

# The Rendering Equation & Irradiance Caching & Photon Mapping

## HW3: Raytracing & Epsilon

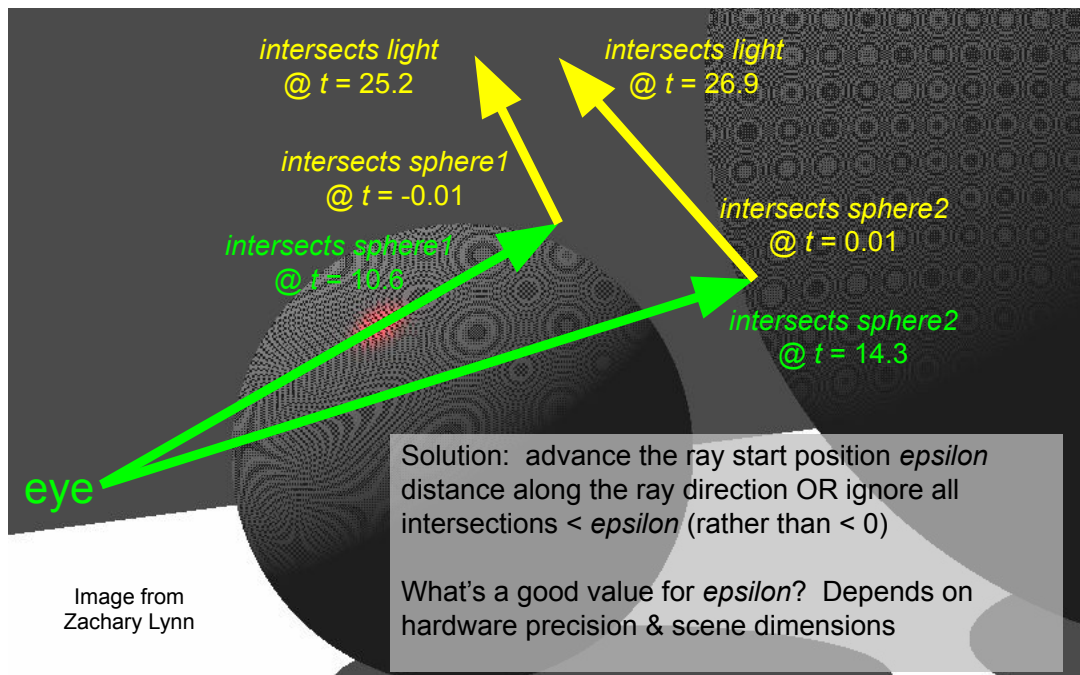


Image from  
Zachary Lynn

# Final Project Brainstorming

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- Each student should post two different ideas for a final project on the forum.
  - For each idea:
    - Briefly describe the idea, your motivation for it, and an example of the potential result.
    - What is the significant/interesting technical implementation challenge?
  - Have you already decided on one idea? Which one?
  - Do you already have a partner? Who?  
(even if you have chosen an idea and/or a partner everyone must post 2 different ideas)
  - Due **Wednesday 3/17 @ 11:59pm**
- Teams of 2 strongly recommended  
(individuals & teams >2 require instructor permission)
- *Projects from prior terms are on the website*

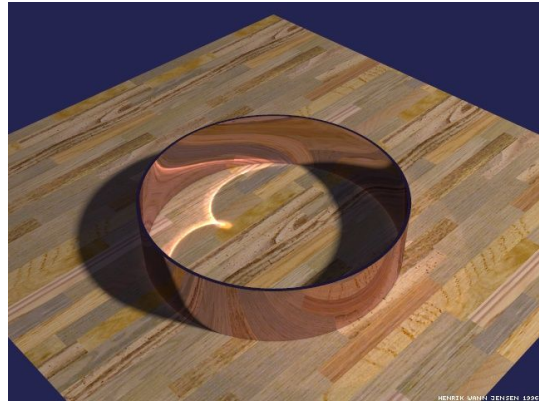
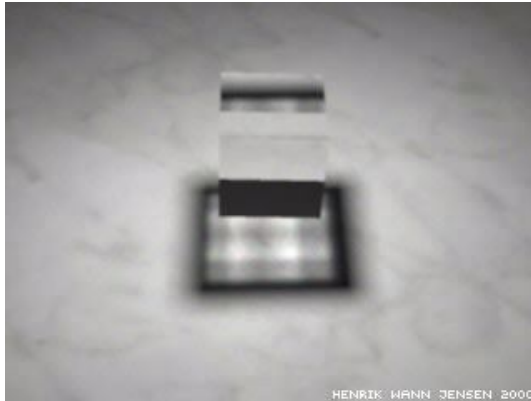
## *The Light of Mies van der Rohe*

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Henrik Wann Jensen, SIGGRAPH 2000

# Is this Traditional Ray Tracing?

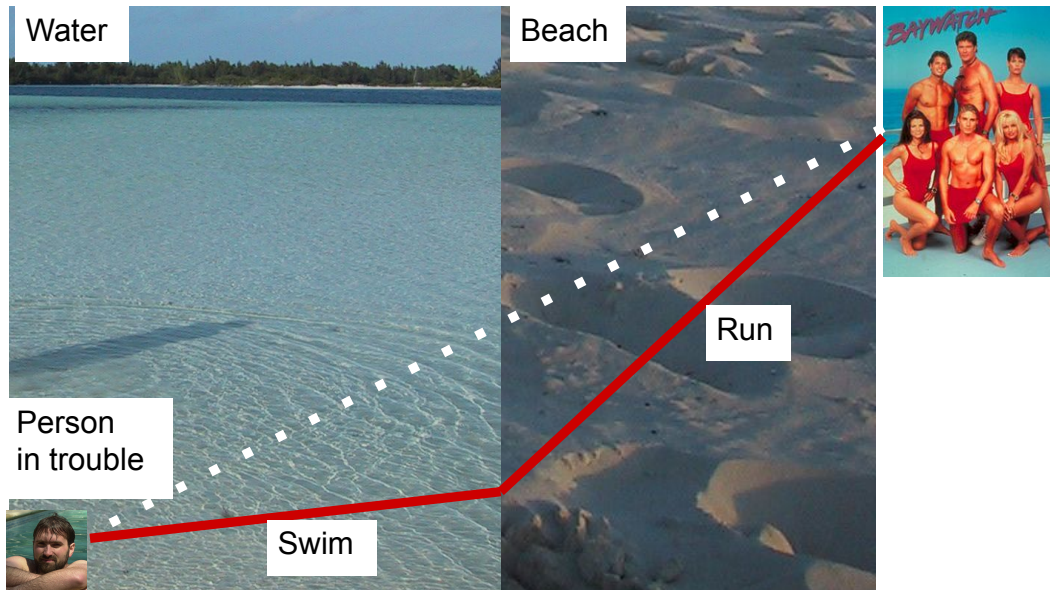


Images by Henrik Wann Jensen

*No. Refraction and complex reflections for illumination are not handled properly in traditional (backward) ray tracing.*

# Refraction and the Lifeguard Problem

- Running is faster than swimming



# Today

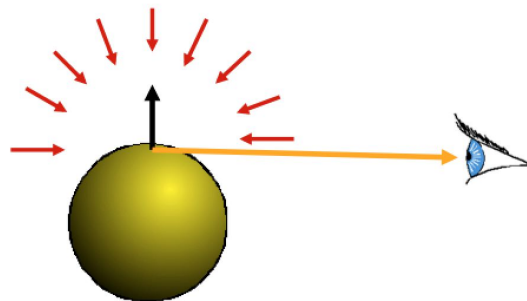
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- **The Rendering Equation**
- Worksheet on Progressive Radiosity
- Ray Casting vs. Ray Tracing vs. Monte-Carlo Ray Tracing vs. Path Tracing
- Irradiance Caching
- Photon Mapping
- Papers for Today
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# The Rendering Equation

