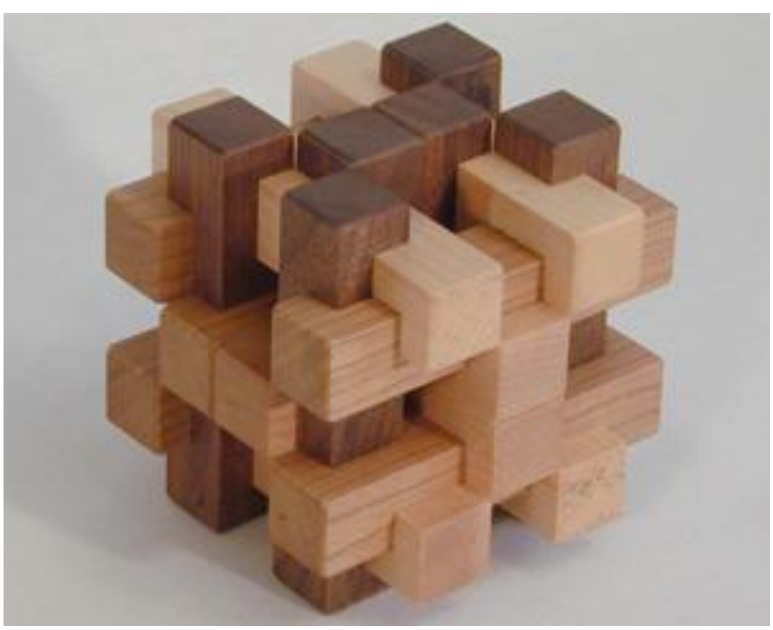
Rigid Body Dynamics, Collision Response, & Deformation

