

# The Rendering Equation

## *Rendering with Natural Light*

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Paul Debevec et. al, SIGGRAPH 1998

## *Rendering with Natural Light*

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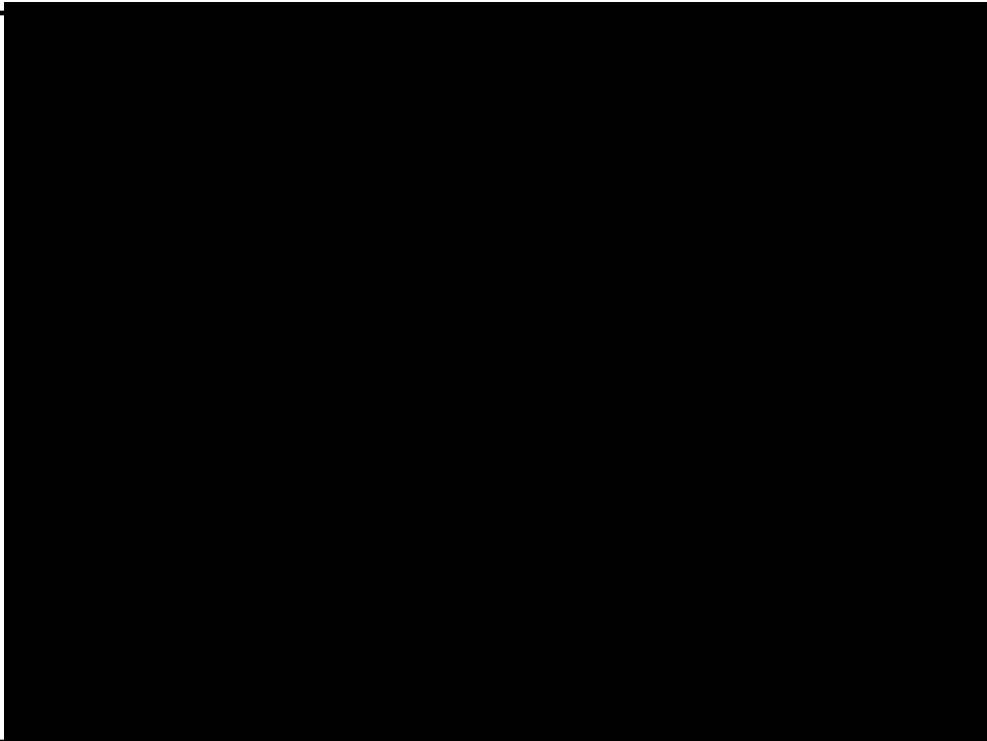
## *Image Based Lighting*

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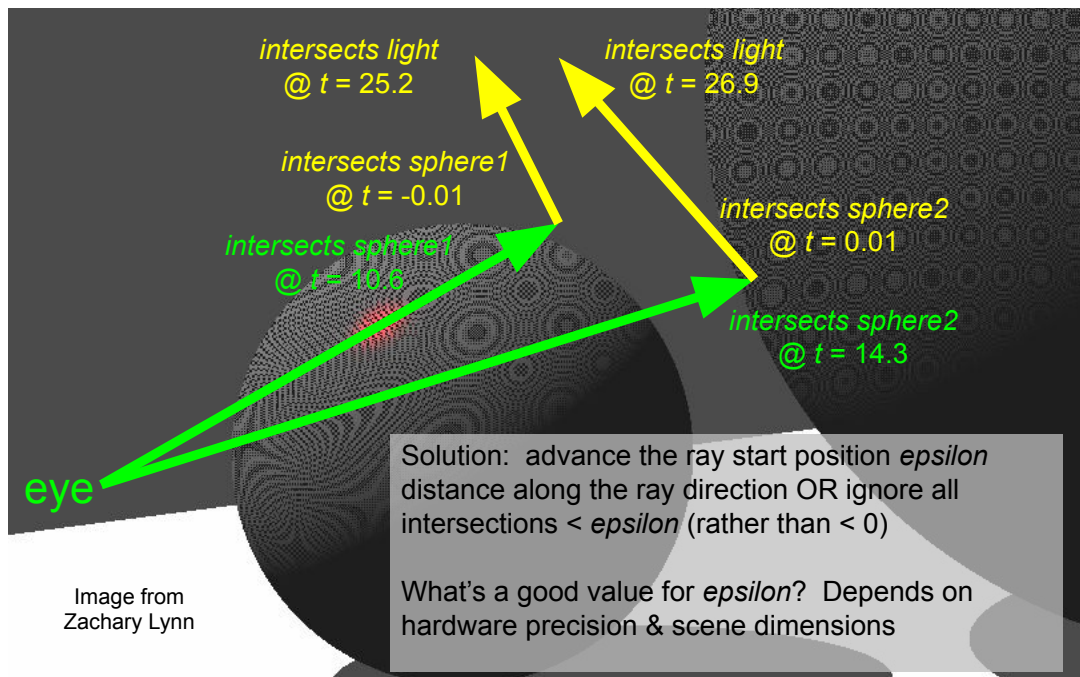


Paul Debevec et al, SIGGRAPH 2000

# Image Based Lighting



## HW3: Raytracing & Epsilon

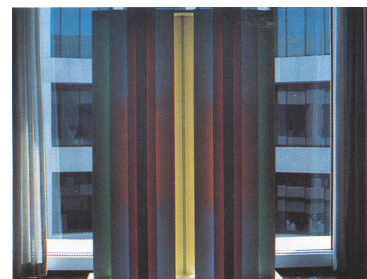
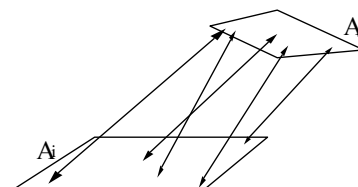
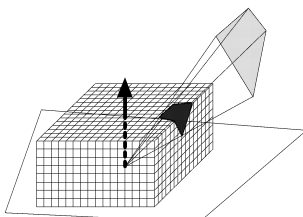
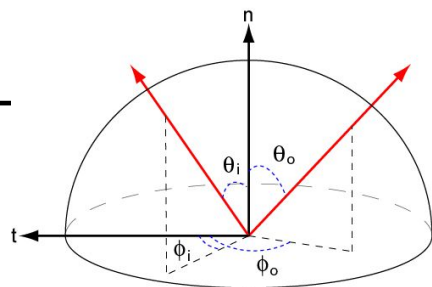


# Final Project Brainstorming

- Each student should post two different ideas for a final project on the forum.
  - For each idea:
    - Briefly describe the idea, your motivation for it, and an example of the potential result.
    - What is the significant/interesting technical implementation challenge?
  - Have you already decided on one idea? Which one?
  - Do you already have a partner? Who?  
(even if you have chosen an idea and/or a partner everyone must post 2 different ideas)
  - Due **Wednesday 3/1 @ 11:59pm**
- Teams of 2 strongly recommended  
(individuals & teams >2 require instructor permission)
- *Projects from prior terms are on the website*

## Last Time?

- Local Illumination
  - BRDF
  - Ideal Diffuse Reflectance
  - Ideal Specular Reflectance
  - The Phong Model
- Radiosity Equation/Matrix
- Calculating the Form Factors



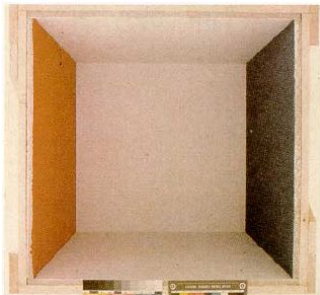
# Today

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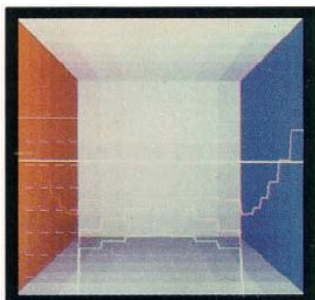
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  - Advanced Radiosity
- Does Ray Tracing Simulate Physics?
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- Worksheet
- Papers for Next Time

