Ray Tracing

https://i.imgur.com/i7Aohc0.jpg

Fiat Lux, Debevec, 1999
Last Time?

- Keyframing
- Procedural Animation
- Physically-Based Animation
- Forward and Inverse Kinematics
- Motion Capture
Today

- Reading for Today
- Ray Casting
- Ray Tracing
- Recursive Ray Tracing
- Distributed Ray Tracing
- Readings for Friday

Reading for Today

Today

- Reading for Today
- Ray Casting
  - Ray-Plane Intersection
  - Ray-Sphere Intersection
  - Point in Polygon
- Ray Tracing
- Recursive Ray Tracing
- Distributed Ray Tracing
- Readings for Friday

Durer’s Ray Casting Machine

- Albrecht Durer, 16th century
Ray Casting

For every pixel
- Construct a ray from the eye
For every object in the scene
  - Find intersection with the ray
  - Keep if closest
Shade depending on light and normal vector

Finding the intersection and normal is the central part of ray casting

A Note on Local Shading

- Surface/Scene Characteristics:
  - surface normal
  - direction to light
  - viewpoint
- Material Properties
  - color/texture
  - diffuse (matte)
  - specular (shiny)
  - ...
- More later!

Diffuse sphere  Specular spheres
Ray Representation?

- Two vectors:
  - Origin
  - Direction (normalized is better)
- Parametric line (explicit representation)
  - \( P(t) = \text{origin} + t \times \text{direction} \)

3D Plane Representation?

- Plane defined by
  - \( P_o = (x,y,z) \)
  - \( n = (A,B,C) \)
- Implicit plane equation
  - \( H(P) = Ax + By + Cz + D = 0 \)
    - \( = n \cdot P + D = 0 \)
- Point-Plane distance?
  - If \( n \) is normalized, distance to plane, \( d = H(P) \)
  - \( d \) is the signed distance!
Explicit vs. Implicit?

• Ray equation is explicit  \( P(t) = R_o + t \cdot R_d \)
  – Parametric
  – Generates points
  – Harder to verify that a point is on the ray

• Plane equation is implicit  \( H(P) = n \cdot P + D = 0 \)
  – Solution of an equation
  – Does not generate points
  – Verifies that a point is on the plane

Ray-Plane Intersection

• Intersection means both are satisfied
• So, insert explicit equation of ray into implicit equation of plane & solve for \( t \)

\[
P(t) = R_o + t \cdot R_d \\
H(P) = n \cdot P + D = 0 \\
n \cdot (R_o + t \cdot R_d) + D = 0 \\
t = -\frac{(D + n \cdot R_o)}{n \cdot R_d}
\]
Additional Housekeeping

- Verify that intersection is closer than previous
  \[ P(t) < t_{\text{current}} \]
- Verify that it is not out of range (behind eye)
  \[ P(t) > t_{\text{min}} \]

Normal at Surface Intersection

- Needed for shading
  - diffuse: dot product between light and normal
- Normal of a plane is constant!
Ray-Triangle Intersection

- Intersect with the plane...
- Then use barycentric coordinates:
  - \( P(\alpha, \beta, \gamma) = \alpha a + \beta b + \gamma c \)
    with \( \alpha + \beta + \gamma = 1 \)
  - If \( 0 < \alpha < 1 \) & \( 0 < \beta < 1 \) & \( 0 < \gamma < 1 \)
    then the point is inside the triangle!

How Do We Compute \( \alpha, \beta, \gamma \) ?

- Ratio of opposite sub-triangle area to total area
  - \( \alpha = A_a / A \) \( \beta = A_b / A \) \( \gamma = A_c / A \)
- Use signed areas for points outside the triangle

But how do I know if the point is outside the triangle?
That’s what I was trying to determine!
Using Cramer’s Rule…

- Used to solve for one variable at a time in system of equations

\[
\beta = \frac{\begin{vmatrix}
  a_x - R_{ox} & a_x - c_x & R_{dx} \\
  a_y - R_{oy} & a_y - c_y & R_{dy} \\
  a_z - R_{oz} & a_z - c_z & R_{dz}
\end{vmatrix}}{|A|}
\]

\[
\gamma = \frac{\begin{vmatrix}
  a_x - b_x & a_x - R_{ox} & R_{dx} \\
  a_y - b_y & a_y - R_{oy} & R_{dy} \\
  a_z - b_z & a_z - R_{oz} & R_{dz}
\end{vmatrix}}{|A|}
\]

\[
t = \frac{\begin{vmatrix}
  a_x - b_x & a_x - c_x & a_x - R_{ox} \\
  a_y - b_y & a_y - c_y & a_y - R_{oy} \\
  a_z - b_z & a_z - c_z & a_z - R_{oz}
\end{vmatrix}}{|A|}
\]

| | denotes the determinant

Can be copied mechanically into code

Sphere Representation?