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- Clean mathematical framework for light-transport simulation
- At each point, outgoing **light in one direction** is the integral of **incoming light in all directions** multiplied by reflectance property



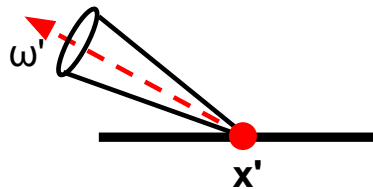
## “The Rendering Equation”, Kajiya, SIGGRAPH 1986

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## The Rendering Equation

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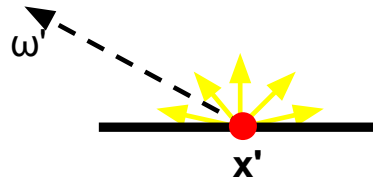
$$L(x', \omega') = E(x', \omega') + \int \rho_{x'}(\omega, \omega') L(x, \omega) G(x, x') V(x, x') dA$$



$L(x', \omega')$  is the radiance from a point on a surface in a given direction  $\omega'$

# The Rendering Equation

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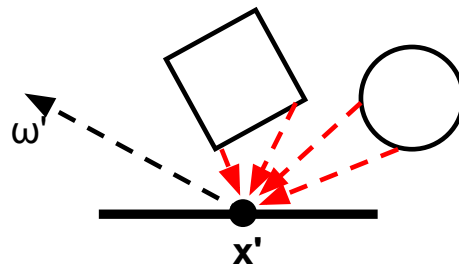
$$L(x', \omega') = E(x', \omega') + \int \rho_{x'}(\omega, \omega') L(x, \omega) G(x, x') V(x, x') dA$$



$E(x', \omega')$  is the emitted radiance from a point:  $E$  is non-zero only if  $x'$  is emissive (a light source)

# The Rendering Equation

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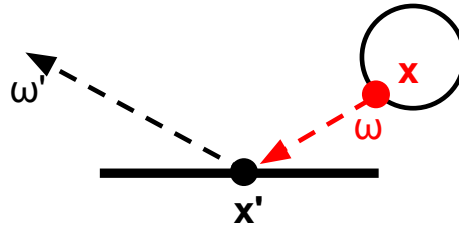


$$L(x', \omega') = E(x', \omega') + \underbrace{\int \rho_{x'}(\omega, \omega') L(x, \omega) G(x, x') V(x, x') dA}$$



Sum the contribution from all of the other surfaces in the scene

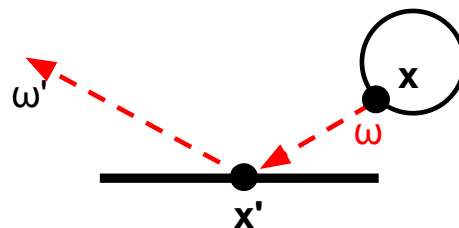
# The Rendering Equation



$$L(x', \omega') = E(x', \omega') + \int \rho_{x'}(\omega, \omega') L(x, \omega) G(x, x') V(x, x') dA$$

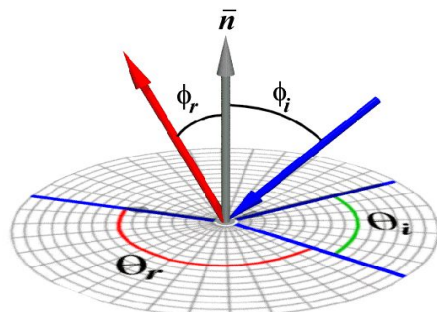
For each  $x$ , compute  $L(x, \omega)$ , the radiance at point  $x$  in the direction  $\omega$  (from  $x$  to  $x'$ )

# The Rendering Equation



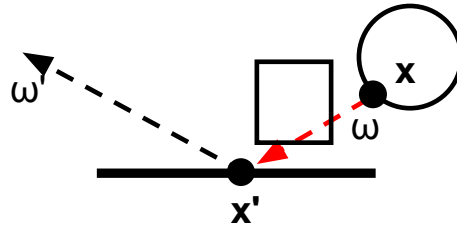
$$L(x', \omega') = E(x', \omega') + \int \rho_{x'}(\omega, \omega') L(x, \omega) G(x, x') V(x, x') dA$$

scale the contribution by  $\rho_{x'}(\omega, \omega')$ , the reflectivity (BRDF) of the surface at  $x'$



# The Rendering Equation

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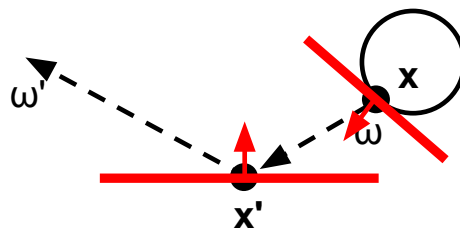


$$L(x', \omega') = E(x', \omega') + \int \rho_x(\omega, \omega') L(x, \omega) G(x, x') V(x, x') dA$$

For each  $x$ , compute  $V(x, x')$ ,  
the visibility between  $x$  and  $x'$ :  
1 when the surfaces are unobstructed  
along the direction  $\omega$ , 0 otherwise

# The Rendering Equation

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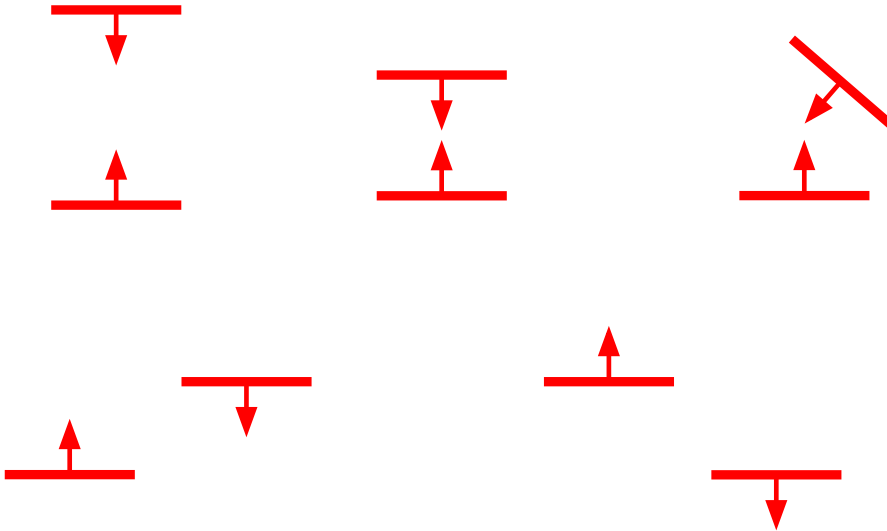
$$L(x', \omega') = E(x', \omega') + \int \rho_x(\omega, \omega') L(x, \omega) G(x, x') V(x, x') dA$$

For each  $x$ , compute  $G(x, x')$ , which  
describes the on the geometric relationship  
between the two surfaces at  $x$  and  $x'$



# Intuition about $G(x,x')$ ?

- Which arrangement of two surfaces will yield the greatest transfer of light energy? Why?



