Rigid Body Dynamics, Collision Response, & Deformation

Burr, Justin Legakis ~1999

18 Piece Burr, Bill Cutler Puzzles
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Bill Cutler Puzzles

Dice, Hitoshi Akayama, SIGGRAPH 2005
Last Time?

• Navier-Stokes Equations
• Conservation of Momentum & Mass
• Incompressible Flow
Today

- Readings for Today
- Flow Simulation Discussion
- Velocity Interpolation Worksheet
- Rigid Body Dynamics
- Collision Response
- Non-Rigid, Deformable Objects
- Finite Element Method
- Papers for Tuesday

Reading for Today

- “Realistic Animation of Liquids”, Foster & Metaxas, 1996

Everyone should read this
(simple fluid model used in HW2)
Optional Reading


Optional Reading

“Preserving Geometry and Topology for Fluid Flows with Thin Obstacles and Narrow Gaps”
Azevedo, Batty, & Oliveira, SIGGRAPH 2016
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Modeling the Air/Water Surface

- Volume-of-fluid tracking

- Marker and Cell (MAC)

- Smoothed Particle Hydrodynamics (SPH)
Comparing Representations

- How do we render the resulting surface?
- Are we guaranteed not to lose mass/volume? (is the simulation incompressible?)
- How is each affected by the grid resolution and timestep?
- Can we guarantee stability?

Volume-of-fluid-tracking

- Each cell stores a scalar floating point value indicating that cell’s “full”-ness

+ preserves volume
  - difficult to render
  - very dependent on grid resolution
Marker and Cell (MAC)

- *Volume marker particles* identify location of fluid within the volume
- (Optional) *surface marker particles* track the detailed shape of the fluid/air boundary
- But… marker particles don’t have or represent a mass/volume of fluid

+ rendering
  - does not preserve volume
  - dependent on grid resolution

Smoothed Particle Hydrodynamics (SPH)

- Each particle represents a specific mass of fluid
- “Meshless” (no voxel grid)
- Repulsive forces between neighboring particles maintain constant volume

+ no grid resolution concerns
  (now accuracy depends on number/size of particles)
+ volume is preserved*
+ render similar to Marker and Cell (MAC)
  - much more expensive (particle-particle interactions)

*Note: Usually a spatial data structure (grid!) is added to reduce the number of particle-particle comparisons!
At each Timestep:

- Identify which cells are Empty, Full, or on the Surface
- Compute new velocities
- Adjust the velocities to maintain an incompressible flow
- Move the particles
  - Interpolate the velocities at the faces
- Render the geometry and repeat!

Empty, Surface & Full Cells

- Empty: no marker particles
- Surface: has a neighbor that is “Empty”
- Full: not “Empty” or “Surface”

For 2-phase simulations, where we enforce incompressibility for only one phase!
At each Timestep:

- Identify which cells are Empty, Full, or on the Surface
- Compute new velocities
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Compute New Velocities

\[
\tilde{u}_{i+1/2,j,k} = u_{i+1/2,j,k} + \delta t \left\{ (1/\delta x)[(u_{i,j,k})^2 - (u_{i+1,j,k})^2] \\
+ (1/\delta y)[(uv)_{i+1/2,j-1/2,k} - (uv)_{i+1/2,j+1/2,k}] \\
+ (1/\delta z)[(uw)_{i+1/2,j,k-1/2} - (uw)_{i+1/2,j,k+1/2}] + g_x \\
+ (1/\delta x)(p_{i,j,k} - p_{i+1,j,k}) + (\nu/\delta x^2)(u_{i+3/2,j,k} \\
- 2u_{i+1/2,j,k} + u_{i-1/2,j,k}) + (\nu/\delta y^2)(u_{i+1/2,j+1,k} \\
- 2u_{i+1/2,j,k} + u_{i+1/2,j-1,k}) + (\nu/\delta z^2)(u_{i+1/2,j,k+1} \\
- 2u_{i+1/2,j,k} + u_{i+1/2,j,k-1}) \right\},
\]

Note: some of these values are the average velocity within the cell rather than the velocity at a cell face
At each Timestep:

- Identify which cells are Empty, Full, or on the Surface
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Adjusting the Velocities