- Implicit sphere equation
  - Assume centered at origin (easy to translate)
  - \( H(P) = P \cdot P - r^2 = 0 \)
Ray-Sphere Intersection

- Insert explicit equation of ray into implicit equation of sphere & solve for $t$

$$P(t) = R_o + tR_d \quad H(P) = P \cdot P - r^2 = 0$$

$$(R_o + tR_d) \cdot (R_o + tR_d) - r^2 = 0$$

$$R_d \cdot R_d t^2 + 2R_d \cdot R_o t + R_o \cdot R_o - r^2 = 0$$

Ray-Sphere Intersection

- Quadratic: $at^2 + bt + c = 0$
  - $a = 1$ (remember, $||R_d|| = 1$)
  - $b = 2R_d \cdot R_o$
  - $c = R_o \cdot R_o - r^2$

- with discriminant $d = \sqrt{b^2 - 4ac}$

- and solutions $t_{\pm} = \frac{-b \pm d}{2a}$

- What does it mean if there are no solutions, 1 solution, or 2 solutions?
Today

- Reading for Today
- Ray Casting
- Ray Tracing
  - Shadows
  - Reflection
  - Refraction
- Recursive Ray Tracing
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How Can We Add Shadows?

Find the point to be shaded
For every light,
   Construct ray from point to light
   For every object
       find intersection of ray with object
   If no objects between point and light
       Add contribution from light

Mirror Reflection

• Cast ray symmetric with respect to the normal
• Multiply by reflection coefficient (color)
Reflection

- Reflection angle = view angle
- \[ R = V - 2 (V \cdot N) N \]

Transparency

- Cast ray in refracted direction
- Multiply by transparency coefficient (color)
Qualitative Refraction

From “Color and Light in Nature” by Lynch and Livingston
Refraction

\[ I = N \cos \theta_i - M \sin \theta_i \]
\[ M = \frac{(N \cos \theta_i - I)}{\sin \theta_i} \]
\[ T = -N \cos \theta_T + M \sin \theta_T \]
\[ = -N \cos \theta_T + (N \cos \theta_i - I) \sin \theta_T / \sin \theta_i \]
\[ = -N \cos \theta_T + (N \cos \theta_i - I) \eta_r \]
\[ = [ \eta_r \cos \theta_i \cos \theta_T ] N - \eta_r I \]
\[ = [ \eta_r \cos \theta_i - \sqrt{1 - \sin^2 \theta_T} ] ] N - \eta_r I \]
\[ = [ \eta_r \cos \theta_i - \sqrt{1 - \eta_r^2 \sin^2 \theta_i} ] ] N - \eta_r I \]
\[ = [ \eta_r (N \cdot I) - \sqrt{1 - \eta_r^2 (1 - \cos^2 \theta_i) } ] ] N - \eta_r I \]

\[ \sin \theta_T = \frac{\eta_i}{\eta_T} = \eta_r \]

- Total internal reflection when the square root is imaginary
- Don’t forget to normalize!

Refraction & the Sidedness of Objects

- Make sure you know whether you’re entering or leaving the transmissive material:

- What about intersecting transparent objects?
• What about intersecting transparent objects?

Total Internal Reflection

Fig. 3.7A The optical manhole. From under water, the entire celestial hemisphere is compressed into a circle only 97.2° across. The dark boundary defining the edges of the manhole is not sharp due to surface waves. The rays are analogous to the crepuscular type seen in hazy air. Section 1.9. (Photo by D. Granger)

Fig. 3.7B The optical manhole. Light from the horizon (angle of incidence = 90°) is refracted downward at an angle of 48.6°. This compresses the sky into a circle with a diameter of 97.2° instead of its usual 180°.

From “Color and Light in Nature” by Lynch and Livingston
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**Ray Tracing**

```plaintext
trace ray
Intersect all objects
color = ambient term
For every light
  cast shadow ray
  color += local shading term
If mirror
  color += color_{refl} * trace reflected ray
If transparent
  color += color_{trans} * trace transmitted ray
```

Does it ever end?

**Stopping criteria:**
- Recursion depth
  - Stop after a number of bounces
- Ray contribution
  - Stop if reflected / transmitted contribution becomes too small
The Ray Tree

- $N_i$: surface normal
- $R_i$: reflected ray
- $L_i$: shadow ray
- $T_i$: transmitted (refracted) ray

Ray Debugging

- Visualize the ray tree for single image pixel
Shadows of Transparent Objects

- Is this physically accurate?

Reading for Next Time

Today

• Reading for Today
• Ray Casting
• Ray Tracing
• Recursive Ray Tracing
• Distributed Ray Tracing
  – Soft shadows
  – Antialiasing (getting rid of jaggies)
  – Glossy reflection
  – Motion blur
  – Depth of field (focus)
• Readings for Friday

Shadows

• one shadow ray per intersection per point light source
Shadows & Light Sources

Soft Shadows

- multiple shadow rays to sample area light source
Antialiasing – Supersampling

- multiple rays per pixel

Point light

Area light

Reflection

- one reflection ray per intersection

Perfect mirror
Glossy Reflection

- multiple reflection rays

polished surface

Motion Blur

- Sample objects temporally

Rob Cook
Depth of Field

- multiple rays per pixel

Ray Tracing Algorithm Analysis

- Ray casting
- Lots of primitives
- Recursive
- Distributed Ray Tracing Effects
  - Soft shadows
  - Anti-aliasing
  - Glossy reflection
  - Motion blur
  - Depth of field

\[ \text{cost} \approx \text{height} \times \text{width} \times \text{num primitives} \times \text{intersection cost} \times \text{size of recursive ray tree} \times \text{num shadow rays} \times \text{num supersamples} \times \text{num glossy rays} \times \text{num temporal samples} \times \text{num focal samples} \times \ldots \]

Can we reduce this?

These can serve double duty
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Reading for Next Time


Everyone should read this paper for HW3
Reading for Next Time *(optional)*

- "Measuring and Modeling Anisotropic Reflection", Ward, SIGGRAPH 1992