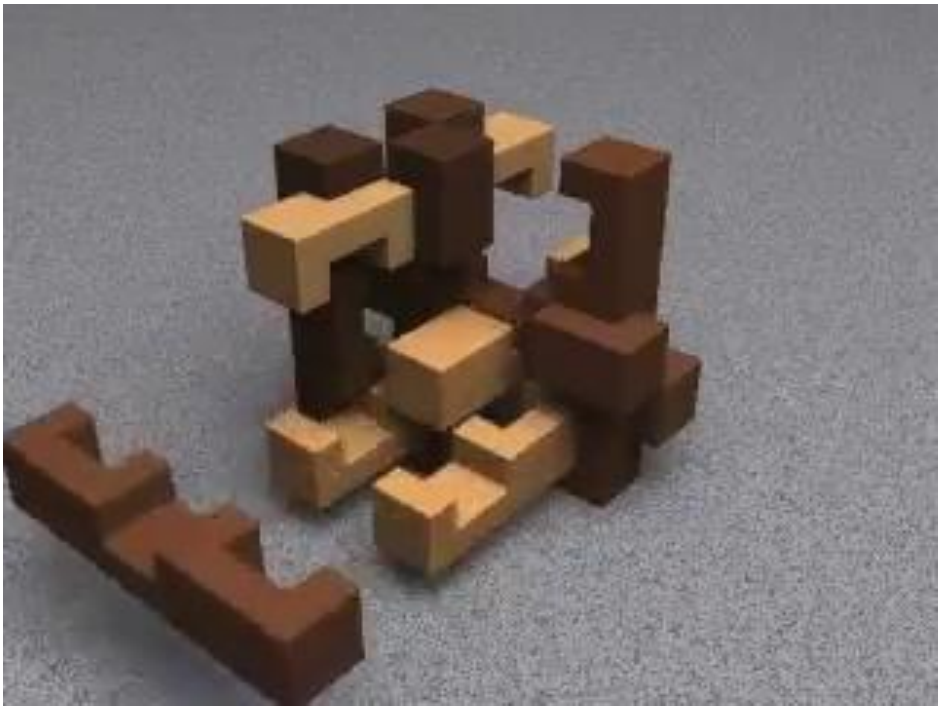
Burr, Justin Legakis ~1999



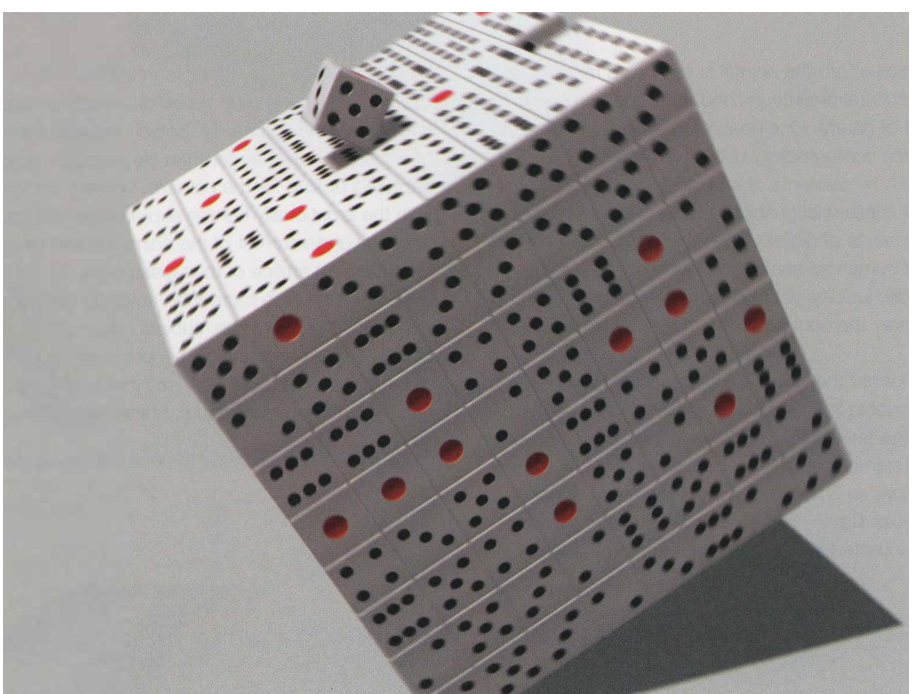
18 Piece Burr, Bill Cutler Puzzles

Burr, Justin Legakis ~1999

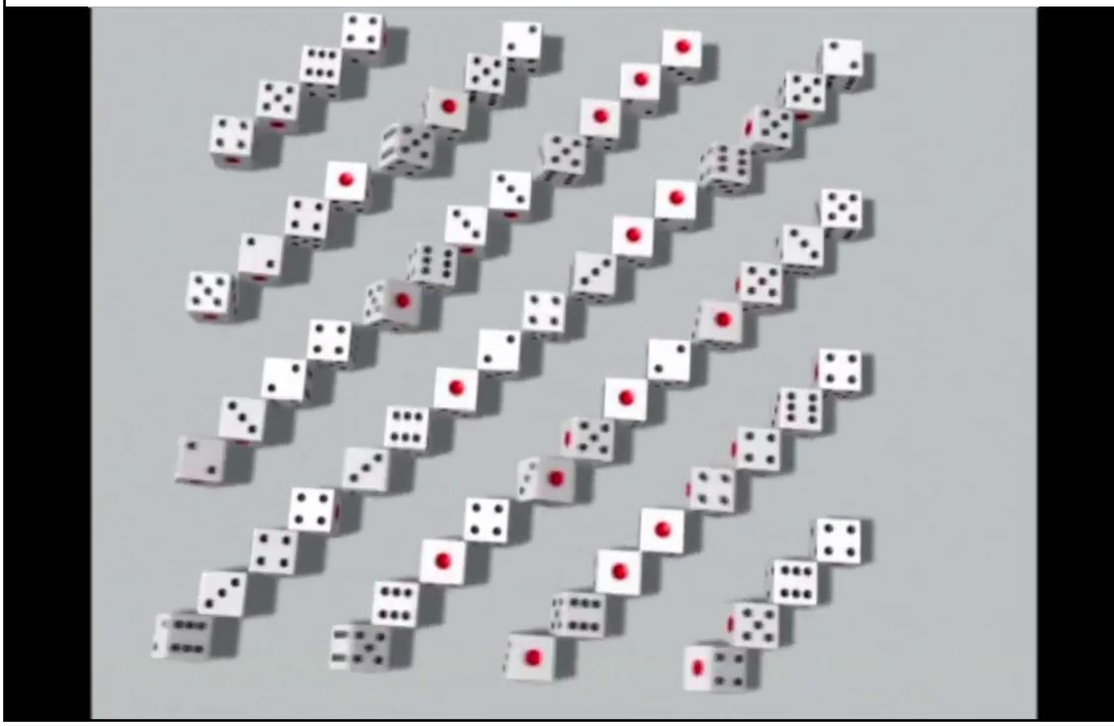
18 Piece Burr
Bill Cutler Puzzles



Dice, Hitoshi Akayama, SIGGRAPH 2005



Dice, Hitoshi Akayama, SIGGRAPH 2005



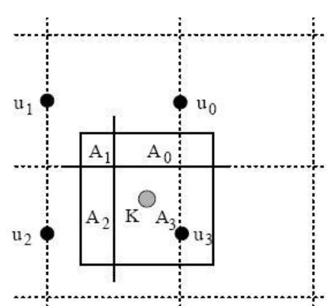
Last Time?

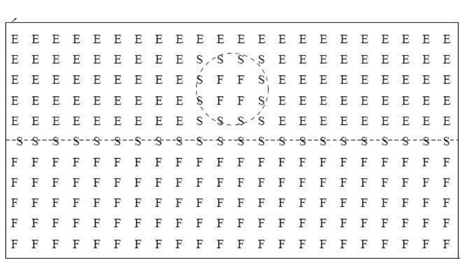
- Navier-Stokes Equations
- Conservation of Momentum & Mass
- Incompressible Flow

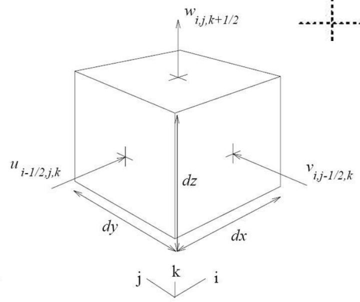
$$\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} = -\frac{\partial p}{\partial x} + g_x + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

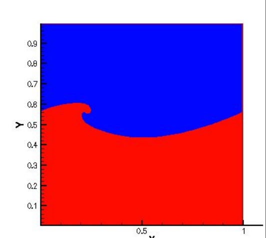
$$\frac{\partial v}{\partial t} + \frac{\partial vu}{\partial x} + \frac{\partial v^2}{\partial y} + \frac{\partial vw}{\partial z} = -\frac{\partial p}{\partial y} + g_y + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$\frac{\partial w}{\partial t} + \frac{\partial wu}{\partial x} + \frac{\partial wv}{\partial y} + \frac{\partial w^2}{\partial z} = -\frac{\partial p}{\partial z} + g_z + \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$









Today

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- Flow Simulation Discussion
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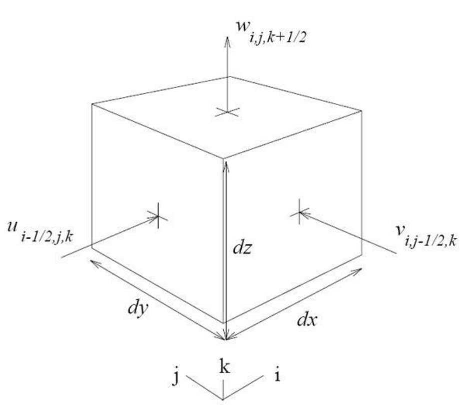
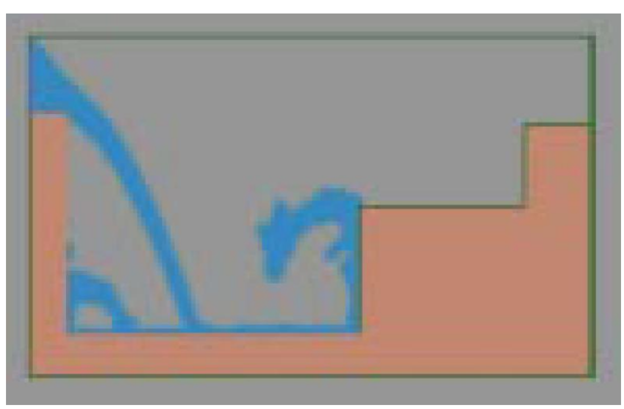
- Finite Element Method
- Papers for Tuesday

Reading for Today

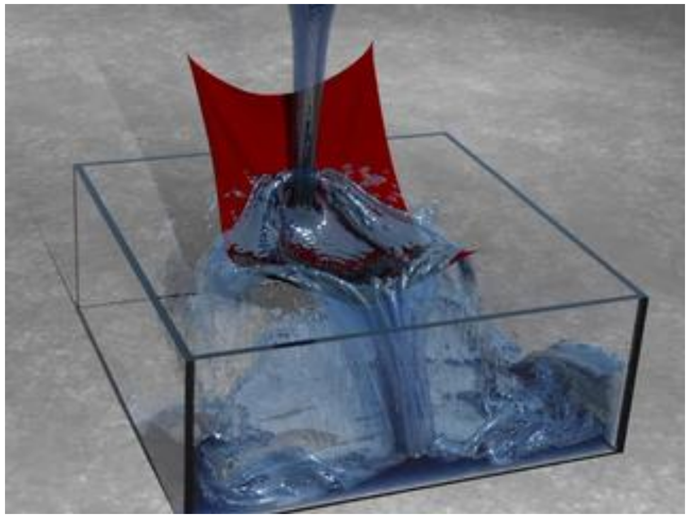
Everyone should read this (simple fluid model used in HW2)

- "Realistic Animation of Liquids", Foster & Metaxas, 1996



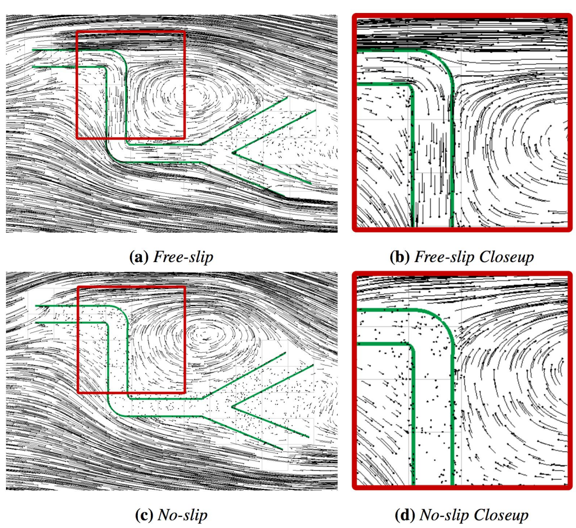


Optional Reading



“Coupling Water and Smoke to Thin Deformable and Rigid Shells”, Guendelman, Selle, Losasso, & Fedkiw, SIGGRAPH 2005.