# Rendering Equation $\rightarrow$ Radiosity

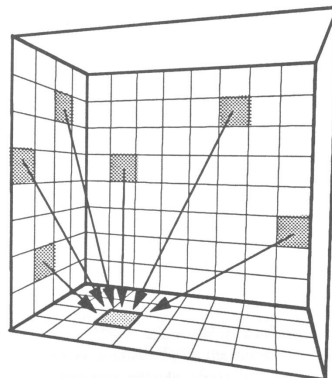
$$L(x',\omega') = E(x',\omega') + \int \rho_{x'}(\omega,\omega') L(x,\omega) G(x,x') V(x,x') dA$$

↓ Radiosity assumption:  
perfectly diffuse surfaces (not directional)

$$B_{x'} = E_{x'} + \rho_{x'} \int B_x G(x,x') V(x,x')$$

↓ discretize

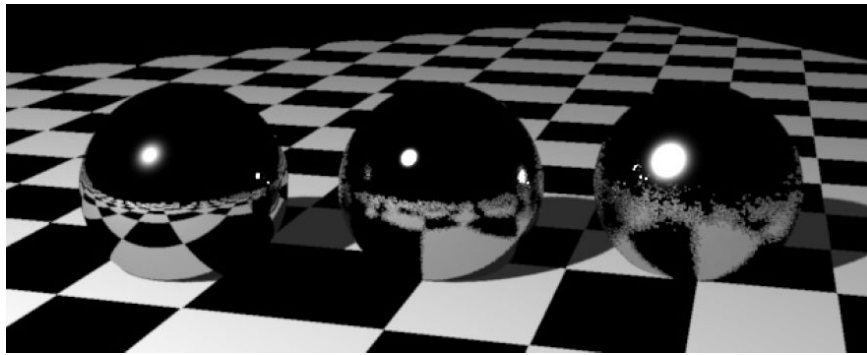
$$B_i = E_i + \rho_i \sum_{j=1}^n F_{ij} B_j$$



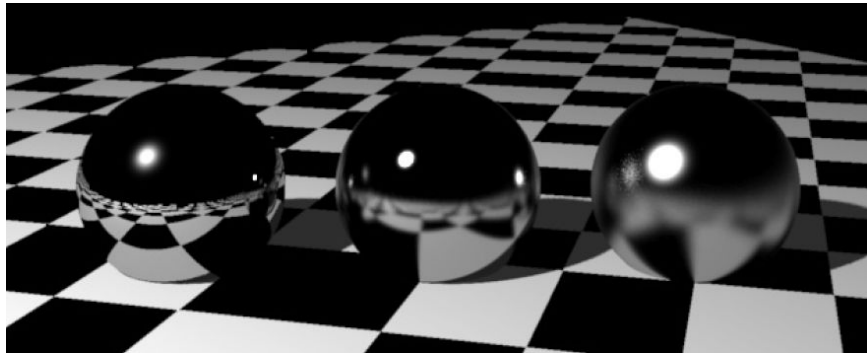
# Questions?

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1 glossy  
sample  
per pixel



256 glossy  
samples  
per pixel



## Today

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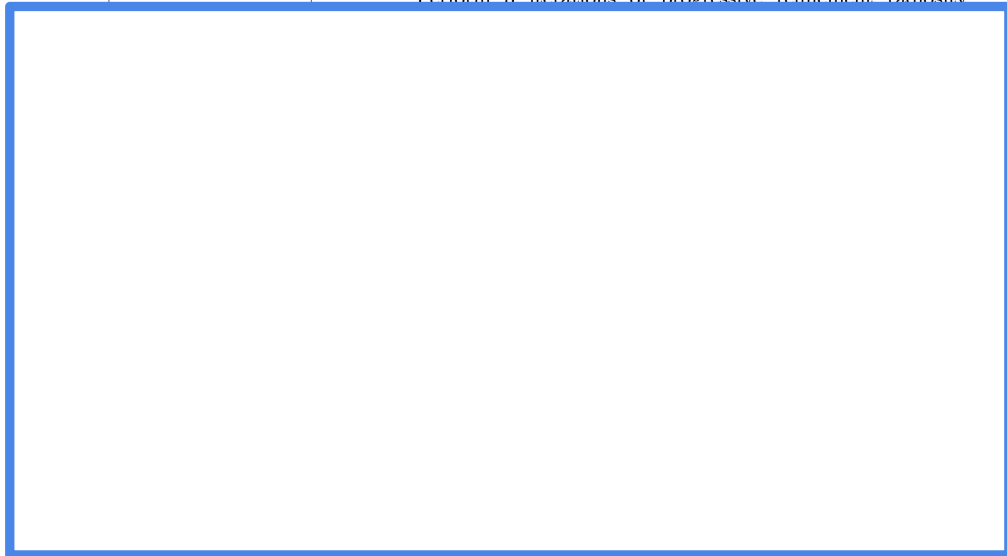
- The Rendering Equation
- **Worksheet on Progressive Radiosity**
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# Pop Worksheet!

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grey wall

Perform 5 iterations of progressive refinement radiosity



radiance @ iter 1 ( 10 , 10 , 10 )

undistributed @ iter 1 ( 0 , 0 , 0 )

## Today

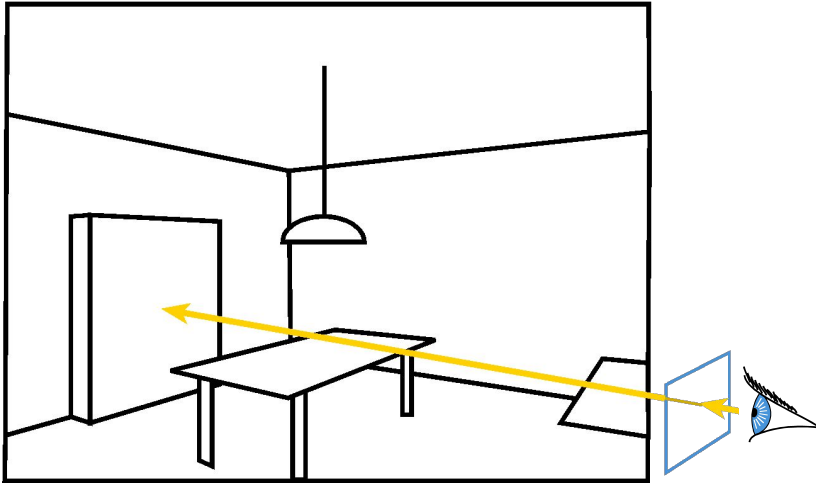
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# Ray Casting

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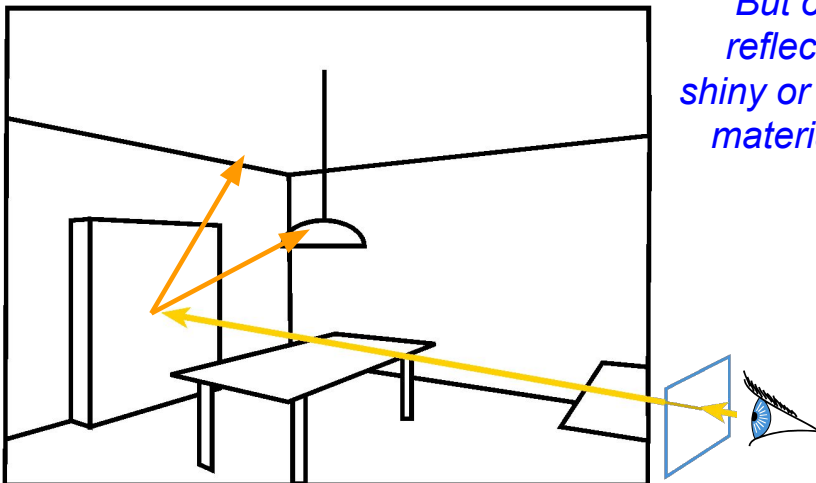
- Cast a ray from the eye through each pixel



# Ray Tracing

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- Cast a ray from the eye through each pixel
- Trace secondary rays (light, reflection, refraction)