- Calculate the divergence of the cell (the extra in/out flow)
- The divergence is used to update the pressure within the cell
- Adjust each face velocity uniformly to bring the divergence to zero
- Iterate across the entire grid until divergence is < ε
Calculating/Eliminating Divergence

Initial flow field

After 1 iteration
(results will vary with different calculation order)

After many iterations

Handing Free Surface with MAC

- Divergence in surface cells:
  - Is divided equally amongst neighboring empty cells
  - Or other similar strategies?
- Zero out the divergence & pressure in empty cells
At each Timestep:

- Identify which cells are Empty, Full, or on the Surface
- Compute new velocities
- Adjust the velocities to maintain an incompressible flow
- **Move the particles**
  - Interpolate the velocities at the faces
- Render the geometry and repeat!

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**Velocity Interpolation**

In 2D: For each axis, find the 4 closest face velocity samples:

\[ u_k = A_0 u_0 + A_1 u_1 + A_2 u_2 + A_3 u_3 \]

(In 3D… Find 8 closest face velocities in each dimension)

**Correct Velocity Interpolation**

NOTE: The complete implementation isn’t particularly elegant… Storing velocities at face midpoints (req’d for conservation of mass) makes the index math messy!

No interpolation (just use the left/bottom face velocity)

Correct Interpolation

Buggy Interpolation

Note the discontinuities in velocity at cell boundaries

Note that the velocity perpendicular to the outer box is zero

Note the clumping particles, and the discontinuities at some of the cell borders (& particles might escape the box!)
Velocity Interpolation

- In 2D: For each axis, find the 4 closest face velocity samples:
- (In 3D… Find 8 closest face velocities in each dimension)

Velocity Interpolation

- It might be simplest to think about one axis at a time
- It doesn’t matter which axis you start with!

- Calculate \( u \), the fraction of the distance along the horizontal axis, \( \text{e.g., } u=0.65 \)
- Then calculate the top & bottom averages:
  \[
  \text{orange} = (1-u) \times \text{red} + u \times \text{yellow} \\
  \text{bluegreen} = (1-u) \times \text{cyan} + u \times \text{green}
  \]

- Calculate \( v \), the fraction of the distance along the vertical axis, \( \text{e.g., } v=0.6 \)
- Then calculate the final average:
  \[
  \text{pukegreen} = (1-v) \times \text{bluegreen} + v \times \text{orange}
  \]
Determine the interpolated 2D velocity (x = horizontal, y = vertical) at the lettered white query dots in the diagram.

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Rigid Body Dynamics

• How do we simulate this object’s motion over time?
• We could discretize the object into many particles…
  – But a rigid body does not deform
  – Only a few degrees of freedom $f_1(t)$
• Instead, we use only one particle at the center of mass
  – Body has velocity and angular velocity
  – Compute net force and net torque

Net Force

Net Torque

$$\mathbf{f}_1(t)$$
$$\mathbf{f}_2(t)$$
$$\mathbf{f}_3(t)$$

$x(t)$

$v(t)$

Degree of Freedom (DOF)

• Rotations:

  1 DOF: knee
  2 DOF: wrist
  3 DOF: arm

• Translations count too… $\rightarrow$ 6 DOF
Energy & Rigid Body Collisions

• Total Energy = Kinetic Energy + Potential Energy + Rotational Energy
• Total Energy stays constant if there is no damping and no friction
• Rotational Energy is constant between collisions

http://www.myphysicslab.com/collision.html

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Collisions

- Detection
- Response
- Overshooting problem (when we enter the solid)

Collision Response

- tangential velocity $v_t$ unchanged
- normal velocity $v_n$ reflects:
  \[ v = v_t + v_n \]
  \[ v \leftarrow v_t - \varepsilon v_n \]
- coefficient of restitution
  - 1 for elastic
  - 0 for plastic
- change of velocity $= -(1+\varepsilon)v$
- change of momentum \textit{Impulse} $= -m(1+\varepsilon)v$
Collisions - Overshooting

• Usually, we detect collision when it’s too late: we’re already inside

• Solutions: back up
  – Compute intersection point
  – Compute response there
  – Advance for remaining fractional time step

