is a total hack!

## Reading for Today (*optional*)

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- "Measuring and Modeling Anisotropic Reflection", Ward, SIGGRAPH 1992



# Questions?

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Lightscape <http://www.lightscape.com>

# Today

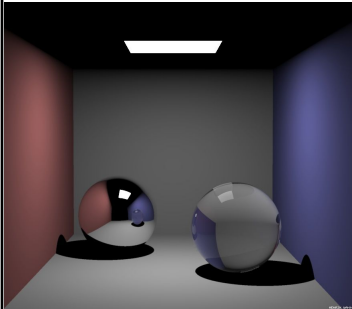
---

- Paper for Today: Distributed Ray Tracing
- Local Illumination
- **Why is Global Illumination Important?**
  - **The Cornell Box**
  - **Radiosity vs. Ray Tracing**
- Radiosity Matrix
- Calculating the Form Factors
- Advanced Radiosity
- Worksheet

# Why Global Illumination?

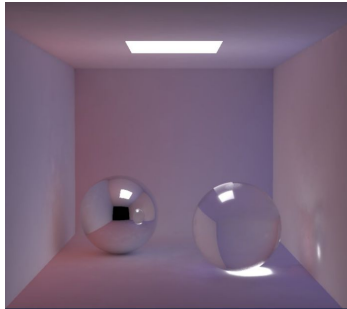
- Simulate all light inter-reflections (indirect lighting)
  - in a room, a lot of the light is indirect: it is reflected by walls.
- How have we dealt with this so far?
  - Ambient term to fake some uniform indirect light

ray tracing



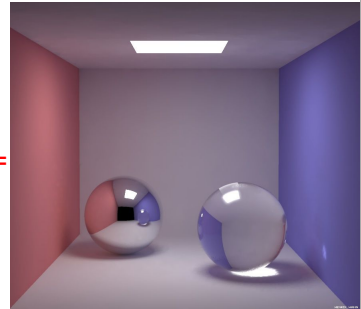
(no ambient term)

indirect illumination



it is smooth,  
but not constant!

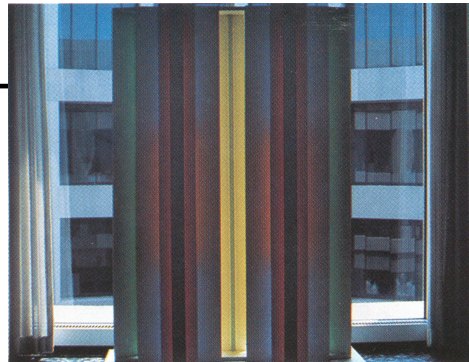
“right” answer



Henrik Wann Jensen

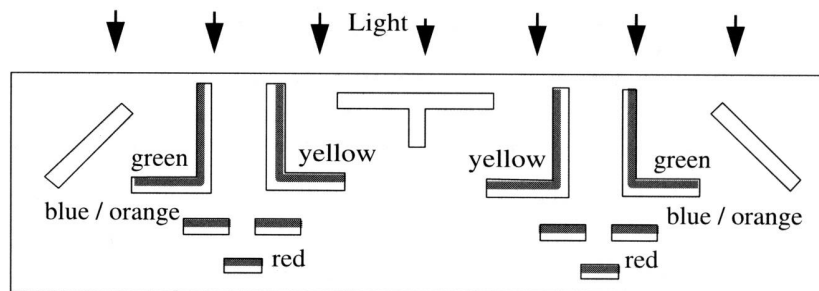
# Why Radiosity?

- Sculpture by John Ferren
- Diffuse panels



photograph:

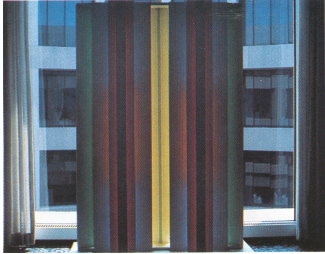
diagram  
from above:



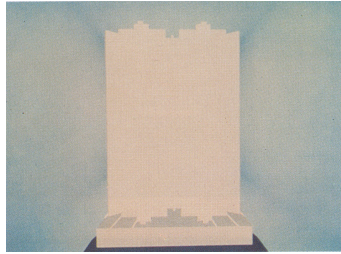
All visible surfaces, white.



# Radiosity vs. Ray Tracing



Original sculpture by John Ferren lit by daylight from behind.



Ray traced image. A standard ray tracer cannot simulate the interreflection of light between diffuse surfaces.

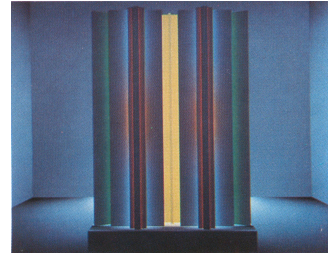
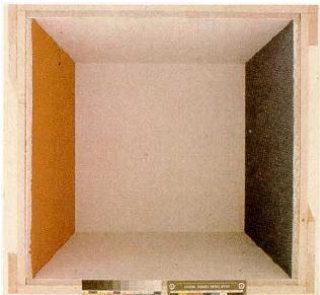
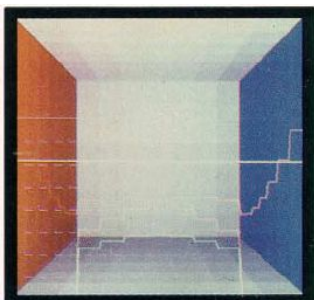


Image rendered with radiosity. Note the color bleeding effects.