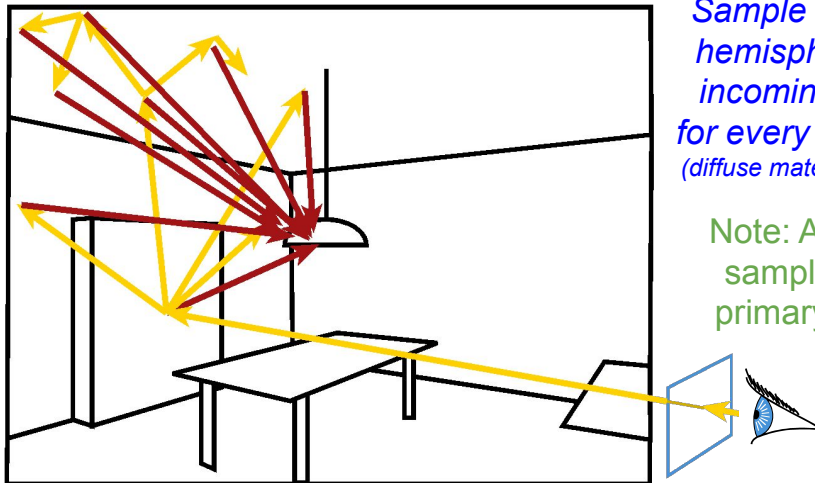


*But only  
reflect off  
shiny or glossy  
materials...*

# Monte Carlo Ray Tracing

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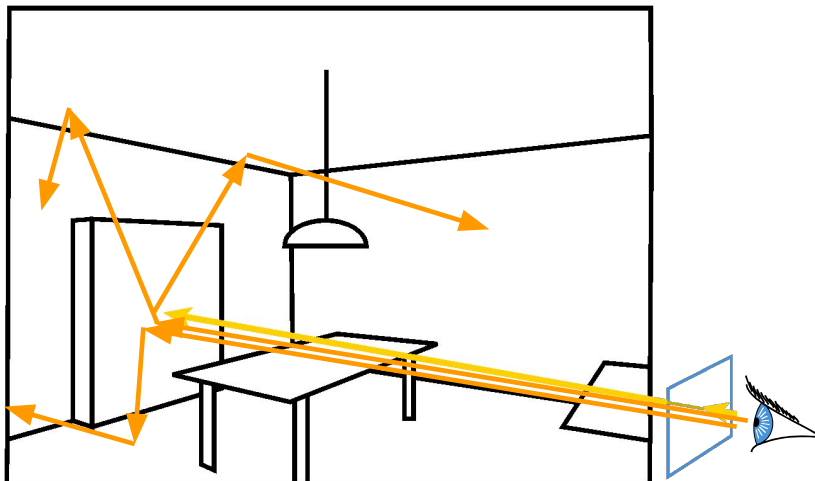
- Cast a ray from the eye through each pixel
- **Cast random rays to accumulate radiance contribution**
  - **Recurse to solve the Rendering Equation**



# (Monte Carlo) Path Tracing

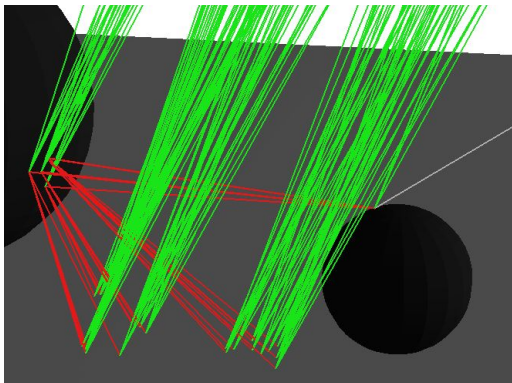
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- **Trace only one secondary ray per recursion**
- **But send many primary rays per pixel (performs antialiasing as well)**



# Ray Tracing vs. Path Tracing

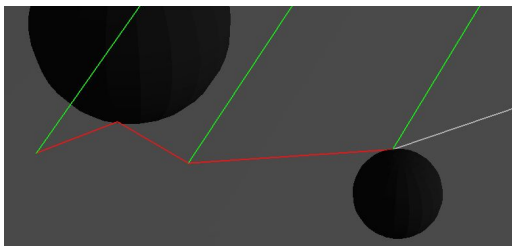
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2 bounces  
5 glossy samples  
5 shadow samples

How many rays cast per pixel?

1 main ray + 5 shadow rays +  
5 glossy rays + 5x5 shadow rays +  
5\*5 glossy rays + 5x5x5 shadow rays  
= 186 rays



How many 3 bounce paths can we trace per pixel for the same cost?

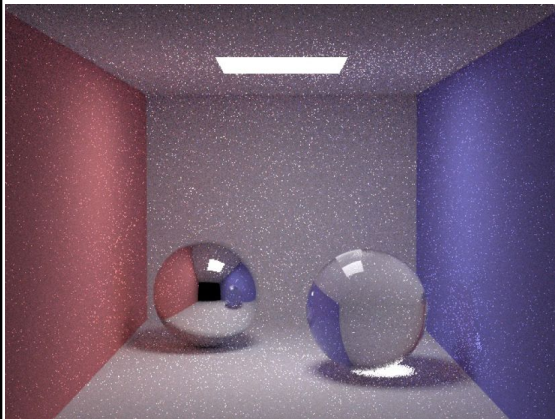
186 rays / 8 ray casts per path  
= ~23 paths

Which will probably have less error?

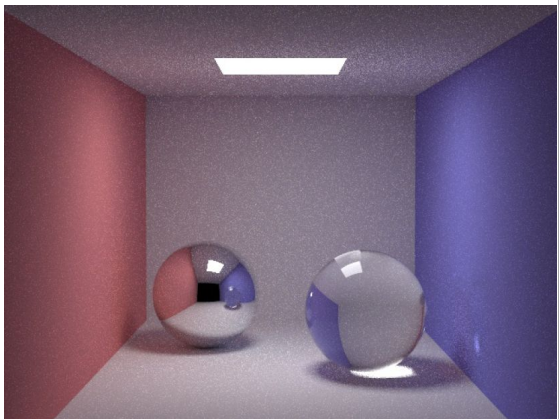
## Questions?

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10 paths/pixel



100 paths/pixel



Images from Henrik Wann Jensen

# Today

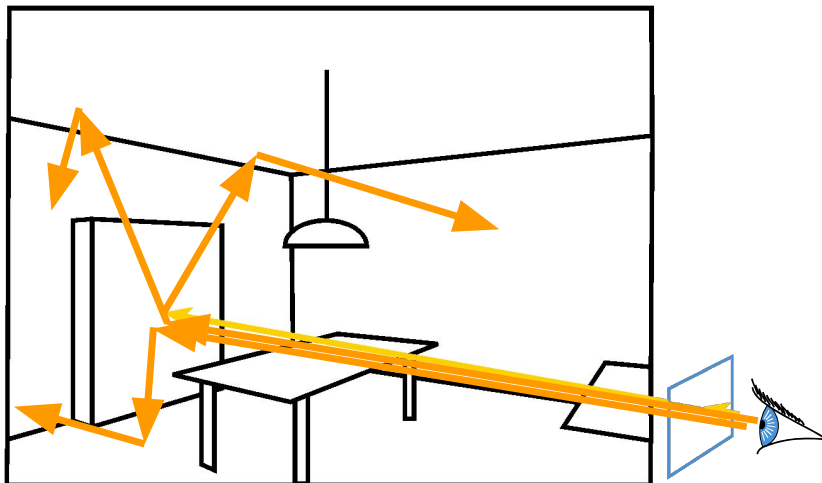
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- **Irradiance Caching**
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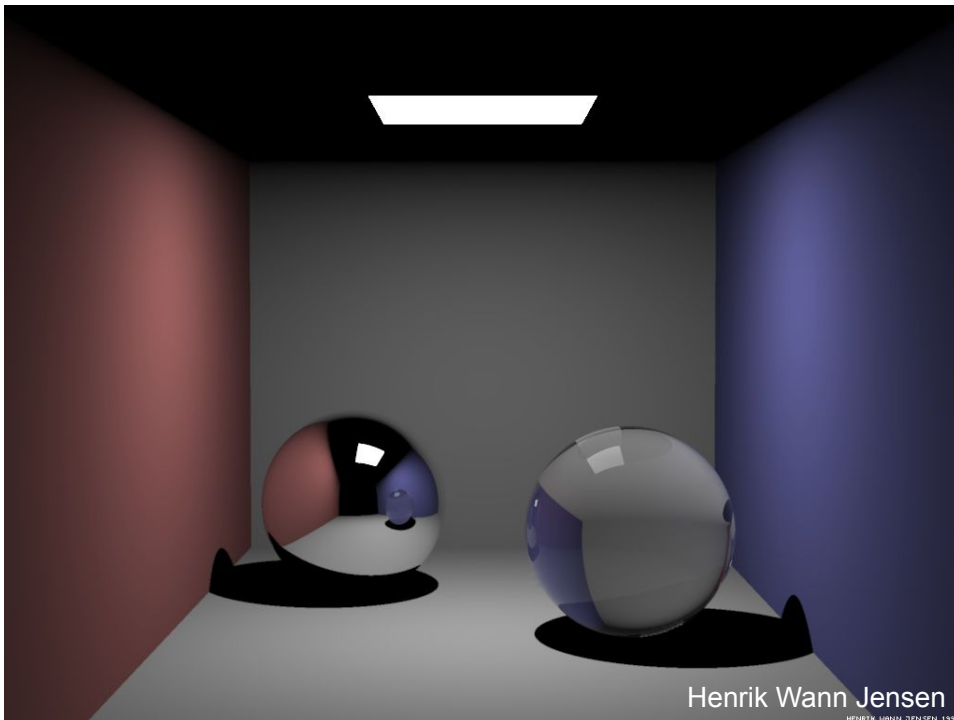
## Path Tracing is costly