Optional Reading



(a) *Free-slip* **(b)** *Free-slip Closeup* **(c)** *No-slip* **(d)** *No-slip Closeup*

“Preserving Geometry and Topology for Fluid Flows with Thin Obstacles and Narrow Gaps”
Azevedo, Batty, & Oliveira, SIGGRAPH 2016

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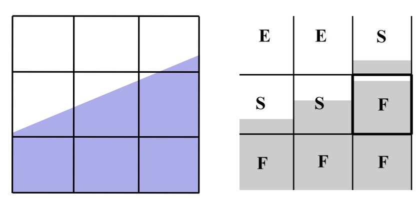
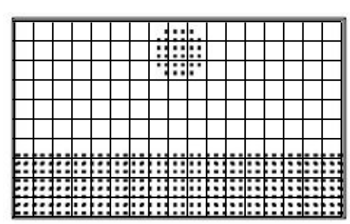
Modeling the Air/Water Surface

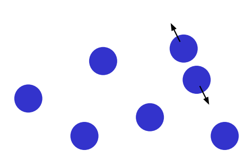
- Volume-of-fluid tracking



- Marker and Cell (MAC)



- Smoothed Particle Hydrodynamics (SPH)

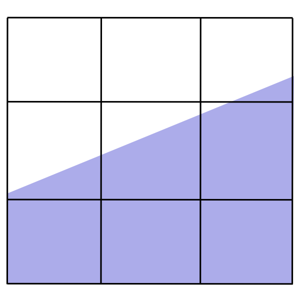


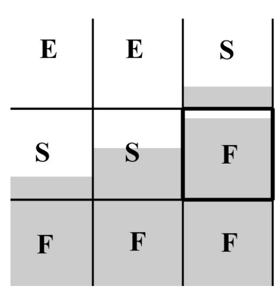
Comparing Representations

- How do we render the resulting surface?
- Are we guaranteed not to lose mass/volume?
  (is the simulation incompressible?)
- How is each affected by the grid resolution and timestep?
- Can we guarantee stability?

Volume-of-fluid-tracking

- Each cell stores a scalar floating point value indicating that cell's "full"-ness

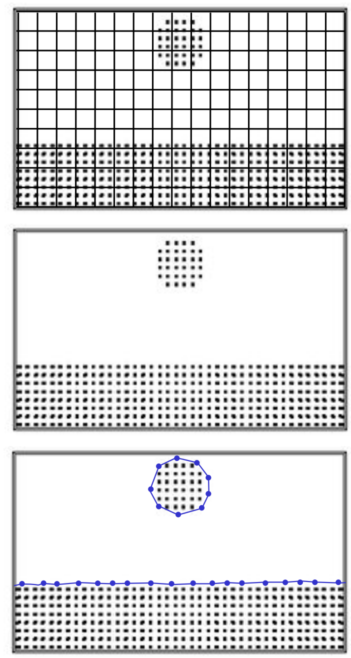




+ preserves volume
– difficult to render
– very dependent on grid resolution

Marker and Cell (MAC)

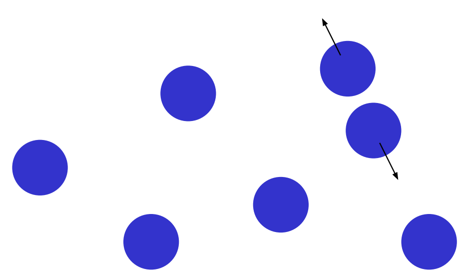
- Harlow & Welch, "Numerical calculation of time-dependent viscous incompressible flow of fluid with free surface", *The Physics of Fluids*, 1965.
- *Volume marker particles* identify location of fluid within the volume
- (Optional) *surface marker particles* track the detailed shape of the fluid/air boundary
- But… marker particles don't have or represent a mass/volume of fluid



+ rendering
– does not preserve volume
– dependent on grid resolution

Smoothed Particle Hydrodynamics (SPH)

- Each particle represents a specific mass of fluid
- "Meshless" (no voxel grid)
- Repulsive forces between neighboring particles maintain constant volume



+ no grid resolution concerns
(now accuracy depends on number/size of particles)
+ volume is preserved*
+ render similar to Marker and Cell (MAC)
– much more expensive (particle-particle interactions)

Note: Usually a spatial data structure (grid!) is added to reduce the number of particle-particle comparisons!

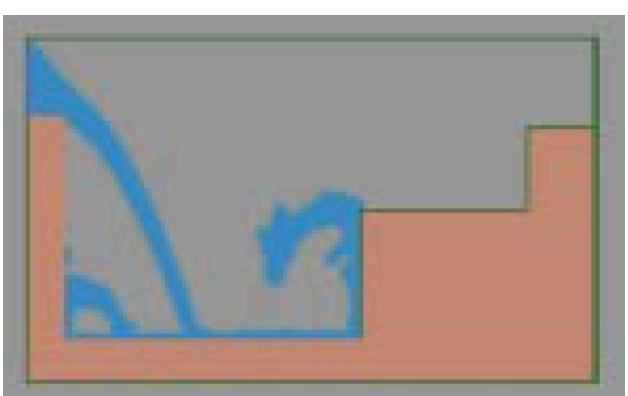
At each Timestep:

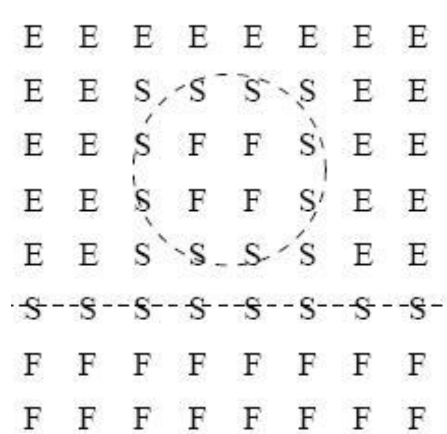
- Identify which cells are Empty, Full, or on the Surface
- Compute new velocities
- Adjust the velocities to maintain an incompressible flow
- Move the particles
  - Interpolate the velocities at the faces
- Render the geometry and repeat!

Empty, Surface & Full Cells

For 2-phase simulations, where we enforce incompressibility for only one phase!