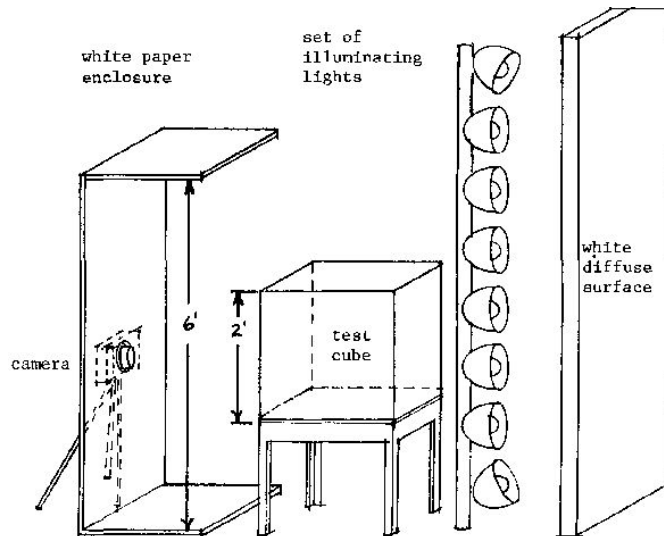
# Reading for Friday - after Quiz 1



photograph



simulation



Goral, Torrance, Greenberg & Battaile  
*Modeling the Interaction of Light Between Diffuse Surfaces* SIGGRAPH '84



# The Cornell Box

---

- Careful calibration and measurement allows for comparison between physical scene & simulation



photograph

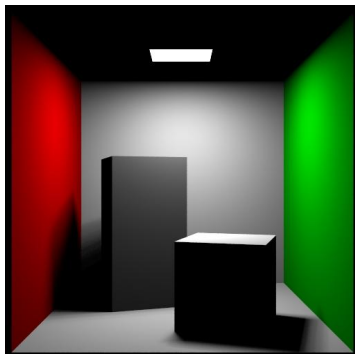


simulation

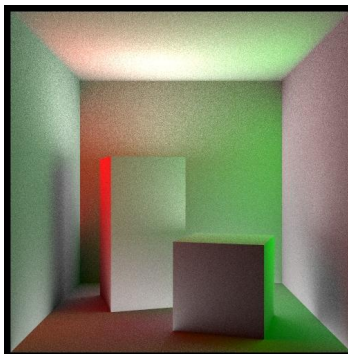
Light Measurement Laboratory  
Cornell University, Program for Computer Graphics

# Visualizing Inter-reflections...

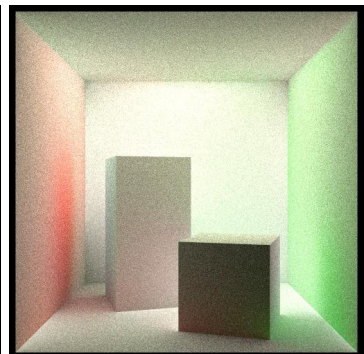
---



direct illumination  
(0 bounces)



1 bounce



2 bounces

*Note: image brightness not constant between images*

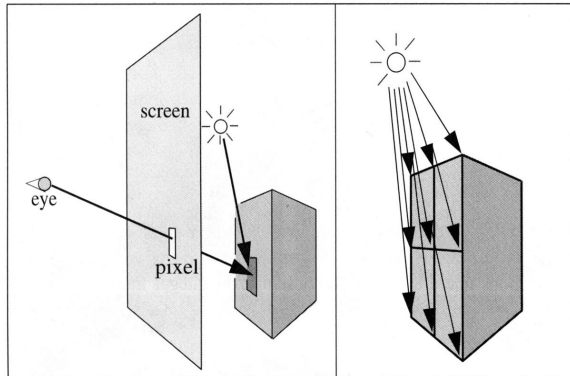
images by Micheal Callahan

[http://www.cs.utah.edu/~shirley/classes/cs684\\_98/students/callahan/bounce/](http://www.cs.utah.edu/~shirley/classes/cs684_98/students/callahan/bounce/)

# Radiosity vs. Ray Tracing

---

- Ray tracing is an *image-space* algorithm
  - If the camera is moved, we have to start over
- Radiosity is computed in *object-space*
  - View-independent (just don't move the light)
  - Can pre-compute complex lighting to allow interactive walkthroughs