# Reading for Today

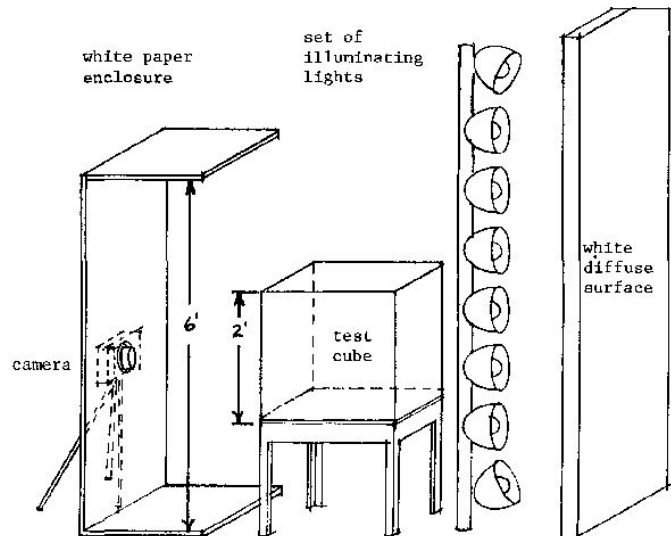
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photograph



simulation



Goral, Torrance, Greenberg & Battaile  
*Modeling the Interaction of Light Between Diffuse Surfaces* SIGGRAPH '84

# The Cornell Box

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- Careful calibration and measurement allows for comparison between physical scene & simulation



photograph



simulation

Light Measurement Laboratory  
Cornell University, Program for Computer Graphics

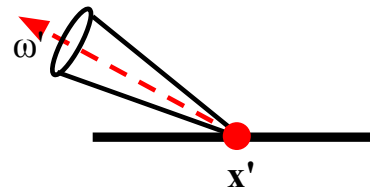
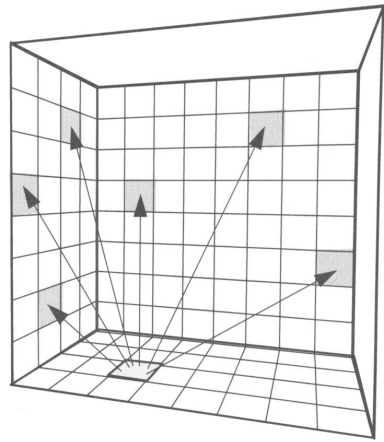
## Today

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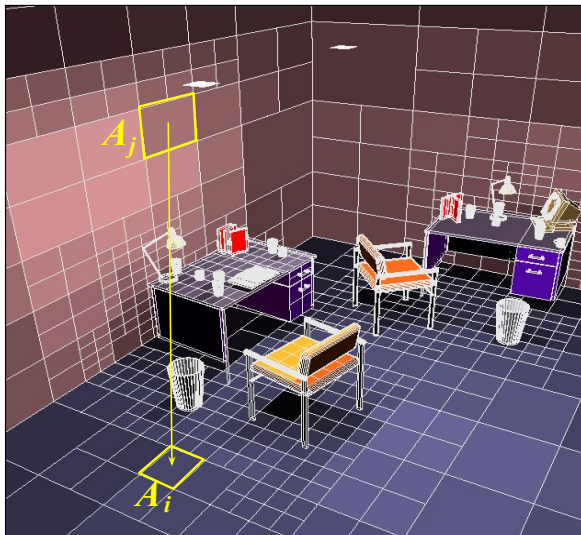
# Radiosity Overview

- Surfaces are assumed to be perfectly Lambertian (diffuse)
  - reflect incident light in all directions with equal intensity
- The scene is divided into a set of small areas, or patches.
- The radiosity,  $B_i$ , of patch  $i$  is the total rate of energy leaving a surface. The radiosity over a patch is constant.
- Units for radiosity: Watts / steradian \* meter<sup>2</sup>



# Discrete Radiosity Equation

Discretize the scene into  $n$  patches, over which the radiosity is constant



$$B_i = E_i + \rho_i \sum_{j=1}^n F_{ij} B_j$$

light leaving patch  $i$  (points to  $B_i$ )  
 material reflectivity (points to  $\rho_i$ )  
 light emitted from patch  $i$  (points to  $E_i$ )  
 form factor (points to  $F_{ij}$ )

The equation is recursive, but it can be solved iteratively

# Radiosity in Matrix Form

$$B_i = E_i + \rho_i \sum_{j=1}^n F_{ij} B_j$$

$n$  simultaneous equations with  $n$  unknown  $B_i$  values can be written in matrix form:

$$\begin{bmatrix} 1 - \rho_1 F_{11} & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1n} \\ -\rho_2 F_{21} & 1 - \rho_2 F_{22} & & \\ \vdots & & \ddots & \\ -\rho_n F_{n1} & \cdots & \cdots & 1 - \rho_n F_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$$

↓ solve for  $B_i$

A solution yields a single radiosity value  $B_i$  for each patch in the environment, a view-independent solution.

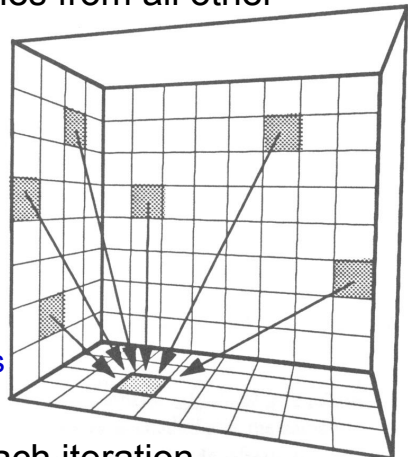
# Solving the Radiosity Matrix

- Initialize all radiosity values to 0
- Each iteration, update the radiosity of each patch by *gathering* the contribution of radiosities from all other

$$\begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_i \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_i \\ \vdots \\ E_n \end{bmatrix} + \begin{bmatrix} \rho_1 F_{11} & \rho_1 F_{12} & \cdots & \rho_1 F_{1n} \\ \rho_2 F_{21} & \rho_2 F_{22} & \cdots & \rho_2 F_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_n F_{n1} & \rho_n F_{n2} & \cdots & \rho_n F_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_i \\ \vdots \\ B_n \end{bmatrix}$$

↑ Radiosity values on iteration  $t+1$

↑ Radiosity values on iteration  $t$

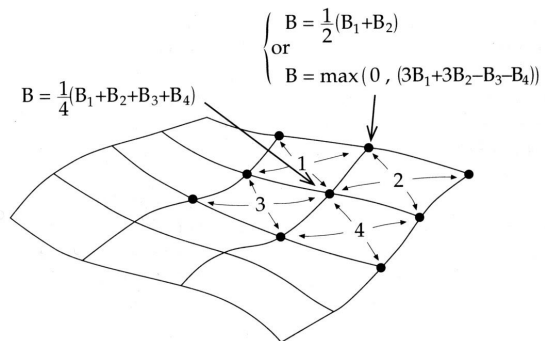
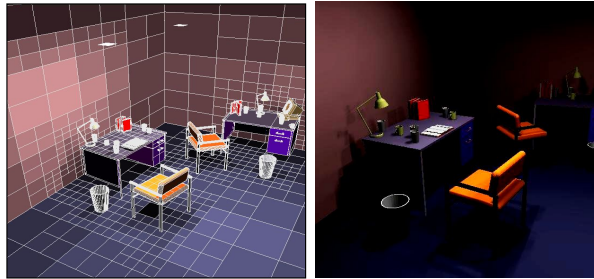


- Radiosity values only increase on each iteration
- This method is fundamentally a Gauss-Seidel relaxation



# Interpolating Vertex Radiosities

- $B_i$  radiosity values are constant over the extent of a patch.
- How are they mapped to the vertex radiosities (intensities) needed by the renderer?
  - Average the radiosities of patches that contribute to the vertex
  - Vertices on the edge of a surface are assigned values extrapolation



## Questions?



Factory simulation. 30,000 patches.  
Program of Computer Graphics, Cornell University.