• Other solution: Quick and dirty fixup
  – Just project back to object closest point

Collision Between Two Objects

• Suppose a vertex on body A is colliding into an edge of body B at point P. Define the following variables:

\[ m_a, m_b = \text{mass of bodies A, B} \]
\[ \mathbf{r}_{ap} = \text{distance vector from center of mass of body A to point P} \]
\[ \mathbf{r}_{bp} = \text{distance vector from center of mass of body B to point P} \]
\[ \omega_a, \omega_b = \text{initial pre-collision angular velocity of bodies A, B} \]
\[ \omega_{a1}, \omega_{b1} = \text{final post-collision angular velocity of bodies A, B} \]
\[ \mathbf{v}_{a1}, \mathbf{v}_{b1} = \text{initial pre-collision velocities of center of mass bodies A, B} \]
\[ \mathbf{v}_{a2}, \mathbf{v}_{b2} = \text{final post-collision velocities of center of mass bodies A, B} \]
\[ \mathbf{v}_{ap1} = \text{initial pre-collision velocity of impact point on body A} \]
\[ \mathbf{v}_{bp1} = \text{initial pre-collision velocity of impact point on body B} \]
\[ \mathbf{n} = \text{normal (perpendicular) vector to edge of body B} \]
\[ e = \text{elasticity (0 = inelastic, 1 = perfectly elastic)} \]

http://www.myphysicslab.com/collision.html
Center of Mass & Moment of Inertia

- Center of Mass: mean location of all mass in the system
- Moment of Inertia: a measure of an object's resistance to changes to its rotation
- If a solid cylinder & a hollow tube have the same radius & the same mass, which will reach the bottom of the ramp first?

http://solomon.physics.sc.edu/~tedeschi/demo/demo12.html
http://hyperphysics.phy-astr.gsu.edu/hbase/hoocyl2.html

Rigid Body Dynamics

- Physics
  - Velocity
  - Acceleration
  - Angular Momentum
- Collisions
- Friction

from: Darren Lewis
http://www-cs-students.stanford.edu/~dalewis/cs448a/rigidbody.html
Advanced Collisions

- What about friction?
- What if the contact between two objects is not a single point?
- What if more than two objects collide simultaneously?

Guendelman, Bridson & Fedkiw
*Nonconvex Rigid Bodies with Stacking*
SIGGRAPH 2003

Resting Collisions

- We know how to simulate bouncing really well
- But resting collisions are harder to manage

Victor J. Milenkovic & Harald Schmidl
*Optimization-Based Animation*
SIGGRAPH 2001

Guendelman, Bridson & Fedkiw
*Nonconvex Rigid Bodies with Stacking*, SIGGRAPH 2003
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Deformation & Level of Detail

Gilles Debunne, Mathieu Desbrun, Marie-Paule Cani, & Alan H. Barr

*Dynamic Real-Time Deformations using Space & Time Adaptive Sampling*

SIGGRAPH 2001
Simulation of Non-Rigid Objects

- We modeled string & cloth using mass-spring systems. Can we do the same?
- Yes…
- But a more physically accurate model uses *volumetric elements*:

![Image from O'Brien et al. 1999](http://en.wikipedia.org/wiki/Image:Stress_tensor.png)

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Strain & Stress

- **Stress**
  - the internal distribution of forces within a body that balance and react to the loads applied to it
  - *normal stress* & *shear stress*
- **Strain**
  - material deformation caused by stress.
  - measured by the change in length of a line or by the change in angle between two lines

$$ \varepsilon = \frac{\Delta l}{l_0} $$
Finite Element Method

- To solve the continuous problem (deformation of all points of the object)
  - Discretize the problem
  - Express the interrelationship
  - Solve a big linear system
- More principled than Mass-Spring

Diagram from Debunne et al. 2001

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Readings for Tuesday… *(pick one)*


- Fracture threshold
- Remeshing
  – need connectivity info!
- Material properties
- Parameter tuning

"Robust eXtended Finite Elements for Complex Cutting of Deformables", Koschier, Bender, & Thuerey, SIGGRAPH 2017
Readings for Tuesday… (pick one)