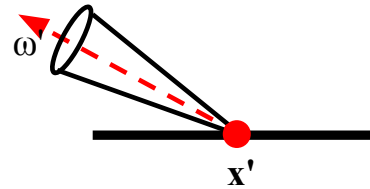
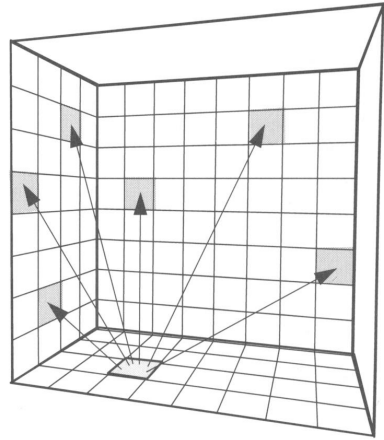
## Today

---

- Paper for Today: Distributed Ray Tracing
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- Why is Global Illumination Important?
- **Radiosity Matrix**
- Calculating the Form Factors
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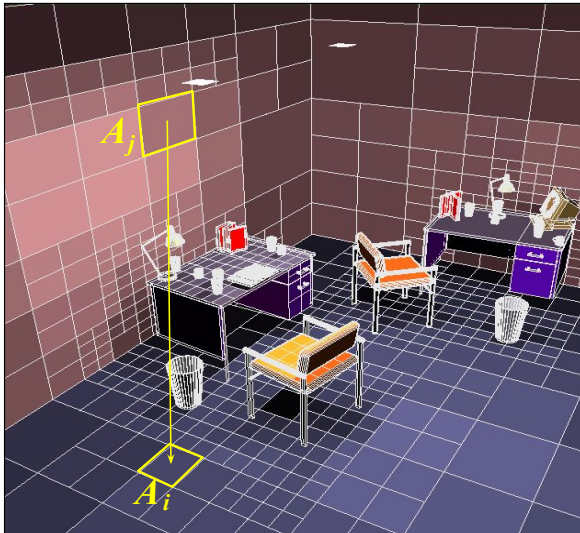
# Radiosity Overview

- Surfaces are assumed to be perfectly Lambertian (diffuse)
  - reflect incident light in all directions with equal intensity
- The scene is divided into a set of small areas, or patches.
- The radiosity,  $B_i$ , of patch  $i$  is the total rate of energy leaving a surface. The radiosity over a patch is constant.
- Units for radiosity: Watts / steradian \* meter<sup>2</sup>



# Discrete Radiosity Equation

Discretize the scene into  $n$  patches, over which the radiosity is constant



$$B_i = E_i + \rho_i \sum_{j=1}^n F_{ij} B_j$$

light leaving patch  $i$  (points to  $B_i$ )  
 material reflectivity (points to  $\rho_i$ )  
 light emitted from patch  $i$  (points to  $E_i$ )  
 form factor (points to  $F_{ij}$ )

The equation is recursive, but it can be solved iteratively

# Radiosity in Matrix Form

$$B_i = E_i + \rho_i \sum_{j=1}^n F_{ij} B_j$$

$n$  simultaneous equations with  $n$  unknown  $B_i$  values can be written in matrix form:

$$\begin{bmatrix} 1 - \rho_1 F_{11} & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1n} \\ -\rho_2 F_{21} & 1 - \rho_2 F_{22} & & \\ \vdots & & \ddots & \\ -\rho_n F_{n1} & \cdots & \cdots & 1 - \rho_n F_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$$

↓ solve for  $B_i$

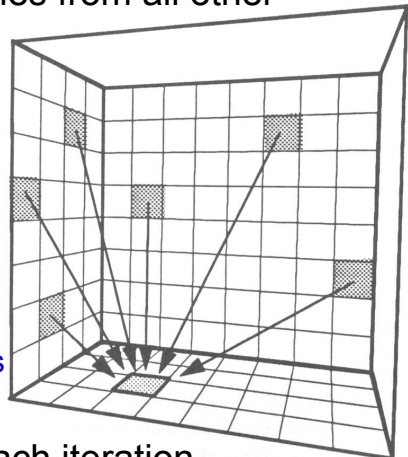
A solution yields a single radiosity value  $B_i$  for each patch in the environment, a view-independent solution.

# Solving the Radiosity Matrix

- Initialize all radiosity values to 0
- Each iteration, update the radiosity of each patch by *gathering* the contribution of radiosities from all other

$$\begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_i \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_i \\ \vdots \\ E_n \end{bmatrix} + \begin{bmatrix} \rho_1 F_{i1} & \rho_1 F_{i2} & \cdots & \rho_1 F_{in} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_i \\ \vdots \\ B_n \end{bmatrix}$$

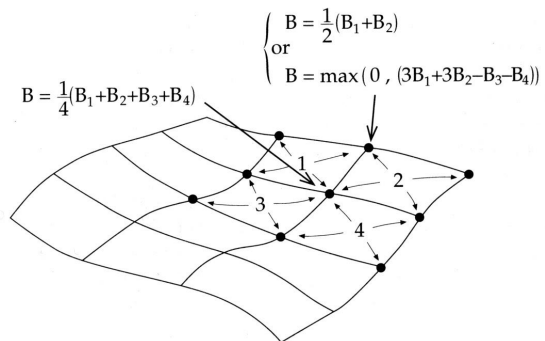
↑ Radiosity values on iteration  $t+1$                       ↑ Radiosity values on iteration  $t$



- Radiosity values only increase on each iteration
- This method is fundamentally a Gauss-Seidel relaxation

# Interpolating Vertex Radiosities

- $B_i$  radiosity values are constant over the extent of a patch.
- How are they mapped to the vertex radiosities (intensities) needed by the renderer?
  - Average the radiosities of patches that contribute to the vertex
  - Vertices on the edge of a surface are assigned values extrapolation



## Questions?



Factory simulation. 30,000 patches.  
Program of Computer Graphics, Cornell University.