# Today

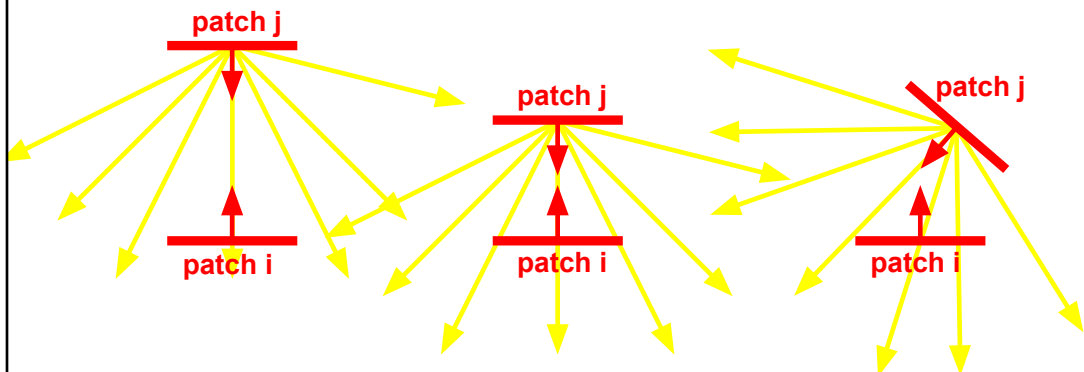
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## Calculating the Form Factor $F_{ij}$

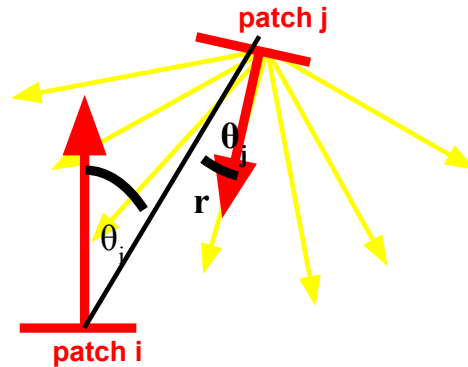
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- $F_{ij}$  = fraction of light energy leaving patch j that arrives at patch i
- Takes account of both:
  - geometry (size, orientation & position)
  - visibility (are there any occluders?)



# Calculating the Form Factor $F_{ij}$

- $F_{ij}$  = fraction of light energy leaving patch j that arrives at patch i

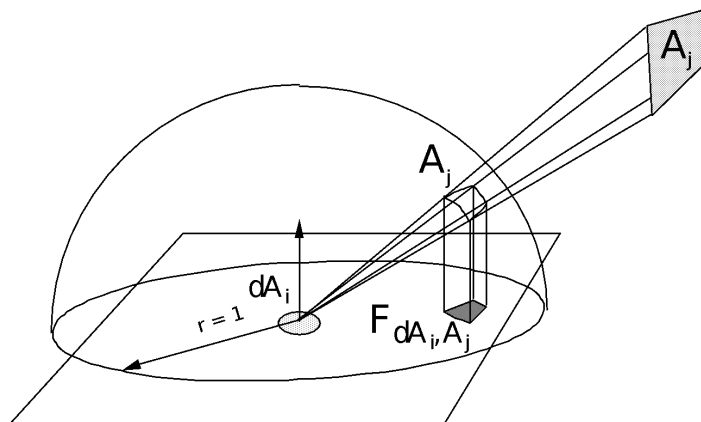


WARNING:  
common typo is  
to flip i & j

$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos \theta_i \cos \theta_j}{\pi r^2} V_{ij} dA_j dA_i$$

## Form Factor Determination

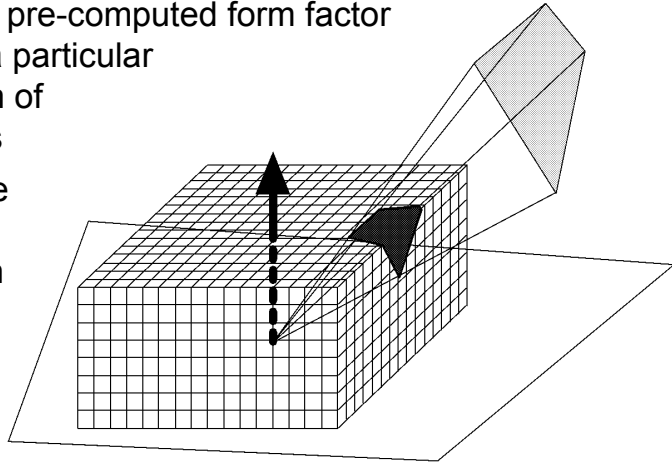
The Nusselt analog: the form factor of a patch is equivalent to the fraction of the unit circle that is formed by taking the projection of the patch onto the hemisphere surface and projecting it down onto the circle.



# Hemicube Algorithm

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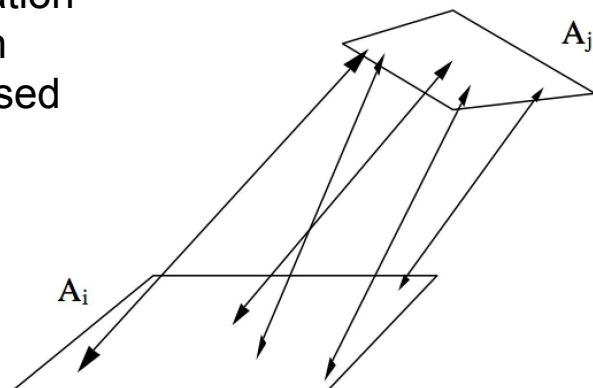
- A hemicube is constructed around the center of each patch
- Faces of the hemicube are divided into "pixels"
- Each patch is projected (rasterized) onto the faces of the hemicube
- Each pixel stores its pre-computed form factor  
The form factor for a particular patch is just the sum of the pixels it overlaps
- Patch occlusions are handled similar to z-buffer rasterization



# Form Factor from Ray Casting

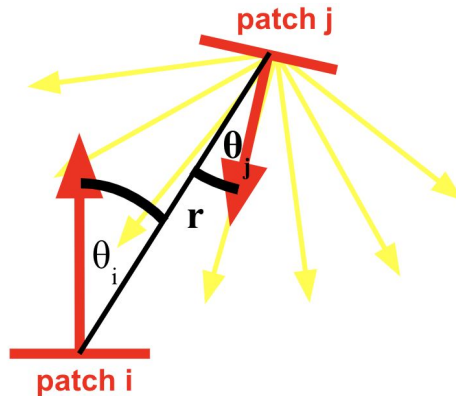
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- Cast  $n$  rays between the two patches
  - Compute visibility (what fraction of rays do not hit an occluder)
  - Integrate the point-to-point form factor
- Permits the computation of the patch-to-patch form factor, as opposed to point-to-patch



# Calculating the Form Factor $F_{ij}$

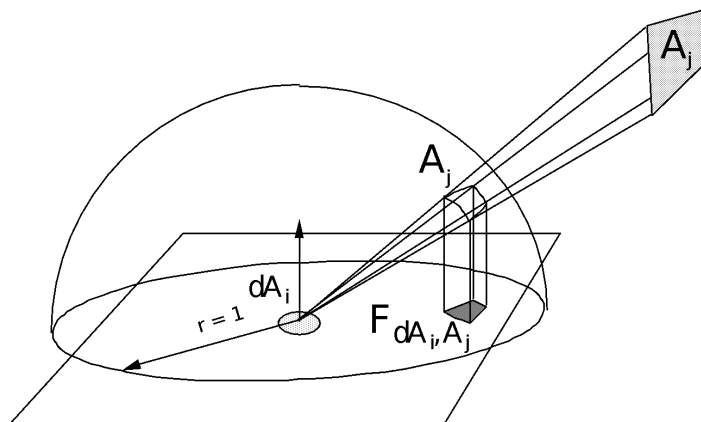
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$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos \theta_i \cos \theta_j}{\pi r^2} V_{ij} dA_j dA_i$$

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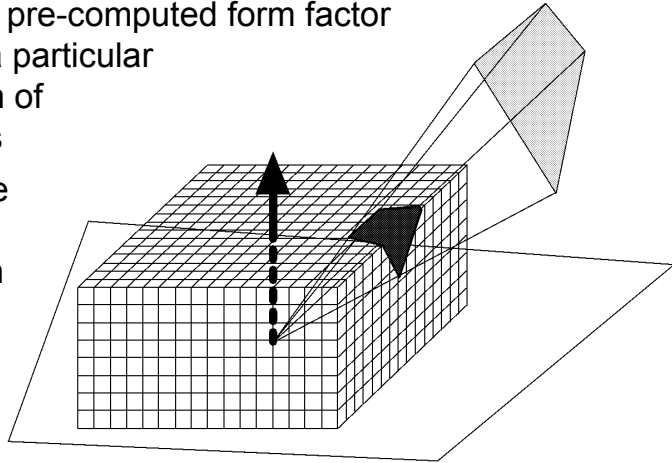
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# Hemicube Algorithm

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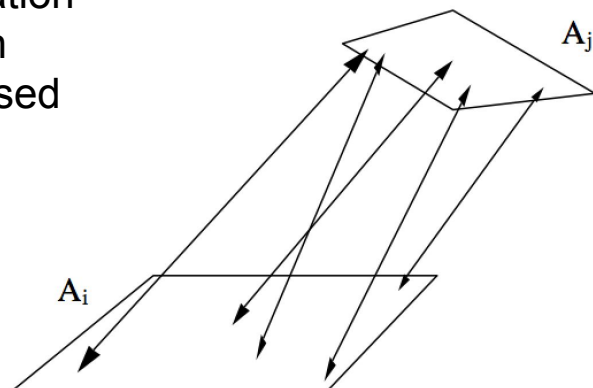
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# Form Factor from Ray Casting

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# Questions?