- Empty: no marker particles
- Surface: has an neighbor that is "Empty"
- Full: not "Empty" or "Surface"





Images from Foster & Metaxas, 1996

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Compute New Velocities

$$\begin{aligned}
\tilde{u}_{i+1/2,j,k} = {} & u_{i+1/2,j,k} + \delta t\{(1/\delta x)[(u_{i,j,k})^2 - (u_{i+1,j,k})^2] \\
& +(1/\delta y)[(uv)_{i+1/2,j-1/2,k} - (uv)_{i+1/2,j+1/2,k}] \\
& +(1/\delta z)[(uw)_{i+1/2,j,k-1/2} - (uw)_{i+1/2,j,k+1/2}] + g_x \\
& +(1/\delta x)(p_{i,j,k} - p_{i+1,j,k}) + (\nu/\delta x^2)(u_{i+3/2,j,k} \\
& -2u_{i+1/2,j,k} + u_{i-1/2,j,k}) + (\nu/\delta y^2)(u_{i+1/2,j+1,k} \\
& -2u_{i+1/2,j,k} + u_{i+1/2,j-1,k}) + (\nu/\delta z^2)(u_{i+1/2,j,k+1} \\
& -2u_{i+1/2,j,k} + u_{i+1/2,j,k-1})\},
\end{aligned}$$

Note: some of these values are the *average velocity* within the cell rather than the velocity at a cell face

At each Timestep:

- Identify which cells are Empty, Full, or on the Surface
- Compute new velocities
- Adjust the velocities to maintain an incompressible flow
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- Render the geometry and repeat!

Adjusting the Velocities

- Calculate the *divergence* of the cell (the extra in/out flow)
- The divergence is used to update the *pressure* within the cell
- Adjust each face velocity uniformly to bring the divergence to zero
- Iterate across the entire grid until divergence is < ε

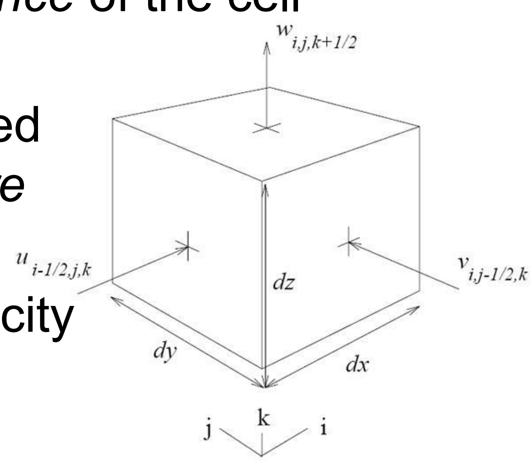
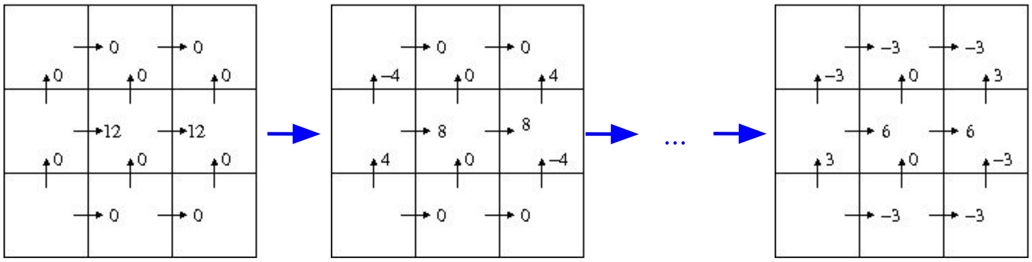


Image from
Foster & Metaxas, 1996

Calculating/Eliminating Divergence



initial flow field *after 1 iteration* *after many iterations*

(results will vary with different calculation order)

0	0	0
–12	0	+12
0	0	0

corresponding divergence

–4	0	4
0	0	0
–4	0	4

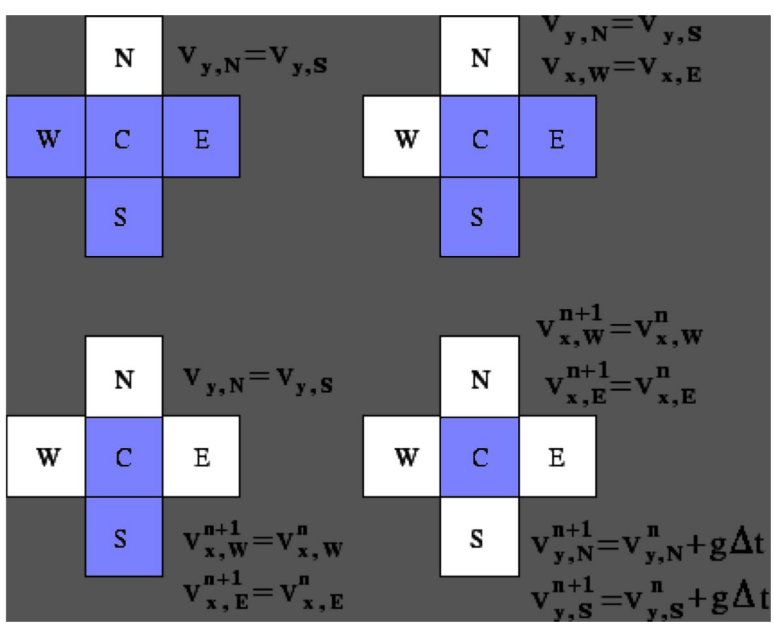
corresponding divergence

0	0	0
0	0	0
0	0	0

corresponding divergence

Handing Free Surface with MAC

- Divergence in surface cells:
  - Is divided equally amongst neighboring empty cells
  - Or other similar strategies?
- Zero out the divergence & pressure in empty cells



At each Timestep:

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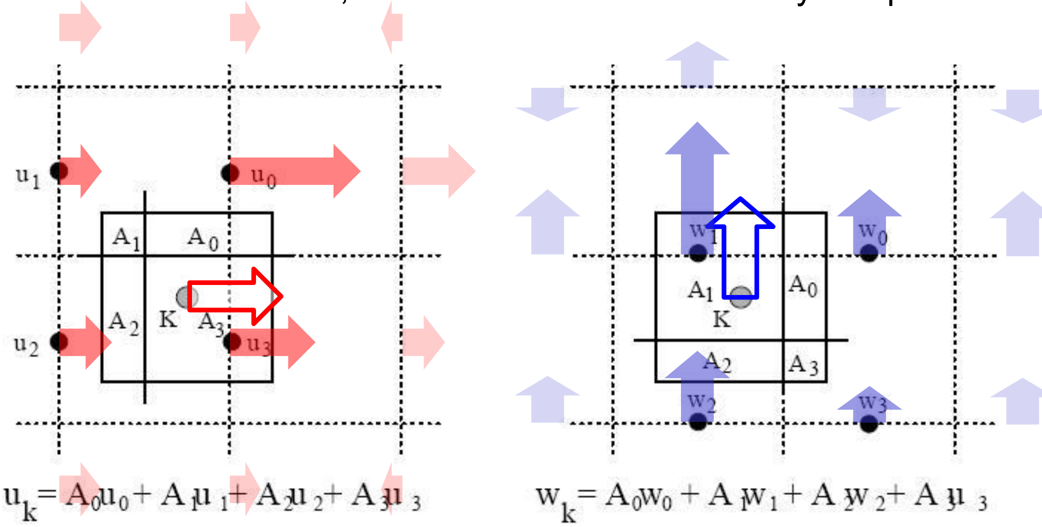
Velocity Interpolation