# Today

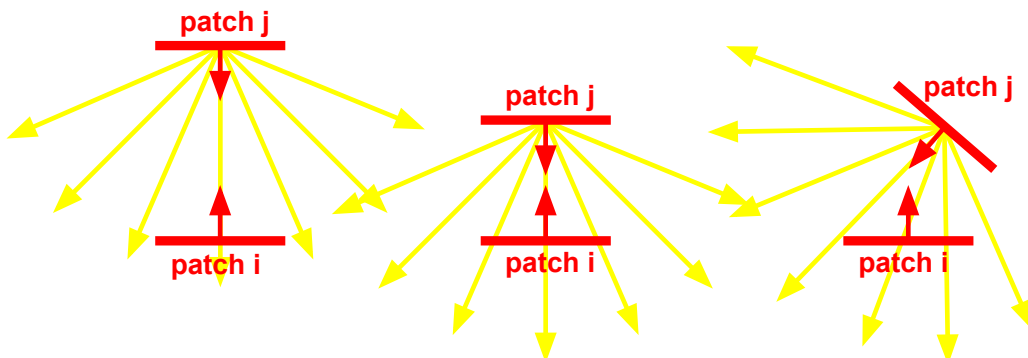
---

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## Calculating the Form Factor $F_{ij}$

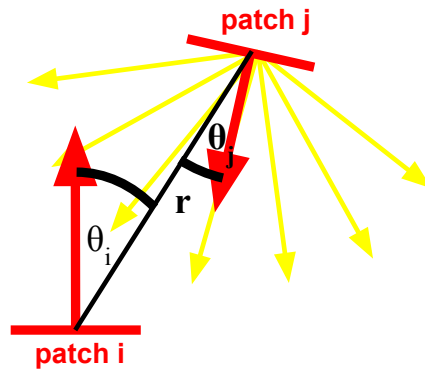
---

- $F_{ij}$  = fraction of light energy leaving patch j that arrives at patch i
- Takes account of both:
  - geometry (size, orientation & position)
  - visibility (are there any occluders?)



# Calculating the Form Factor $F_{ij}$

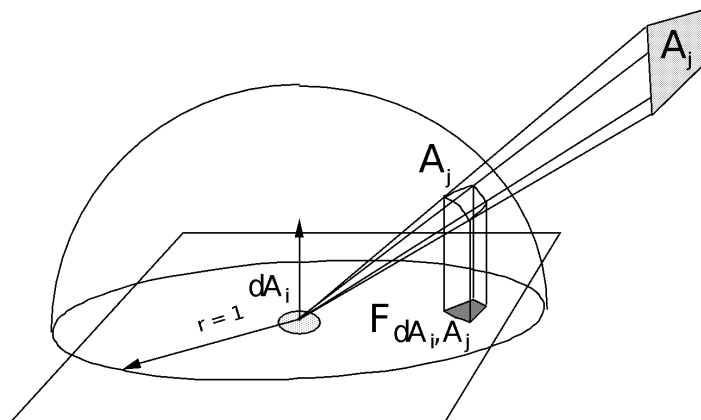
- $F_{ij}$  = fraction of light energy leaving patch j that arrives at patch i



$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos \theta_i \cos \theta_j}{\pi r^2} V_{ij} dA_j dA_i$$

## Form Factor Determination

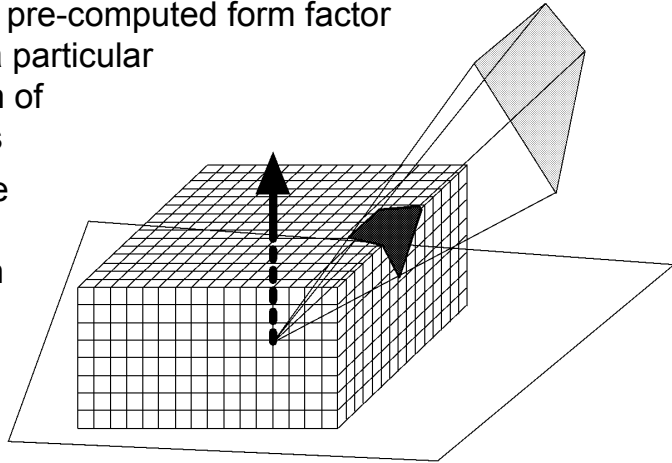
The Nusselt analog: the form factor of a patch is equivalent to the fraction of the unit circle that is formed by taking the projection of the patch onto the hemisphere surface and projecting it down onto the circle.