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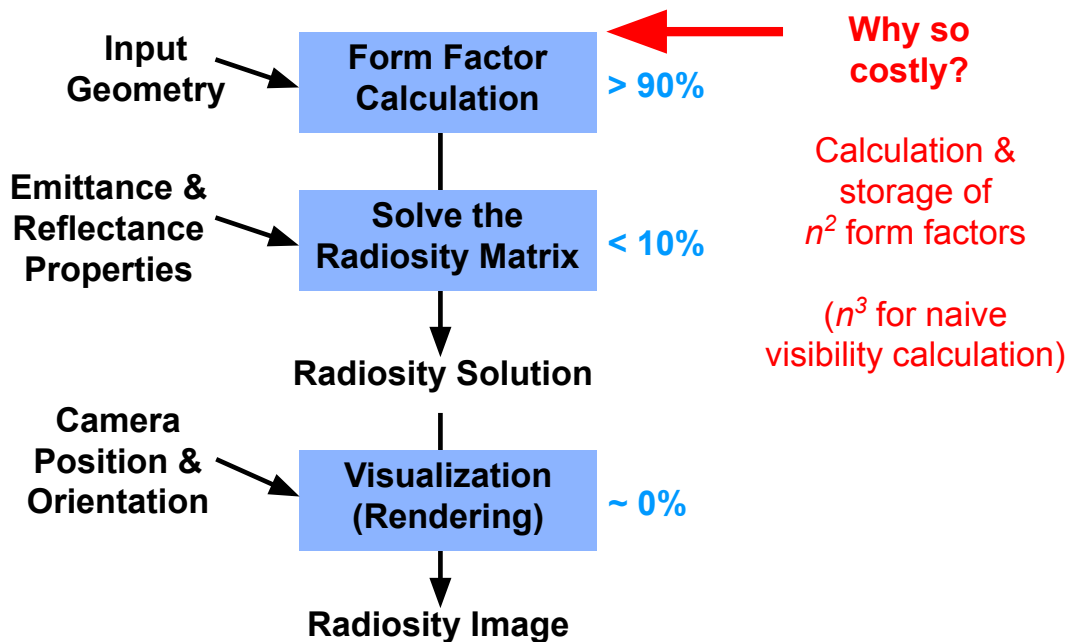
Lightscape <http://www.lightscape.com>

# Today

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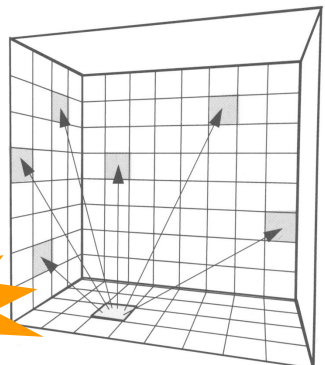
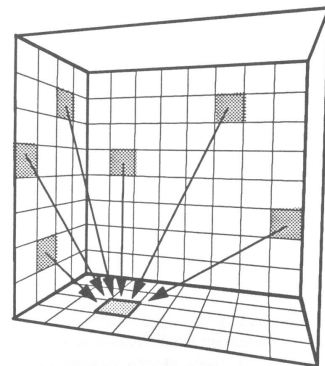
- Paper for Today
- Leftover from Last Time:
  - Radiosity Overview
  - Calculating the Form Factors
  - **Advanced Radiosity**
    - **Progressive Radiosity**
    - Adaptive Subdivision
    - Discontinuity Meshing
    - Hierarchical Radiosity
- Does Ray Tracing Simulate Physics?
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# Stages in a Radiosity Solution



## Progressive Refinement

- Goal: Provide frequent and timely updates to the user during computation
- Key Idea: Update the entire image at every iteration, rather than a single patch
- How? Instead of summing the light received by one patch, distribute the radiance of the patch with the most *undistributed radiance*.



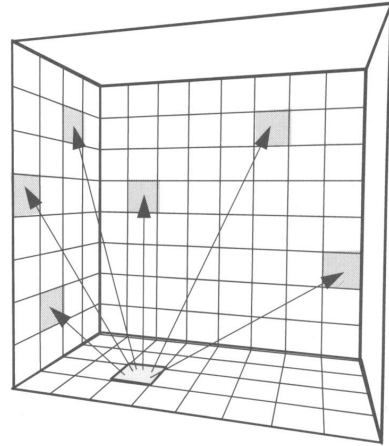
Use this for HW3!



# Reordering the Solution for PR

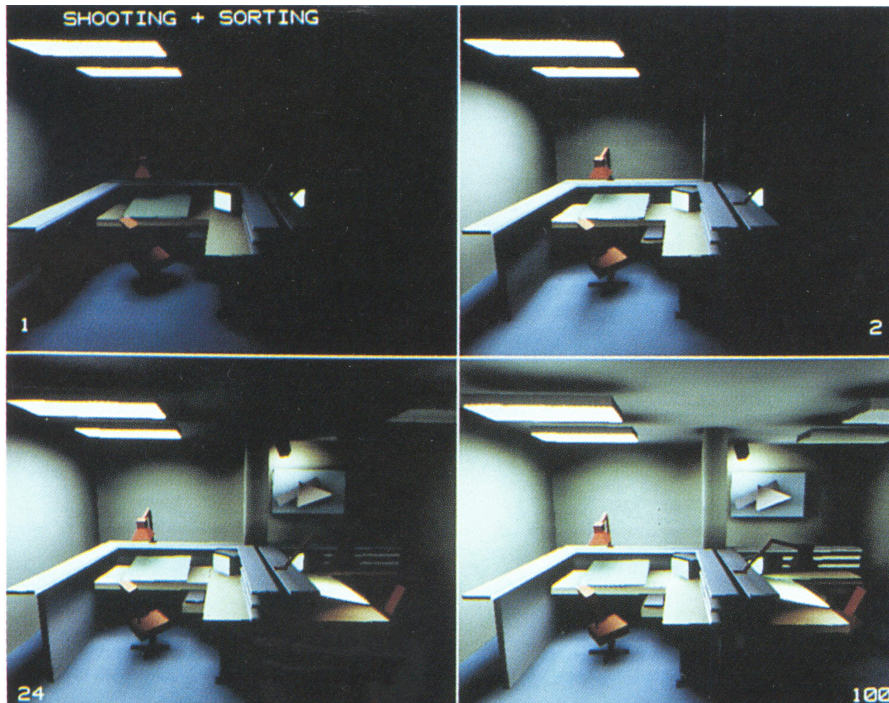
*Shooting*: the radiosity of all patches is updated for each iteration:

$$\begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ \vdots \\ B_n \end{bmatrix} + \begin{bmatrix} \dots & \rho_1 F_{1i} & \dots \\ \dots & \rho_2 F_{2i} & \dots \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ \dots & \rho_n F_{ni} & \dots \end{bmatrix} \begin{bmatrix} \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \end{bmatrix}$$