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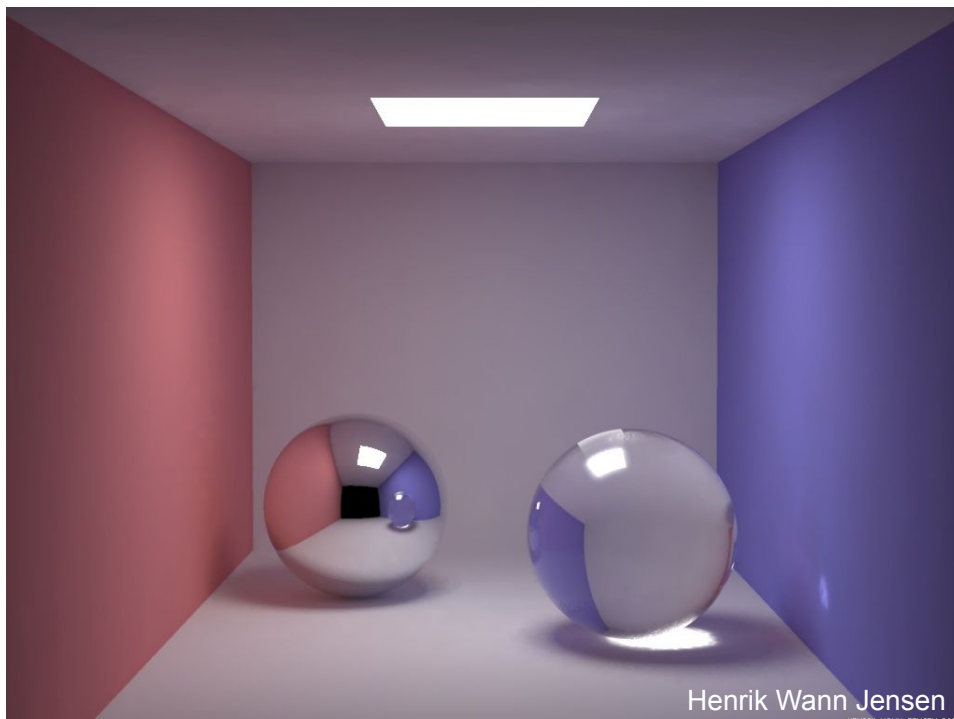
- Needs tons of rays per pixel



# Direct Illumination



# Global Illumination



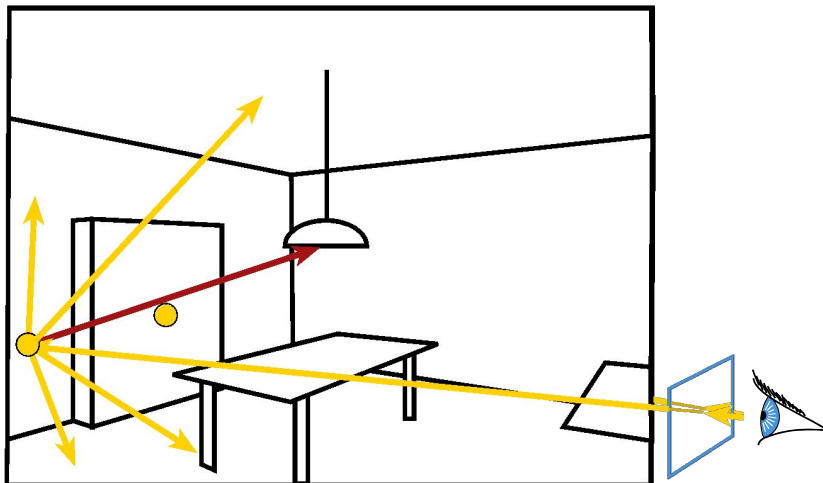


## Indirect Illumination: smooth



## Irradiance Cache

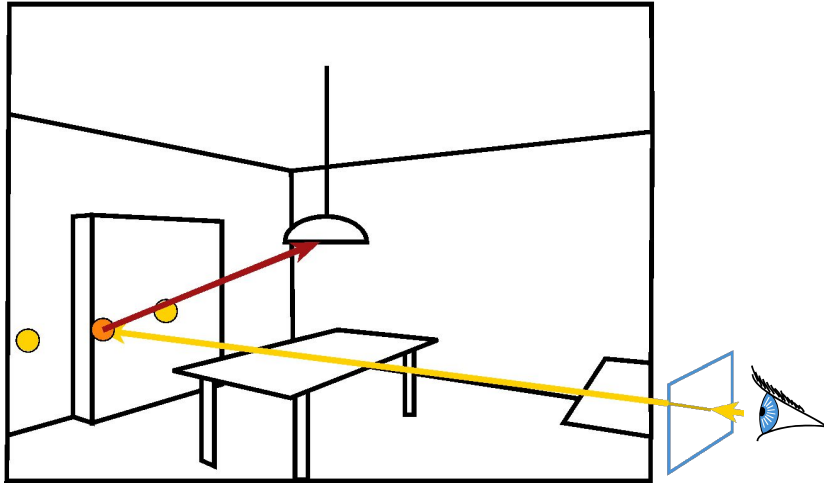
- The indirect illumination is smooth
- Store the indirect illumination