# Hemicube Algorithm

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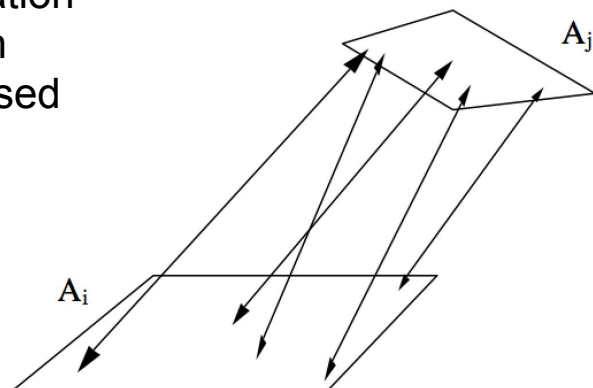
- A hemicube is constructed around the center of each patch
- Faces of the hemicube are divided into "pixels"
- Each patch is projected (rasterized) onto the faces of the hemicube
- Each pixel stores its pre-computed form factor  
The form factor for a particular patch is just the sum of the pixels it overlaps
- Patch occlusions are handled similar to z-buffer rasterization



# Form Factor from Ray Casting

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- Cast  $n$  rays between the two patches
  - Compute visibility (what fraction of rays do not hit an occluder)
  - Integrate the point-to-point form factor
- Permits the computation of the patch-to-patch form factor, as opposed to point-to-patch





# Questions?

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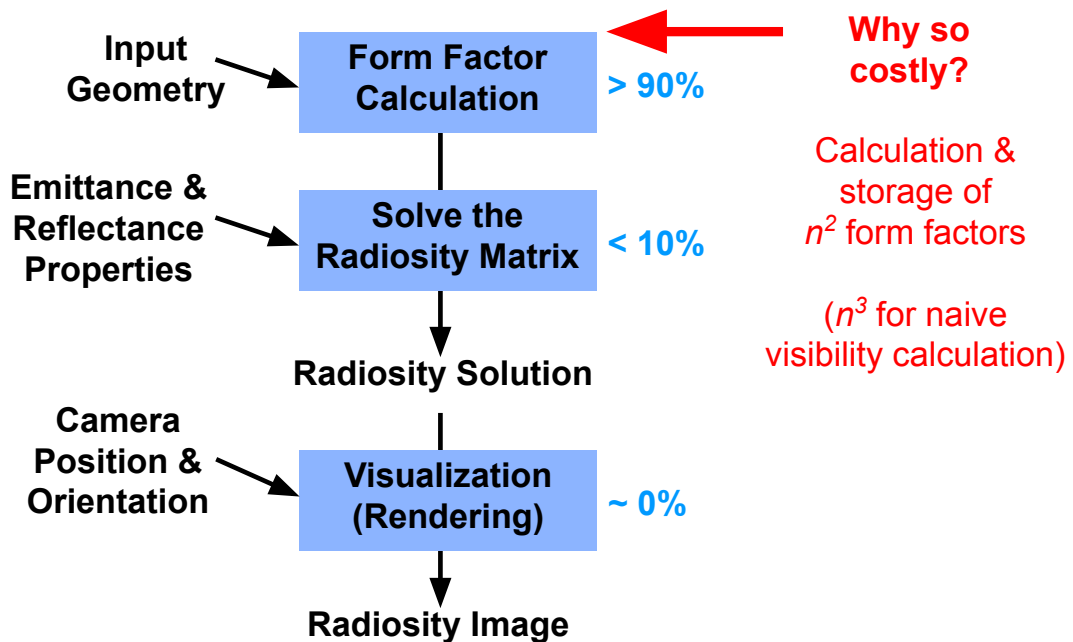
Lightscape <http://www.lightscape.com>

## Today

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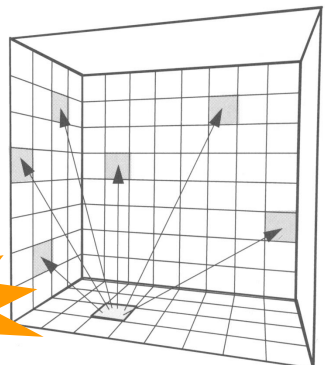
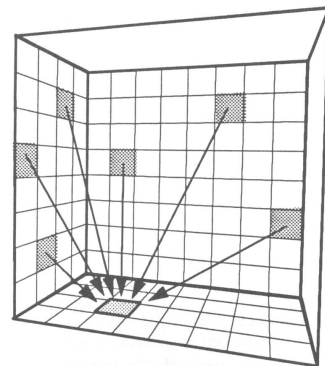
- Paper for Today: Distributed Ray Tracing
- Local Illumination
- Why is Global Illumination Important?
- Radiosity Matrix
- Calculating the Form Factors
- **Advanced Radiosity**
  - **Progressive Radiosity**
  - Adaptive Subdivision
  - Discontinuity Meshing
  - Hierarchical Radiosity
- Worksheet

# Stages in a Radiosity Solution



## Progressive Refinement

- Goal: Provide frequent and timely updates to the user during computation
- Key Idea: Update the entire image at every iteration, rather than a single patch
- How? Instead of summing the light received by one patch, distribute the radiance of the patch with the most *undistributed radiance*.