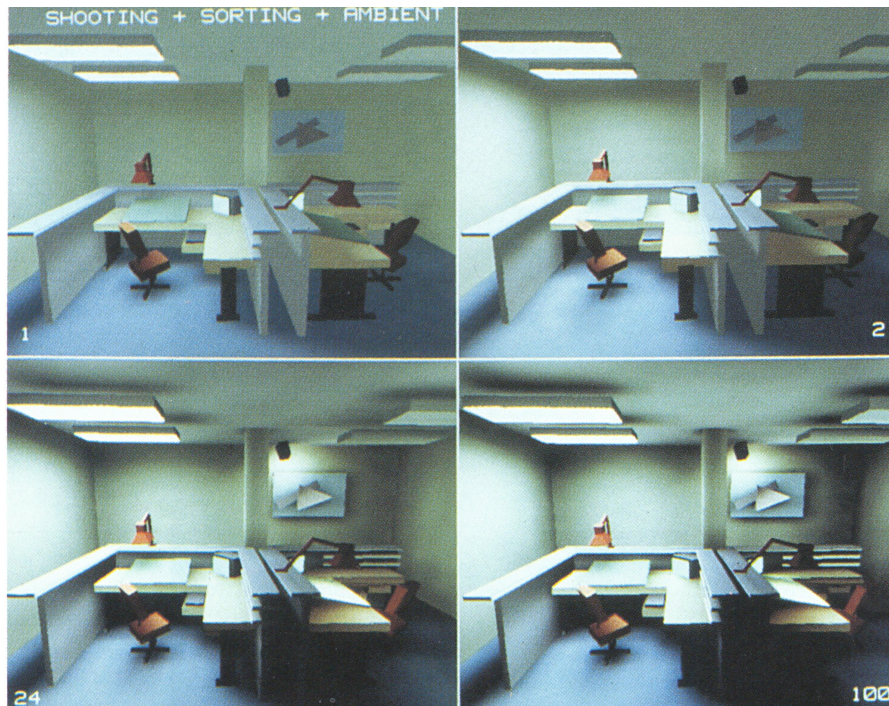


This method is fundamentally a Southwell relaxation

# Progressive Refinement w/out Ambient Term



# Progressive Refinement with Ambient Term



# Questions?



Lightscape

<http://www.lightscape.com>

# Today

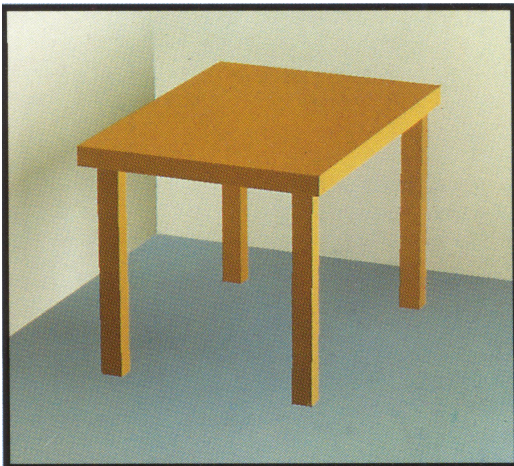
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## Increasing the Accuracy of the Solution

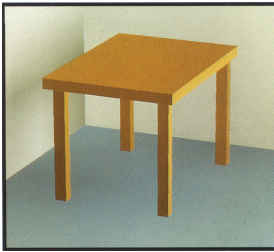
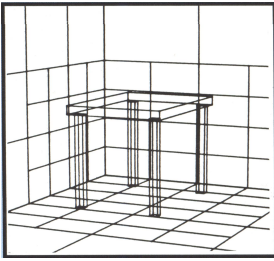
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What's wrong with this picture?

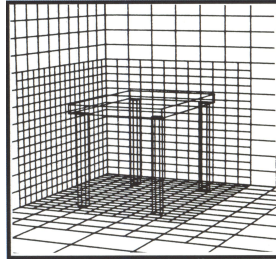


- Image quality is a function of patch size
- Compute a solution on a uniform initial mesh, then refine the mesh in areas that exceed some error tolerance:
  - shadow boundaries
  - other areas with a high radiosity gradient

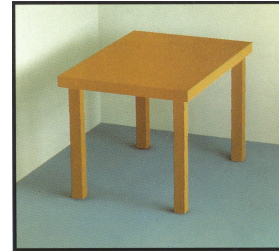
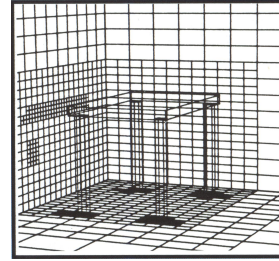
# Adaptive Subdivision of Patches



Coarse patch solution  
(145 patches)



Improved solution  
(1021 subpatches)

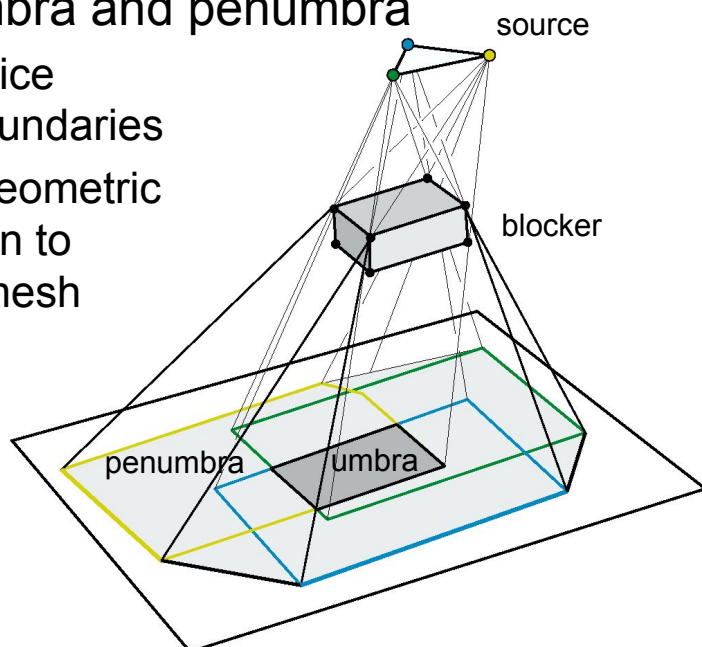


Adaptive subdivision  
(1306 subpatches)

# Discontinuity Meshing

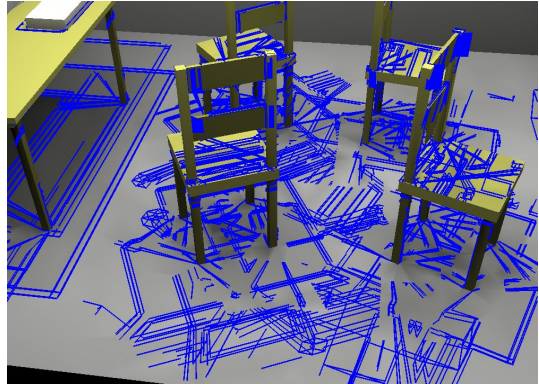
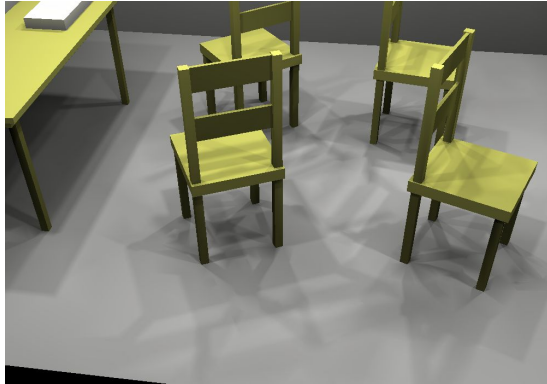
- Limits of umbra and penumbra

- Captures nice shadow boundaries
- Complex geometric computation to construct mesh



## “Fast and Accurate Hierarchical Radiosity Using Global Visibility” Durand, Drettakis, & Puech 1999

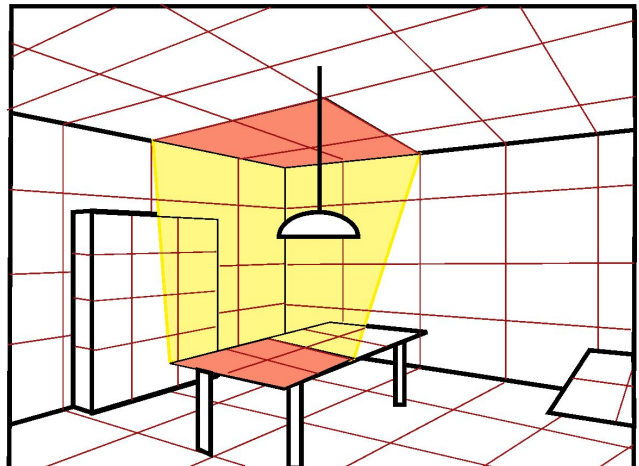
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## Hierarchical Radiosity