# Irradiance Cache

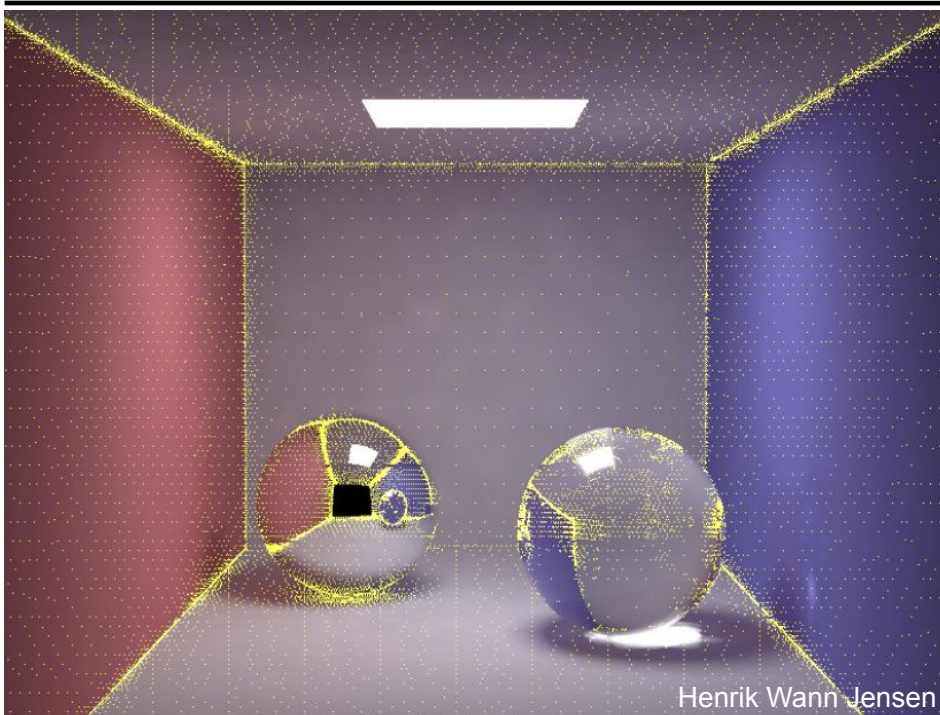
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- Interpolate nearby cached values
- But do full calculation for direct lighting



# Irradiance Cache

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# Questions?

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- Why do we need “good” random numbers?
  - With a fixed random sequence, we see the structure in the error



# Today

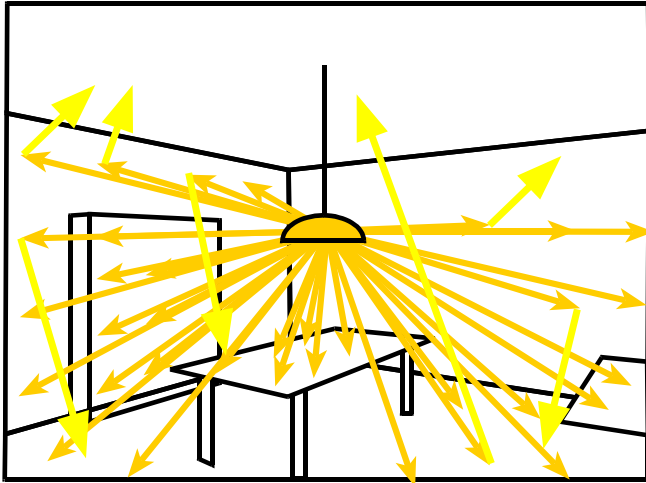
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# Photon Mapping

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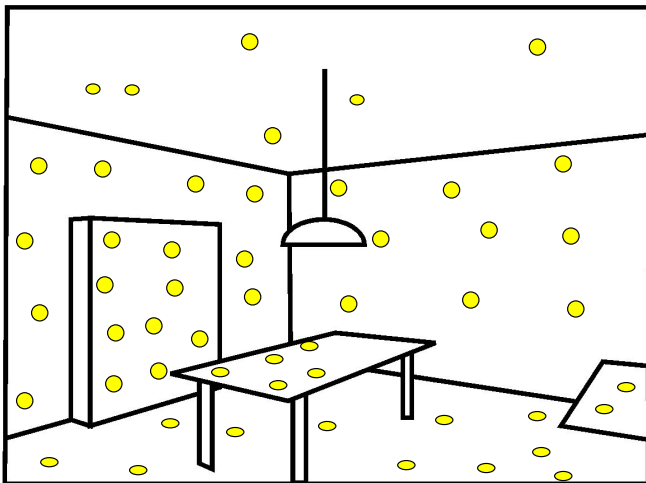
- Preprocess: cast rays from light sources
  - independent of viewpoint



# Photon Mapping

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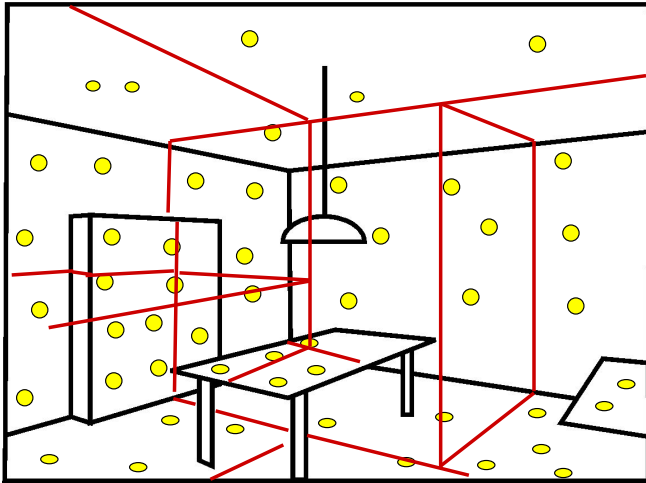
- Store photons
  - position + light power + incoming direction



# Storing the Photon Map

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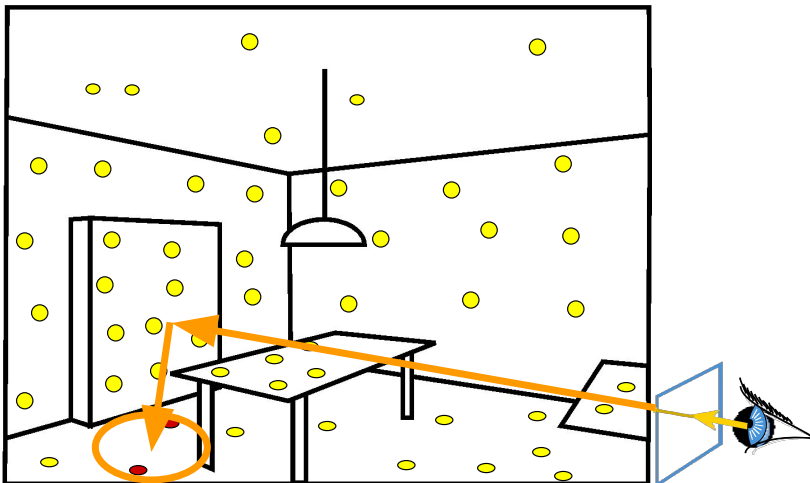
- Efficiently store photons for fast access
- Use hierarchical spatial structure (kd-tree)



# Rendering with Photon Map

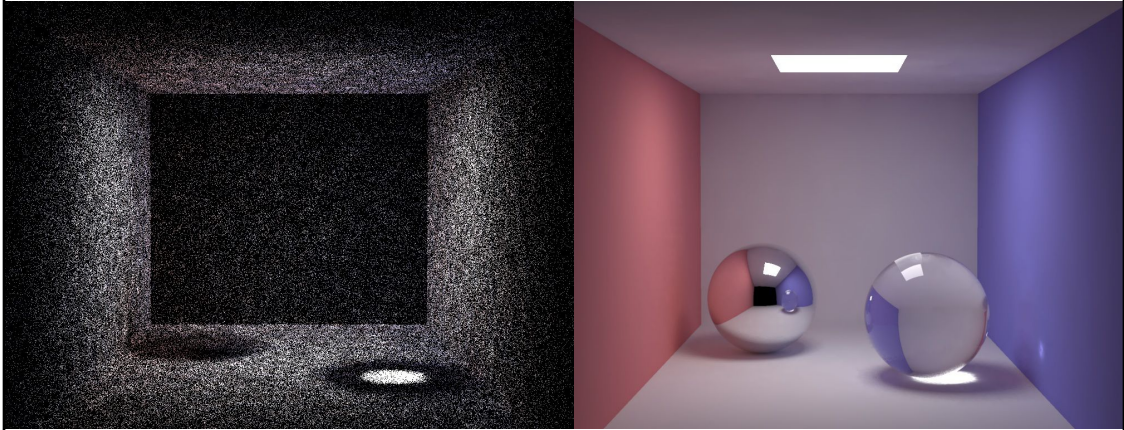
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- Cast primary rays
- For secondary rays: reconstruct irradiance using  $k$  closest photons
- Combine with irradiance caching and other techniques



# Photon Map Results

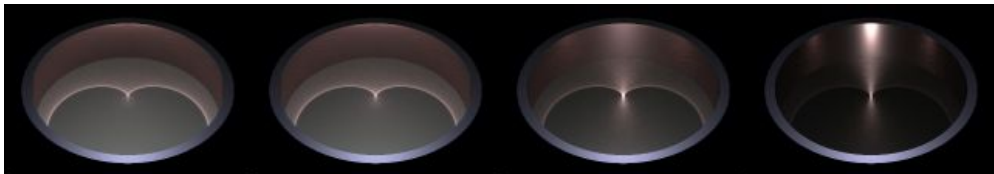
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## Readings for Today:

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- “Rendering Caustics on Non-Lambertian Surfaces”,  
Henrik Wann Jensen, *Graphics Interface* 1996.