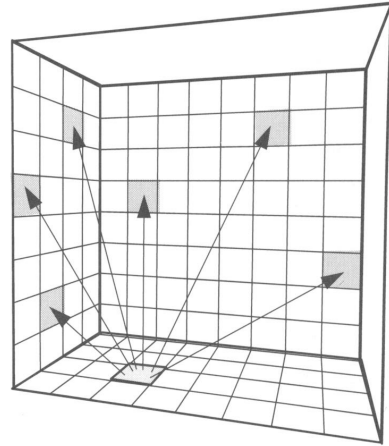


Use this for HW3!

# Reordering the Solution for PR

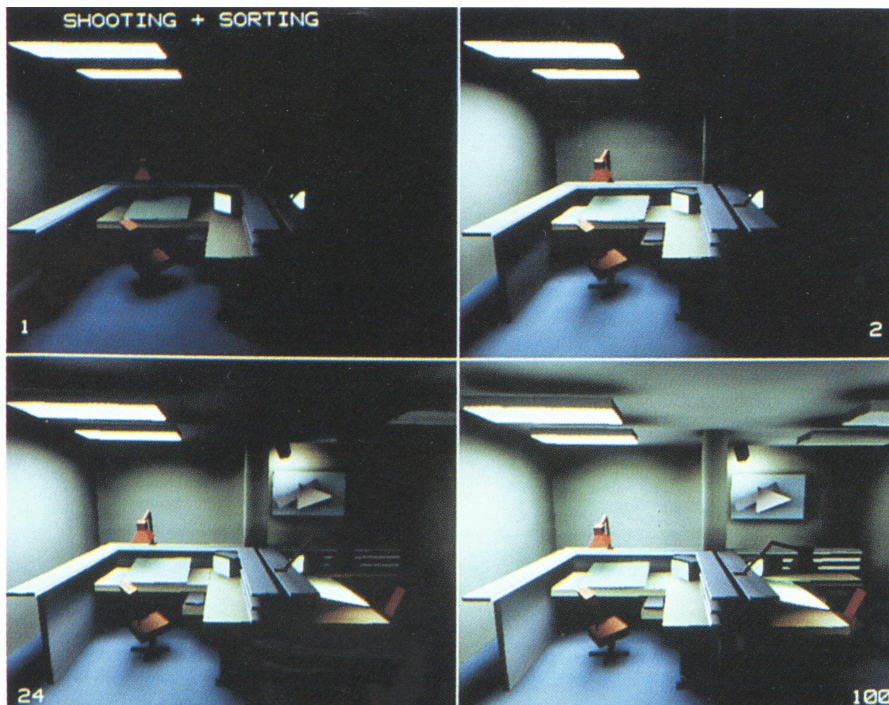
*Shooting*: the radiosity of all patches is updated for each iteration:

$$\begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ \vdots \\ B_n \end{bmatrix} + \begin{bmatrix} \dots & \rho_1 F_{1i} & \dots \\ \dots & \rho_2 F_{2i} & \dots \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ \dots & \rho_n F_{ni} & \dots \end{bmatrix} \begin{bmatrix} \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \end{bmatrix}$$