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- Group elements when the light exchange is not important
  - Breaks the quadratic complexity
  - Control non trivial, memory cost

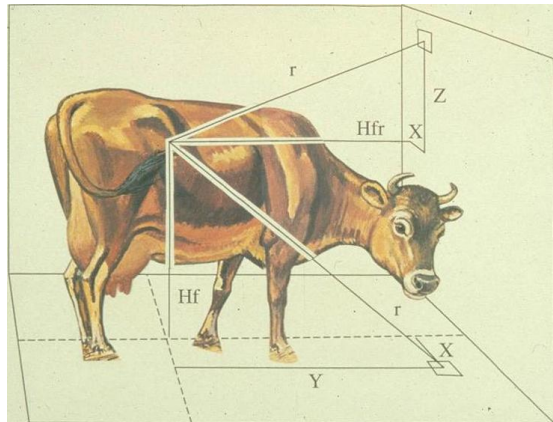


# Practical Problems with Radiosity

- Meshing
  - memory
  - robustness

- Form factors
  - computation

- Diffuse limitation
  - extension to specular takes too much memory



Cow-cow form factor?

## Questions?



Rendered using the Lightscape Visualization System.  
Courtesy of and copyright (c) 1996 Design Visualization Partners (Santa Monica, CA).

Lightscape

<http://www.lightscape.com>

# Today

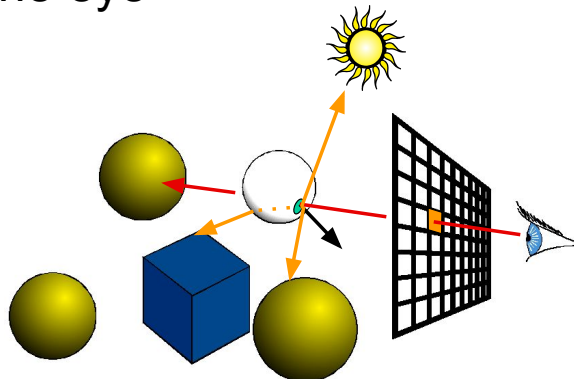
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## Does Ray Tracing Simulate Physics?

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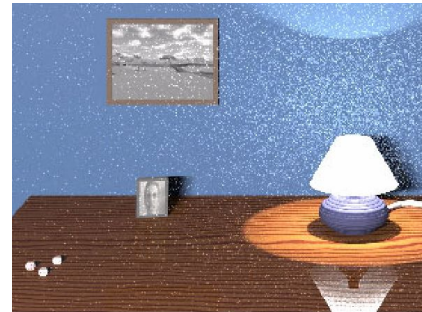
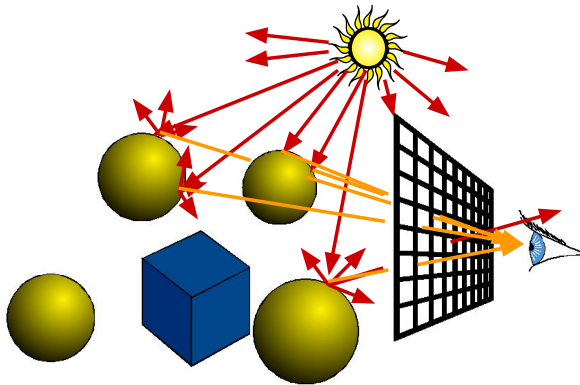
- No.... traditional ray tracing is also called "*backward*" ray tracing
- In reality, photons actually travel from the light to the eye



# Forward Ray Tracing

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- Start from the light source
  - But very, very low probability to reach the eye
- What can we do about it?
  - Always send a ray to the eye.... still not efficient

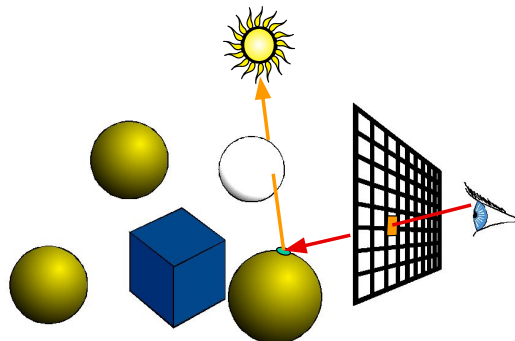


Henrik Wann Jensen

# Transparent Shadows?

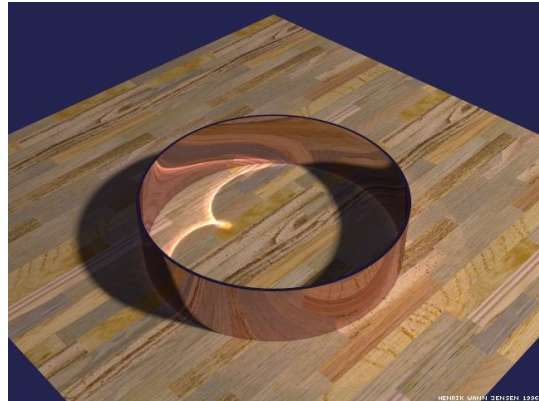
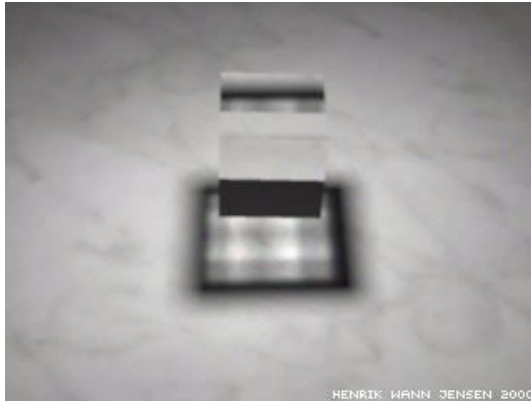
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- What to do if the shadow ray sent to the light source intersects a transparent object?
  - Pretend it's opaque?
  - Multiply by transparency color?  
(ignores refraction & does not produce caustics)
- Unfortunately, ray tracing is full of dirty tricks





# Is this Traditional Ray Tracing?

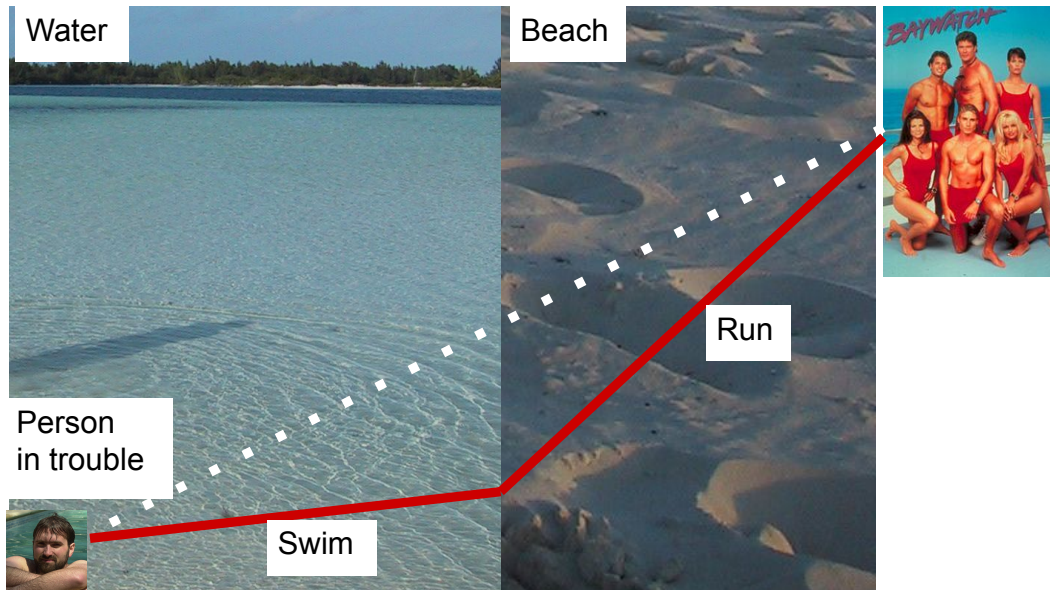


Images by Henrik Wann Jensen

*No. Refraction and complex reflections for illumination are not handled properly in traditional (backward) ray tracing.*