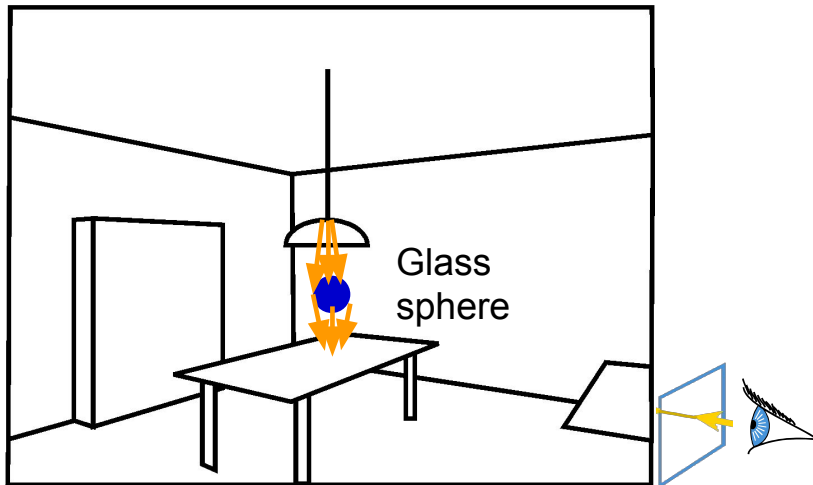
- “Global Illumination using Photon Maps”,  
Henrik Wann Jensen, *Rendering Techniques* 1996.



# Photon Mapping - Caustics

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- Special photon map for specular reflection and refraction

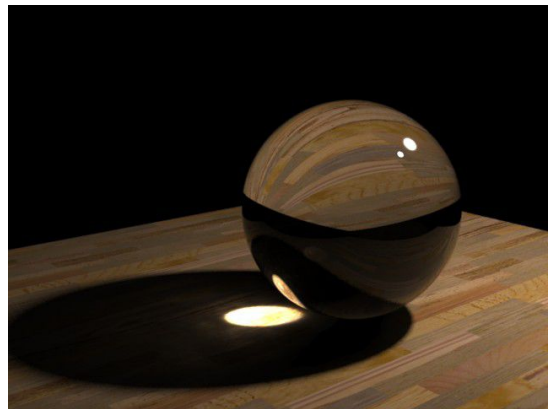


## Comparison

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Path Tracing  
1000 paths/pixel

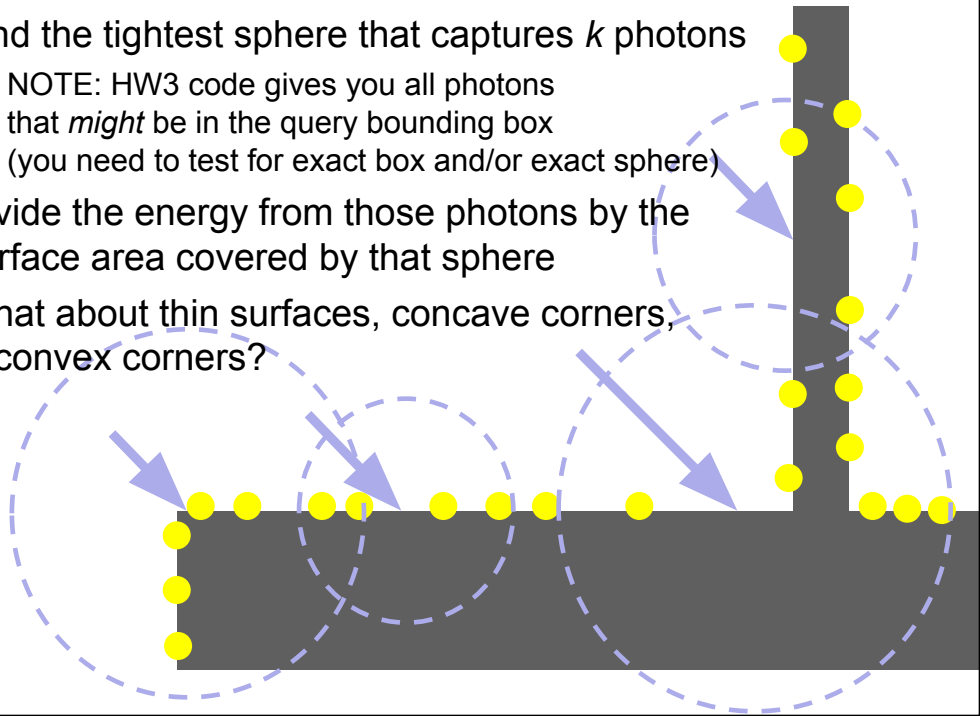
Photon mapping



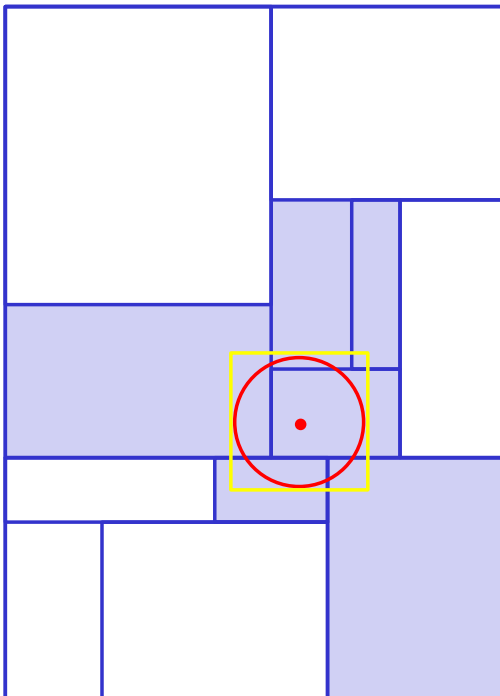
*(similar rendering time)*

# Closest Photon Details

- Find the tightest sphere that captures  $k$  photons
  - NOTE: HW3 code gives you all photons that *might* be in the query bounding box (you need to test for exact box and/or exact sphere)
- Divide the energy from those photons by the surface area covered by that sphere
- What about thin surfaces, concave corners, & convex corners?