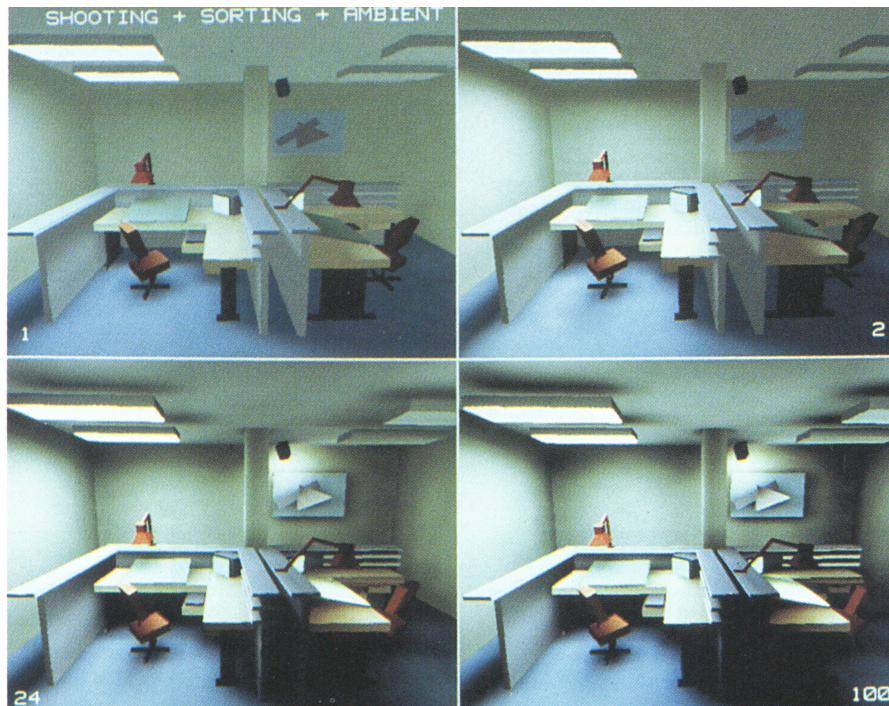


This method is fundamentally a Southwell relaxation

# Progressive Refinement w/out Ambient Term



# Progressive Refinement with Ambient Term



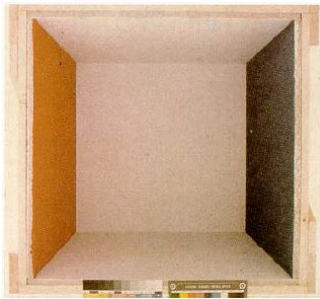
# Questions?



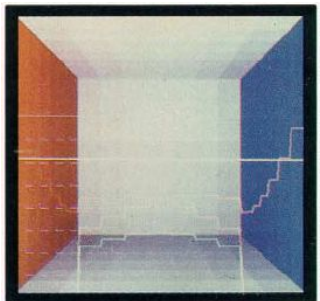
Lightscape

<http://www.lightscape.com>

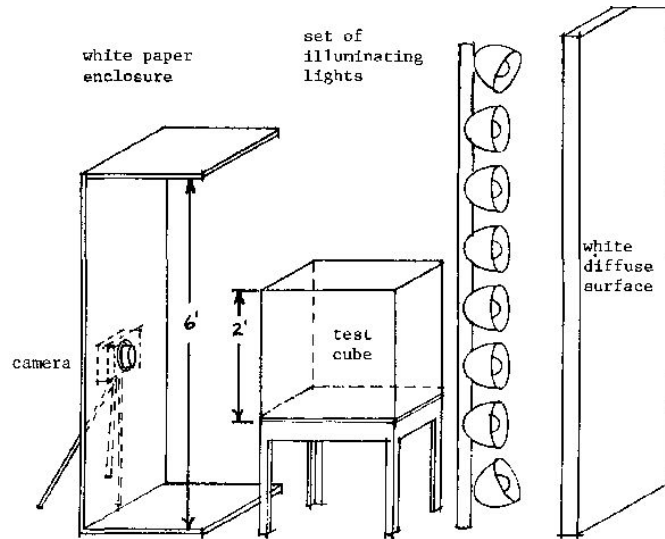
# Reading for Next Time:



photograph



simulation



Goral, Torrance, Greenberg & Battaile  
*Modeling the Interaction of Light Between Diffuse Surfaces* SIGGRAPH '84

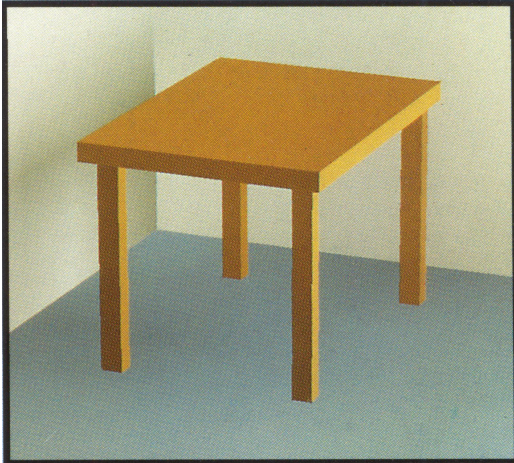
# Today

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# Increasing the Accuracy of the Solution

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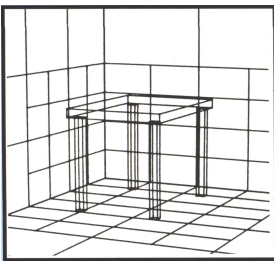
What's wrong with this picture?



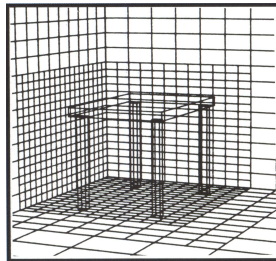
- Image quality is a function of patch size
- Compute a solution on a uniform initial mesh, then refine the mesh in areas that exceed some error tolerance:
  - shadow boundaries
  - other areas with a high radiosity gradient

# Adaptive Subdivision of Patches

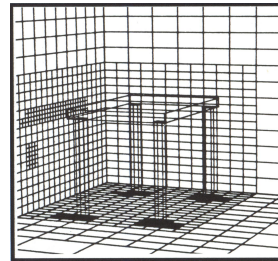
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Coarse patch solution  
(145 patches)



Improved solution  
(1021 subpatches)