Original image from Foster & Metaxas, 1996

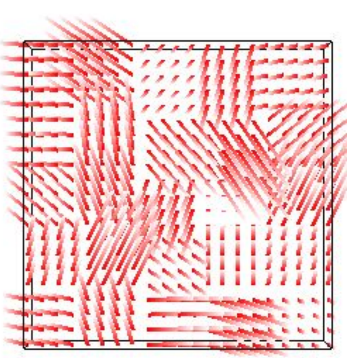
- In 2D: For each axis, find the 4 closest face velocity samples:



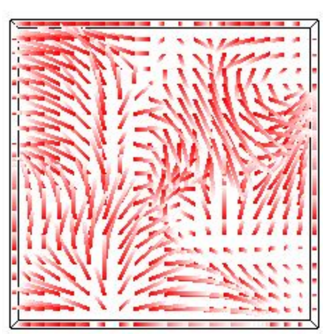
- (In 3D… Find 8 closest face velocities in each dimension)

Correct Velocity Interpolation

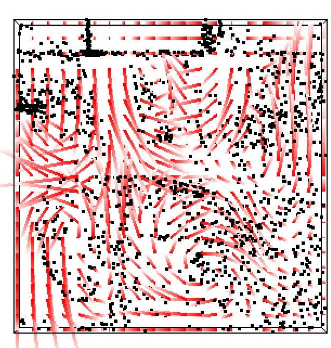
- NOTE: The complete implementation isn't particularly elegant… Storing velocities at face midpoints (req'd for conservation of mass) makes the index math messy!



No interpolation (just use the left/bottom face velocity)
Note the discontinuities in velocity at cell boundaries



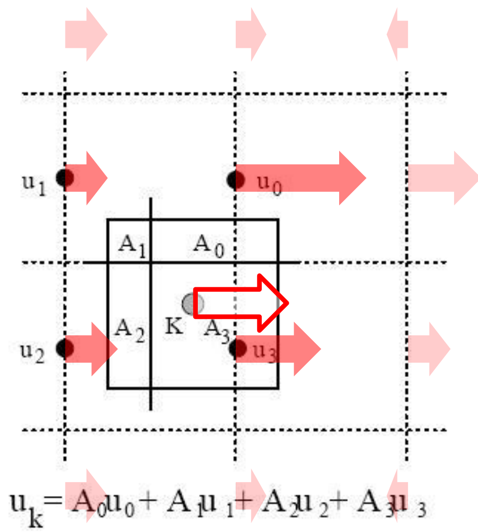
Correct Interpolation
Note that the velocity perpendicular to the outer box is zero

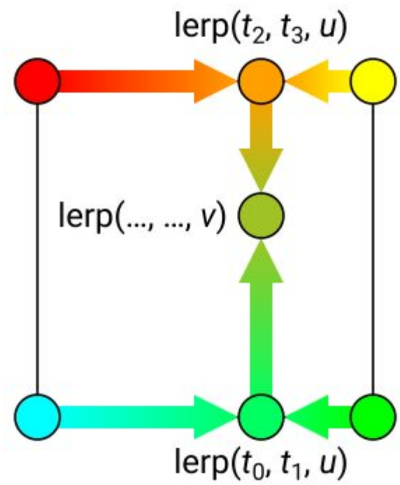


Buggy Interpolation
Note the clumping particles, and the discontinuities at some of the cell borders (& particles might escape the box!)

Velocity Interpolation

- In 2D: For each axis, find the 4 closest face velocity samples:
- (In 3D… Find 8 closest face velocities in each dimension)



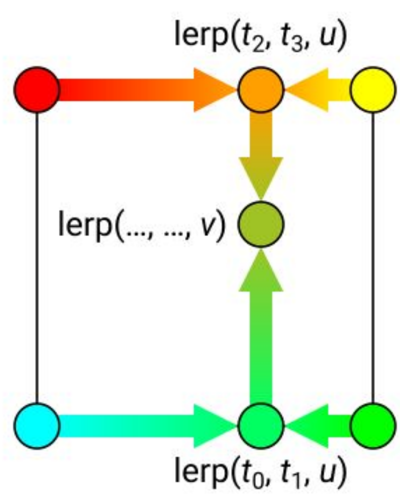


http://reedbeta.com/blog/quadrilateral-interpolation-part-2/

Bilinear Interpolation

- It might be simplest to think about one axis at a time
- It doesn't matter which axis you start with!

- Calculate *u*, the fraction of the distance along the horizontal axis, ***e.g., u=0.65***
- Then calculate the top & bottom averages:
  *orange = (1-u)*red + u*yellow*
  *bluegreen = (1-u)*cyan + u*green*
- Calculate *v*, the fraction of the distance along the vertical axis, ***e.g., v=0.6***
- Then calculate the final average:
  *pukegreen = (1-v)*bluegreen + v*orange*



http://reedbeta.com/blog/quadrilateral-interpolation-part-2/

Pop Worksheet!

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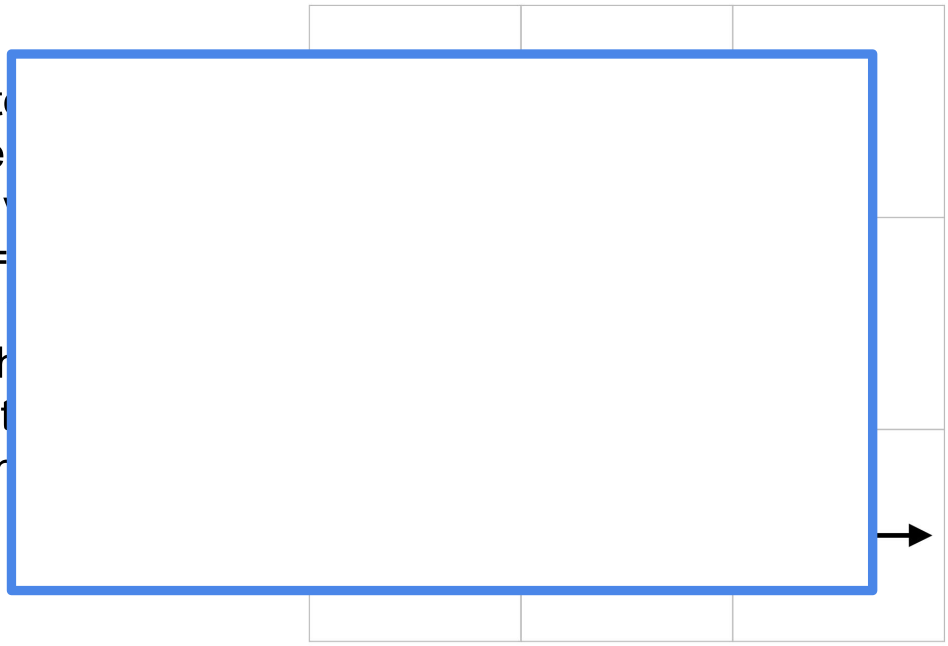