# HW3: Photons in the k-d tree



- You start with query point & radius (red)
- You give the `KDTree::CollectPhotonsInBox` function a bounding box (yellow)
- The algorithm finds all k-d tree cells that overlap with bounding box (blue)
- The function returns all photons in those cells
- *You need to discard all photons not in your original query radius*



# Today

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# Ray Grammar

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- Classify local interaction:

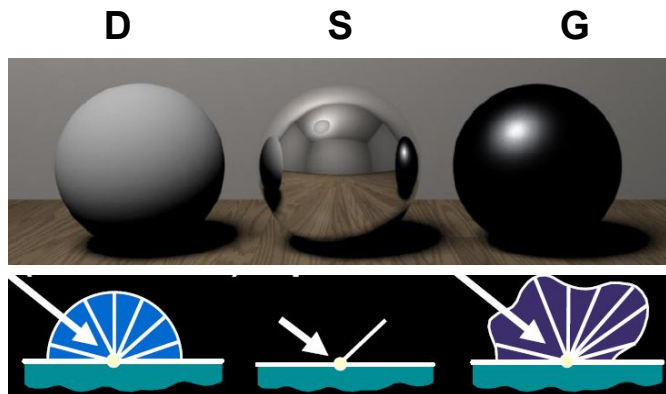
E = eye

L = light

S = perfect specular reflection or refraction

G = glossy scattering

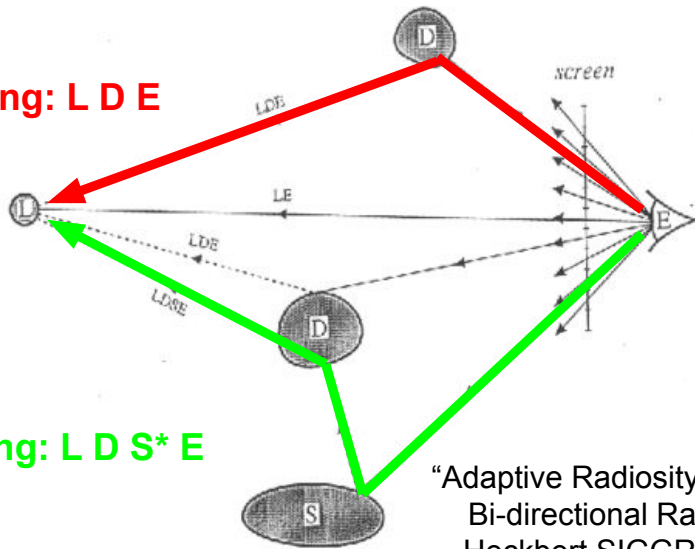
D = diffuse scattering



From Dutre et al.'s slides

# Classic Ray Casting/Tracing

Ray casting: L D E

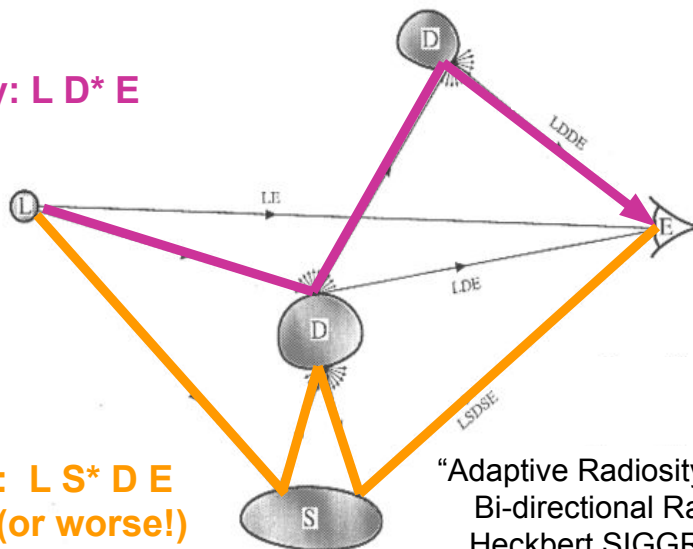


Ray tracing: L D S\* E

“Adaptive Radiosity Textures for Bi-directional Ray Tracing”  
Heckbert SIGGRAPH 1990

# Photon Tracing

Radiosity: L D\* E

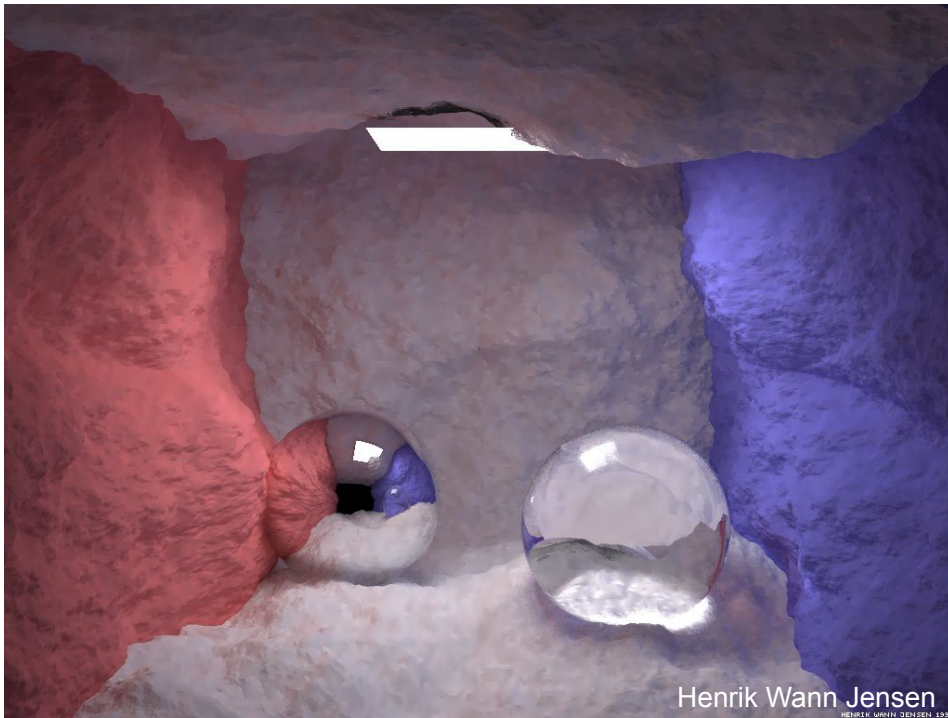


Caustics: L S\* D E  
(or worse!)

“Adaptive Radiosity Textures for Bi-directional Ray Tracing”  
Heckbert SIGGRAPH 1990

# Questions?

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## Readings for Next Time: *(pick one)*

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### “Correlated Multi-Jittered Sampling”, Andrew Kensler, Pixar Technical Memo, 2013

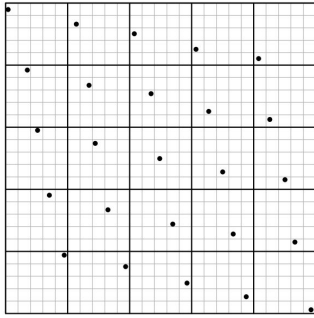


Figure 1: The canonical arrangement. Heavy lines show the boundaries of the 2D jitter cells. Light lines show the horizontal and vertical substrata of N-rooks sampling. Samples are jittered within the subcells.

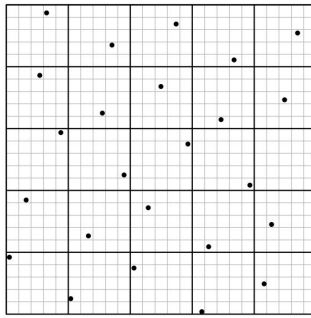


Figure 3: With correlated shuffling.

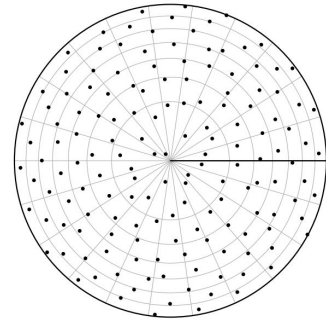


Figure 9: Polar warp with  $m = 22$ ,  $n = 7$ .

<sup>9</sup>G. J. Ward and P. S. Heckbert. Irradiance gradients. In *Third Eurographics Rendering Workshop*, pages 85–98, May 1992.

## Readings for Next Time: *(pick one)*

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### “Implicit Visibility and Antiradiance for Interactive Global Illumination”

Dachsbacher,  
Stamminger,  
Drettakis, and  
Durand  
Siggraph 2007