# Refraction and the Lifeguard Problem

- Running is faster than swimming



# Today

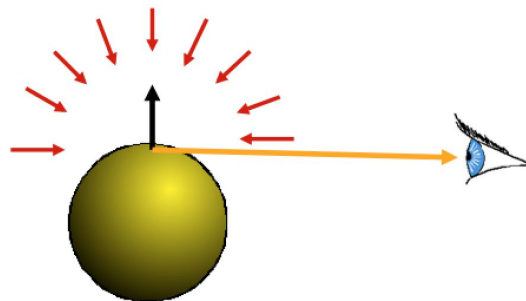
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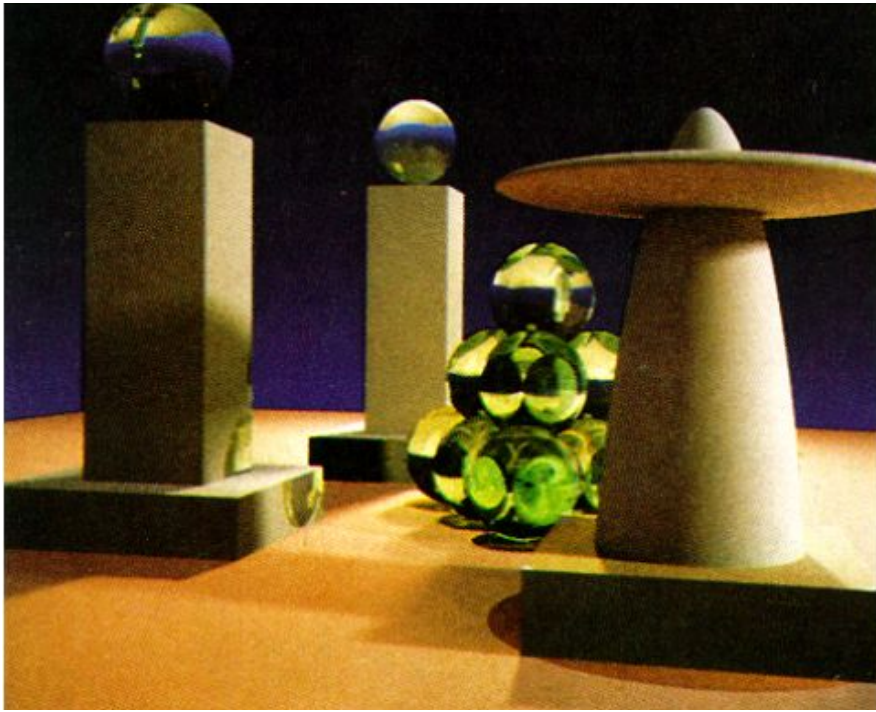
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- Clean mathematical framework for light-transport simulation
- At each point, outgoing **light in one direction** is the integral of **incoming light in all directions** multiplied by reflectance property



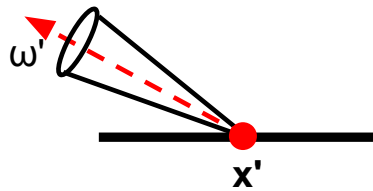
## “The Rendering Equation”, Kajiya, SIGGRAPH 1986

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## The Rendering Equation

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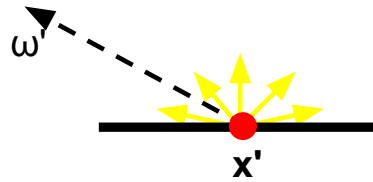
$$L(x', \omega') = E(x', \omega') + \int \rho_x(\omega, \omega') L(x, \omega) G(x, x') V(x, x') dA$$



$L(x', \omega')$  is the radiance from a point on a surface in a given direction  $\omega'$

# The Rendering Equation

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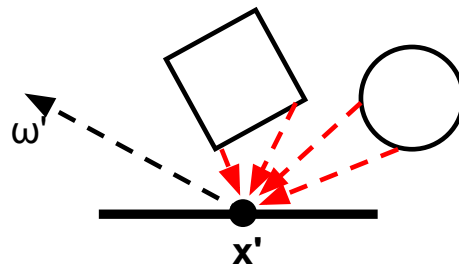
$$L(x', \omega') = E(x', \omega') + \int \rho_{x'}(\omega, \omega') L(x, \omega) G(x, x') V(x, x') dA$$



$E(x', \omega')$  is the emitted radiance from a point:  $E$  is non-zero only if  $x'$  is emissive (a light source)

# The Rendering Equation

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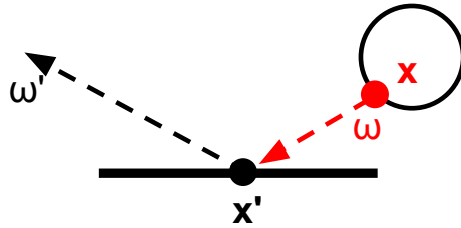


$$L(x', \omega') = E(x', \omega') + \underbrace{\int \rho_{x'}(\omega, \omega') L(x, \omega) G(x, x') V(x, x') dA}$$



Sum the contribution from all of the other surfaces in the scene

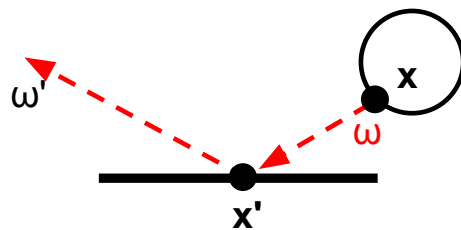
# The Rendering Equation



$$L(x', \omega') = E(x', \omega') + \int \rho_{x'}(\omega, \omega') L(x, \omega) G(x, x') V(x, x') dA$$

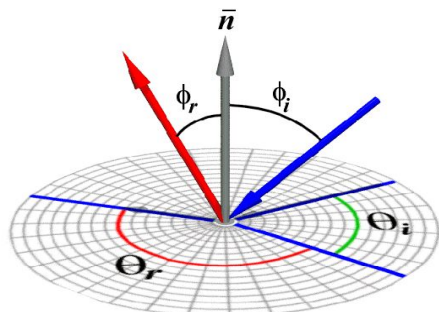
For each  $x$ , compute  $L(x, \omega)$ , the radiance at point  $x$  in the direction  $\omega$  (from  $x$  to  $x'$ )

# The Rendering Equation



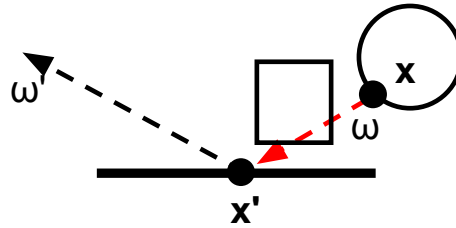
$$L(x', \omega') = E(x', \omega') + \int \rho_{x'}(\omega, \omega') L(x, \omega) G(x, x') V(x, x') dA$$

scale the contribution by  $\rho_{x'}(\omega, \omega')$ , the reflectivity (BRDF) of the surface at  $x'$



# The Rendering Equation

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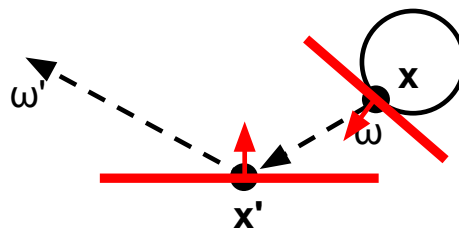


$$L(x', \omega') = E(x', \omega') + \int \rho_{x'}(\omega, \omega') L(x, \omega) G(x, x') V(x, x') dA$$

For each  $x$ , compute  $V(x, x')$ ,  
the visibility between  $x$  and  $x'$ :  
1 when the surfaces are unobstructed  
along the direction  $\omega$ , 0 otherwise