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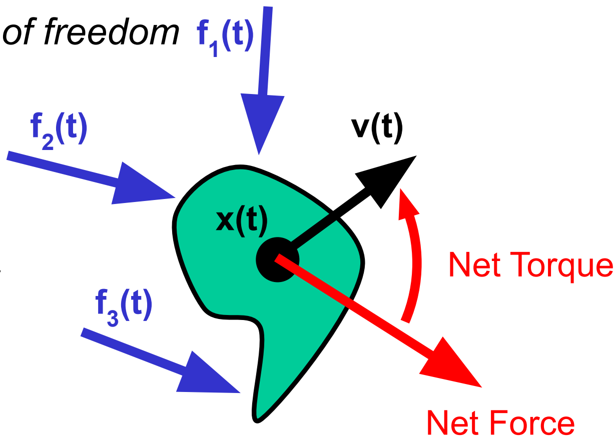
Rigid Body Dynamics

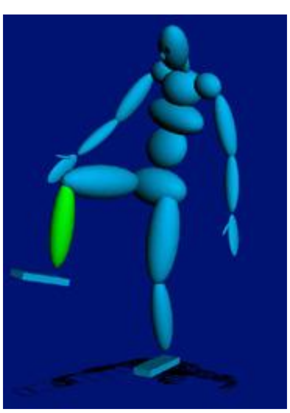
- How do we simulate this object's motion over time?
- We could discretize the object into many particles…
  - But a rigid body does *not* deform
  - Only a few *degrees of freedom*
- Instead, we use only one particle at the center of mass
  - Body has velocity *and* angular velocity
  - Compute net force & net torque



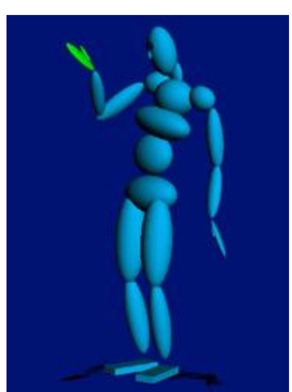
Nice Reference Material: http://www.pixar.com/companyinfo/research/pbm2001/

Degree of Freedom (DOF)

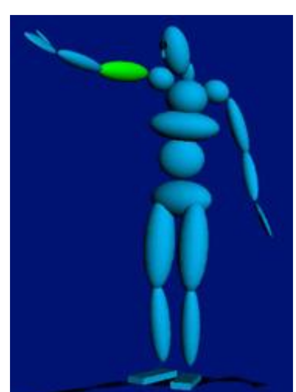
- Rotations:



1 DOF: knee



2 DOF: wrist

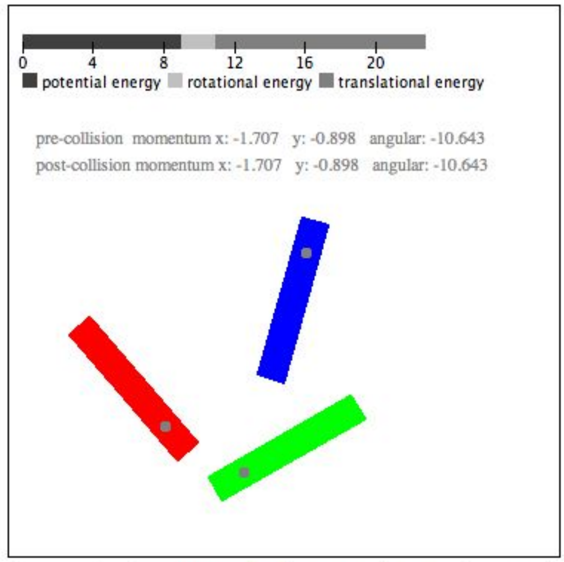


3 DOF: arm

- Translations count too… ➔ 6 DOF

Energy & Rigid Body Collisions

- Total Energy = Kinetic Energy + Potential Energy + Rotational Energy
- Total Energy stays constant if there is no damping and no friction
- Rotational Energy is constant between collisions



http://www.myphysicslab.com/collision.html

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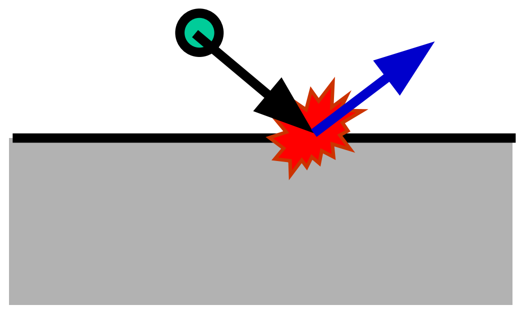
Collisions

- Detection
- Response
- Overshooting problem (when we enter the solid)

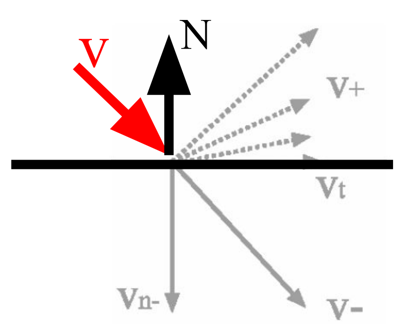


Collision Response

- tangential velocity v_t unchanged
- normal velocity v_n reflects:

$$v = v_t + v_n$$

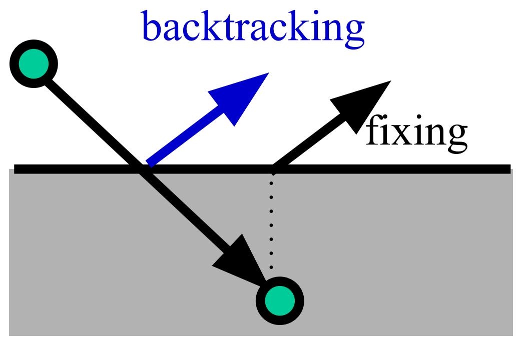
$$v \leftarrow v_t - \varepsilon v_n$$



- coefficient of restitution
  - 1 for elastic
  - 0 for plastic
- change of velocity = *-(1+ϵ)v*
- change of momentum *Impulse = -m(1+ϵ)v*

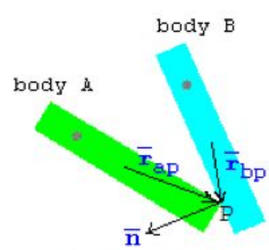
Collisions - Overshooting

- Usually, we detect collision when it’s too late: we’re already inside
- Solutions: back up
  - Compute intersection point
  - Compute response there
  - Advance for remaining fractional time step
- Other solution: Quick and dirty fixup
  - Just project back to object closest point



Collision Between Two Objects

- Suppose a vertex on body A is colliding into an edge of body B at point P. Define the following variables:



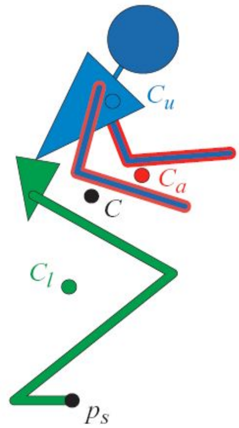
vectors involved in collision

m_a, m_b = mass of bodies A, B
$\bar{r}_{ap}$ = distance vector from center of mass of body A to point P
$\bar{r}_{bp}$ = distance vector from center of mass of body B to point P
ω_{a1}, ω_{b1} = initial pre-collision angular velocity of bodies A, B
ω_{a2}, ω_{b2} = final post-collision angular velocity of bodies A, B
$\bar{v}_{a1}, \bar{v}_{b1}$ = initial pre-collision velocities of center of mass bodies A, B
$\bar{v}_{a2}, \bar{v}_{b2}$ = final post-collision velocities of center of mass bodies A, B
$\bar{v}_{ap1}$ = initial pre-collision velocity of impact point on body A
$\bar{v}_{bp1}$ = initial pre-collision velocity of impact point on body B
$\bar{n}$ = normal (perpendicular) vector to edge of body B
e = elasticity (0 = inelastic, 1 = perfectly elastic)

http://www.myphysicslab.com/collision.html

Center of Mass & Moment of Inertia

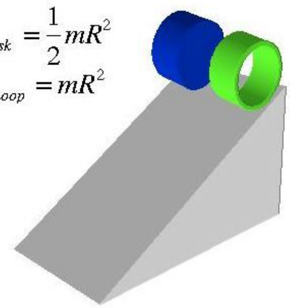
- Center of Mass: mean location of all mass in the system
- Moment of Inertia: a measure of an object's resistance to changes to its rotation
- If a solid cylinder & a hollow tube have the ame radius & the same mass, which will reach the bottom of the ramp first?