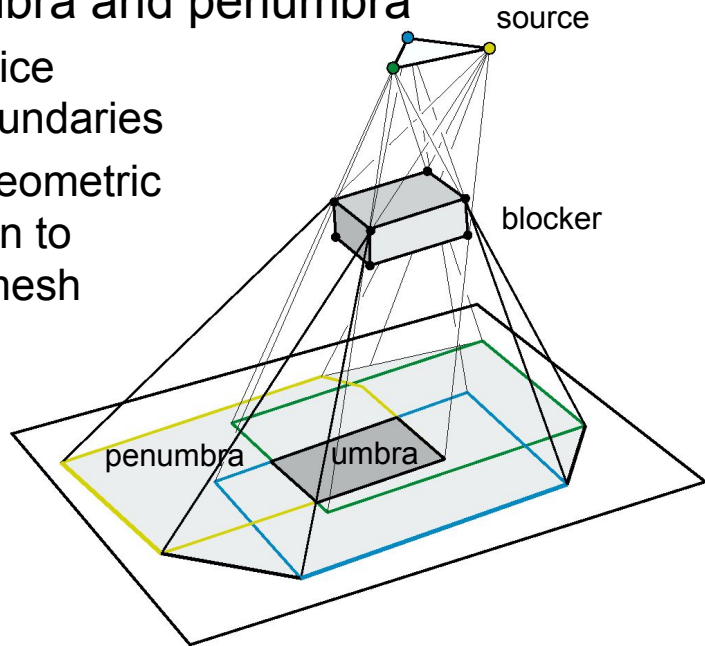
Adaptive subdivision  
(1306 subpatches)

# Discontinuity Meshing

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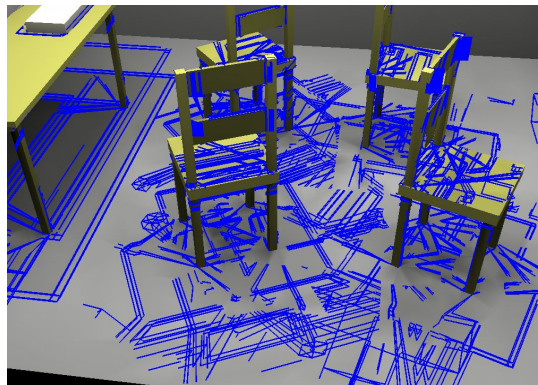
- Limits of umbra and penumbra

- Captures nice shadow boundaries
- Complex geometric computation to construct mesh



## Optional Reading for Next Time:

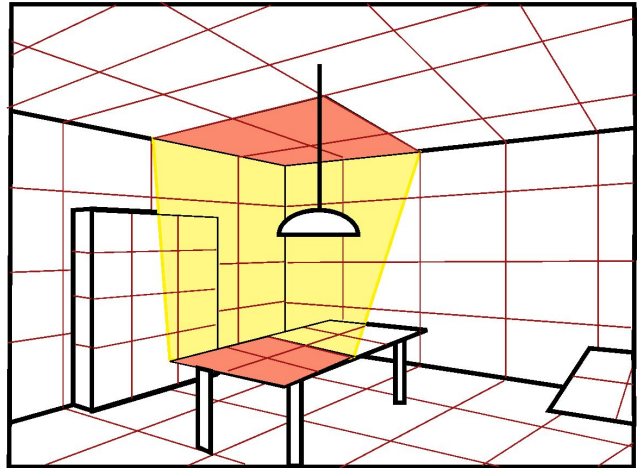
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“Fast and Accurate Hierarchical  
Radiosity Using Global Visibility”  
Durand, Drettakis, & Puech 1999

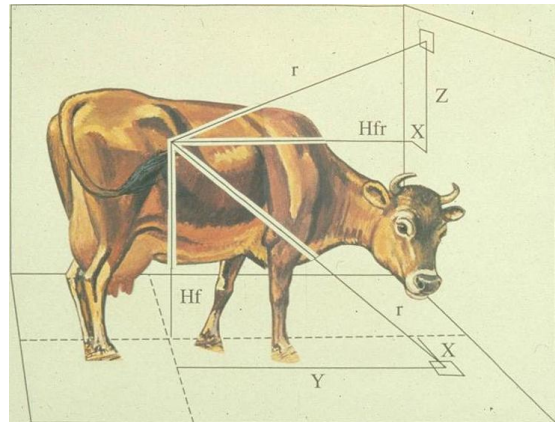
# Hierarchical Radiosity

- Group elements when the light exchange is not important
  - Breaks the quadratic complexity
  - Control non trivial, memory cost



# Practical Problems with Radiosity

- Meshing
  - memory
  - robustness
- Form factors
  - computation
- Diffuse limitation
  - extension to specular takes too much memory



Cow-cow form factor?



# Questions?

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Lightscape

<http://www.lightscape.com>

## Today

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# Pop Worksheet!

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multicolored painted diffuse (matte) mural wall  
function  $M(x,y,z)$  returns the RGB color at the specified location.