# The Rendering Equation

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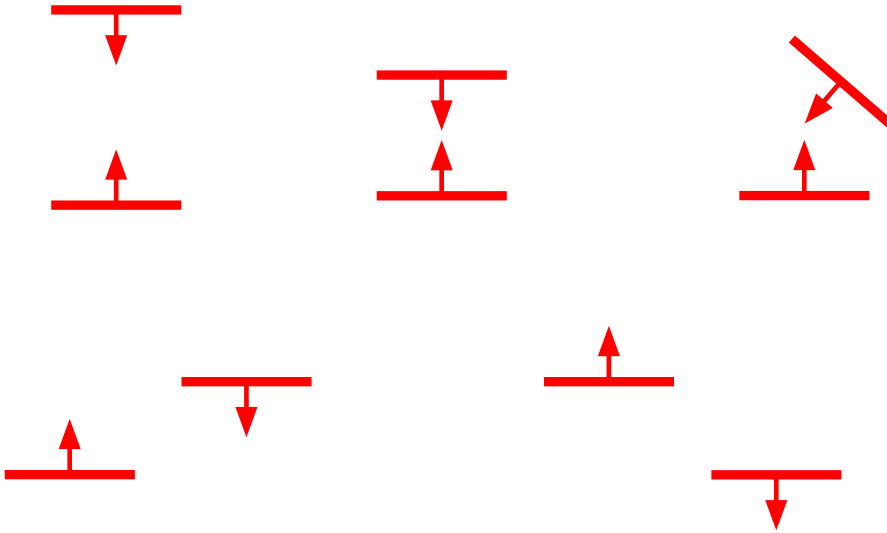


$$L(x', \omega') = E(x', \omega') + \int \rho_{x'}(\omega, \omega') L(x, \omega) G(x, x') V(x, x') dA$$

For each  $x$ , compute  $G(x, x')$ , which  
describes the on the geometric relationship  
between the two surfaces at  $x$  and  $x'$

# Intuition about $G(x,x')$ ?

- Which arrangement of two surfaces will yield the greatest transfer of light energy? Why?



# Rendering Equation $\rightarrow$ Radiosity

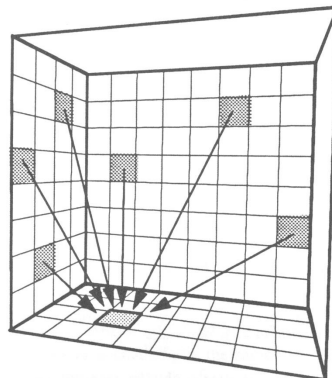
$$L(x',\omega') = E(x',\omega') + \int \rho_{x'}(\omega,\omega') L(x,\omega) G(x,x') V(x,x') dA$$

↓ Radiosity assumption:  
perfectly diffuse surfaces (not directional)

$$B_{x'} = E_{x'} + \rho_{x'} \int B_x G(x,x') V(x,x')$$

↓ discretize

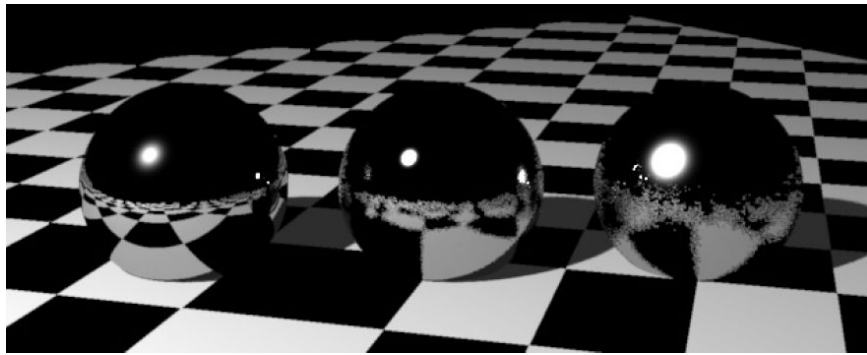
$$B_i = E_i + \rho_i \sum_{j=1}^n F_{ij} B_j$$



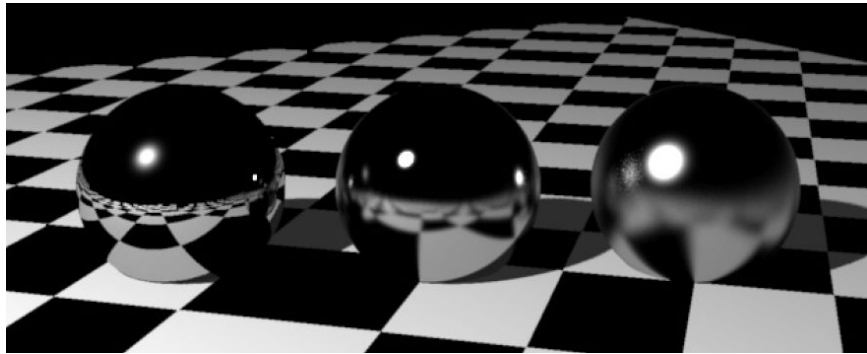
# Questions?

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1 glossy  
sample  
per pixel



256 glossy  
samples  
per pixel



## Today

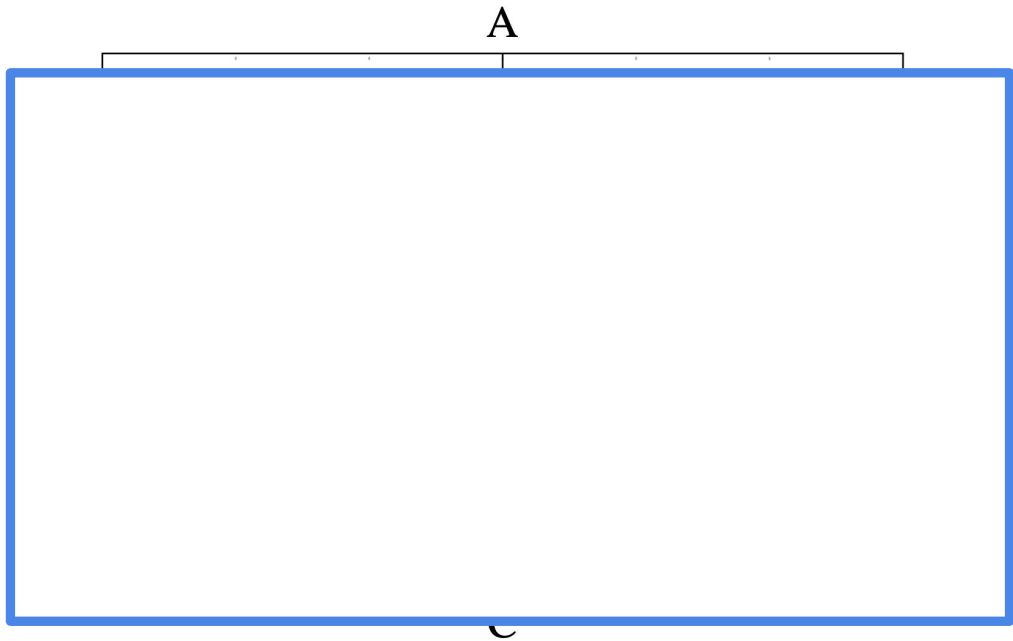
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- Paper for Today
- Leftover from Last Time:
  - Radiosity Overview
  - Calculating the Form Factors
  - Advanced Radiosity
- Does Ray Tracing Simulate Physics?
- The Rendering Equation
- **Worksheet**
- Papers for Next Time



# Pop Worksheet!

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## Today

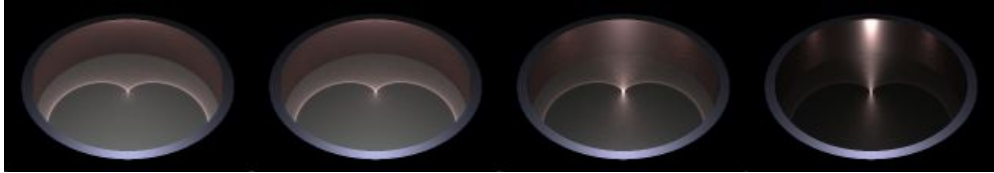
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- Paper for Today
- Leftover from Last Time:
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  - Advanced Radiosity
- Does Ray Tracing Simulate Physics?
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- Worksheet
- Papers for Next Time

# Readings for Next Time:

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- “Rendering Caustics on Non-Lambertian Surfaces”,  
Henrik Wann Jensen, *Graphics Interface* 1996.



- “Global Illumination using Photon Maps”,  
Henrik Wann Jensen, *Rendering Techniques* 1996.