Liu & Popović

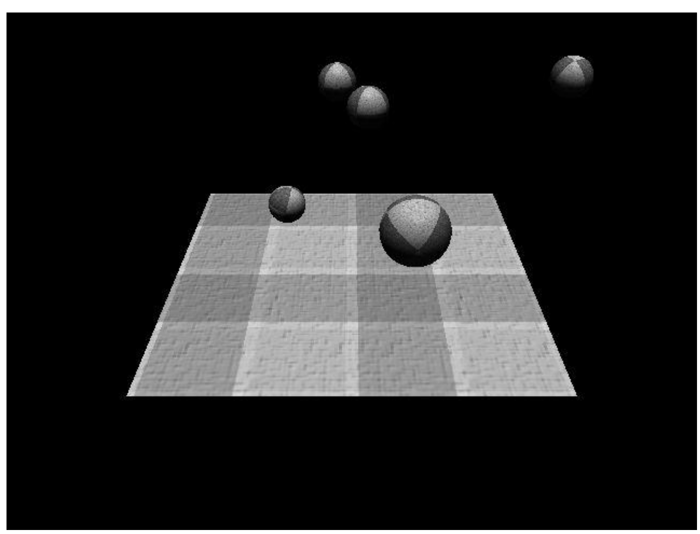

http://en.wikipedia.org/wiki/Fosbury_Flop

$$I_{disk} = \frac{1}{2} mR^2$$
$$I_{hoop} = mR^2$$


http://solomon.physics.sc.edu/~tedeschi/demo/demo12.html
http://hyperphysics.phy-astr.gsu.edu/hbase/hoocyl2.html

Rigid Body Dynamics

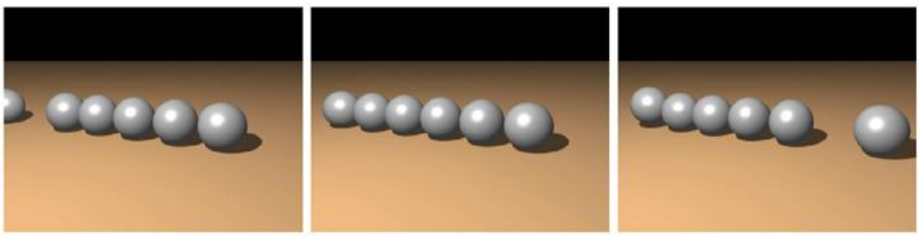
- Physics
  - Velocity
  - Acceleration
  - Angular Momentum
- Collisions
- Friction



from: Darren Lewis
http://www-cs-students.stanford.edu/~dalewis/cs448a/rigidbody.html

Advanced Collisions

- What about friction?
- What if the contact between two objects is not a single point?
- What if more than two objects collide simultaneously?

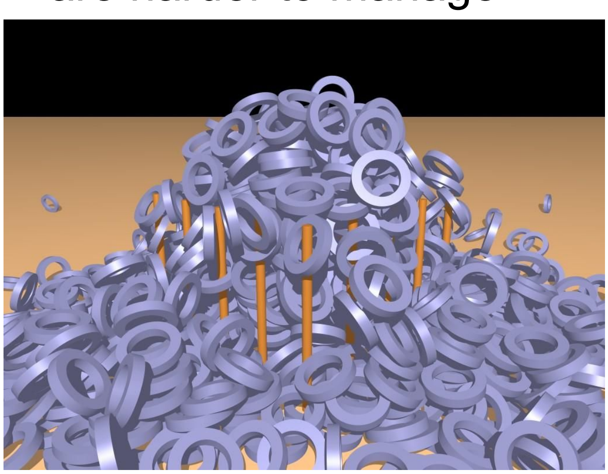


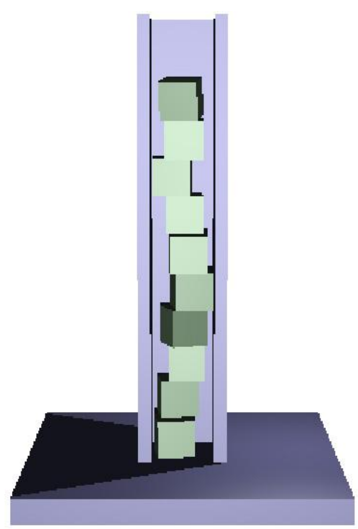
Guendelman, Bridson & Fedkiw
Nonconvex Rigid Bodies with Stacking
SIGGRAPH 2003

Resting Collisions

Victor J. Milenkovic & Harald Schmidl
Optimization-Based Animation
SIGGRAPH 2001

- We know how to simulate bouncing really well
- But resting collisions are harder to manage





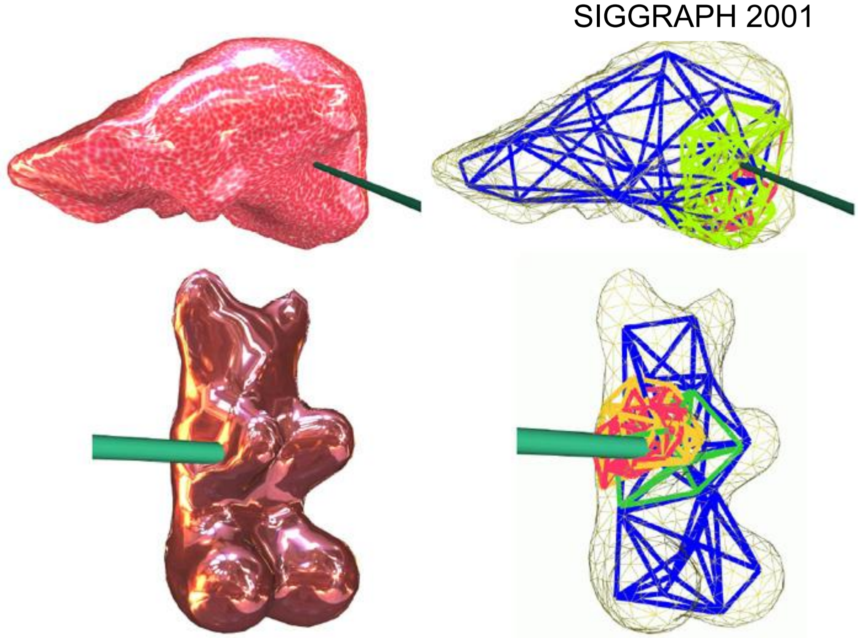
Guendelman, Bridson & Fedkiw
Nonconvex Rigid Bodies with Stacking, SIGGRAPH 2003

Today

- Readings for Today
- Flow Simulation Discussion
- Velocity Interpolation Worksheet
- Rigid Body Dynamics
- Collision Response
- Non-Rigid, Deformable Objects
- Finite Element Method
- Papers for Tuesday

Deformation & Level of Detail

Gilles Debunne , Mathieu Desbrun, Marie-Paule Cani, & Alan H. Barr
Dynamic Real-Time Deformations using Space & Time Adaptive Sampling
SIGGRAPH 2001



Simulation of Non-Rigid Objects

- We modeled string & cloth using mass-spring systems. Can we do the same?
- Yes…
- But a more physically accurate model uses *volumetric elements:*

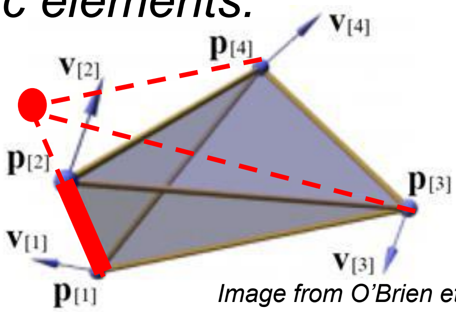
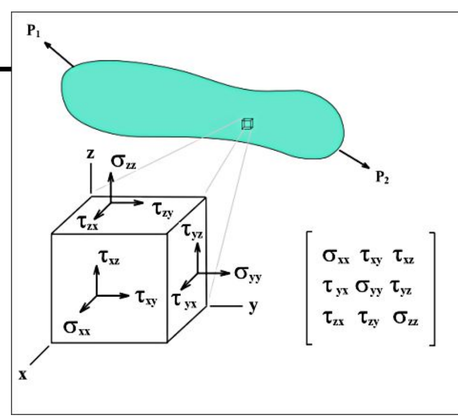


Image from O'Brien et al. 1999

Strain & Stress

- Stress
  - the internal distribution of forces within a body that balance and react to the loads applied to it
  - *normal stress* & *shear stress*



http://en.wikipedia.org/wiki/Image:Stress_tensor.png

- Strain
  - material deformation caused by stress.
  - measured by the change in length of a line or by the change in angle between two lines

$$\varepsilon = \frac{\Delta_l}{l_0}$$

Finite Element Method

- To solve the continuous problem (deformation of all points of the object)
  - Discretize the problem
  - Express the interrelationship
  - Solve a big linear system
- More principled than Mass-Spring

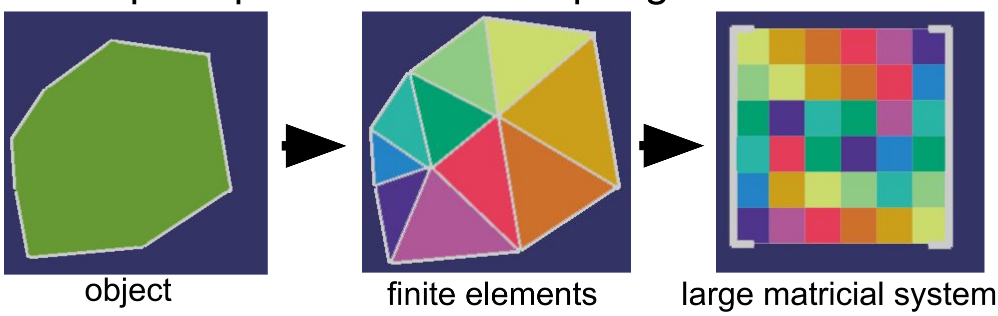


Diagram from Debunne et al. 2001

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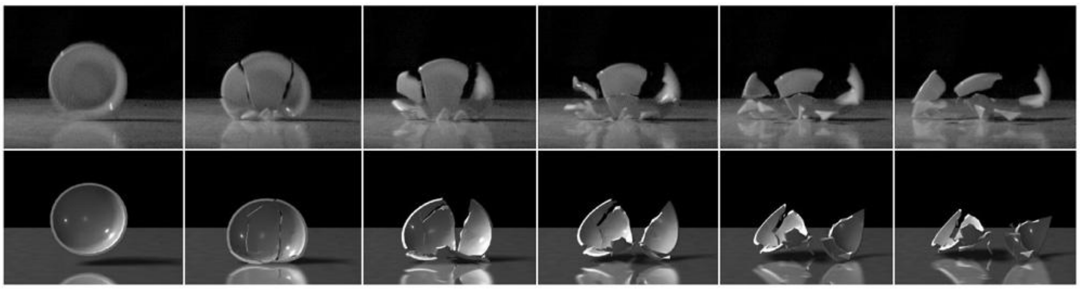
Readings for Tuesday… *(pick one)*

- James O'Brien & Jessica Hodgins "*Graphical Modeling and Animation of Brittle Fracture*" SIGGRAPH 1999.

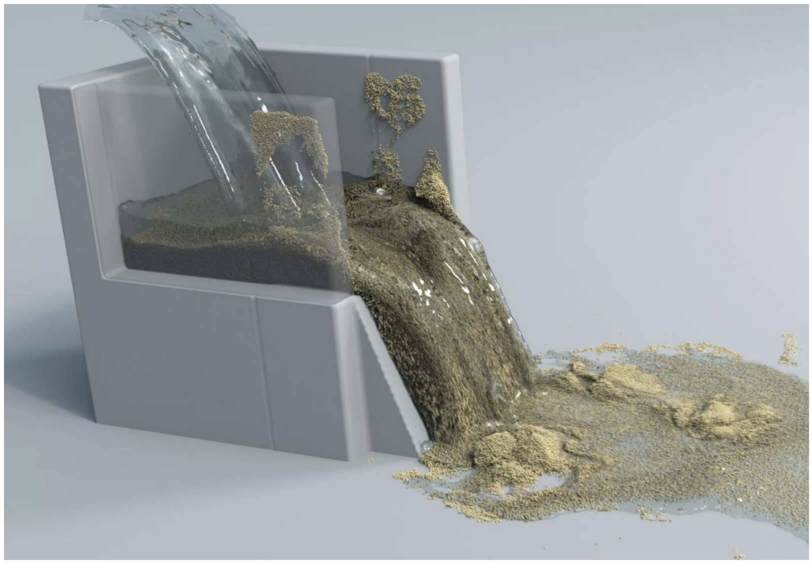


- Fracture threshhold
- Remeshing
  - need connectivity info!
- Material properties
- Parameter tuning

Readings for Tuesday… *(pick one)*

"Robust eXtended Finite Elements for Complex Cutting of Deformables", Koschier, Bender, & Thuerey, SIGGRAPH 2017

Readings for Tuesday… *(pick one)*



"Multi-species simulation of porous sand and water mixtures", Pradhana, Gast, Klar, Fu, Teran, Jiang, and Museth, SIGGRAPH 2017.