

# CSCI 4530/6530 Advanced Computer Graphics

<https://www.cs.rpi.edu/~cutler/classes/advancedgraphics/S25/>

## Lecture 20: Color

*some slides from Fredo Durand*



Elsbeth McLean <https://www.elsbethmclean.com/stones>

# *For the Birds*, Pixar, 2000

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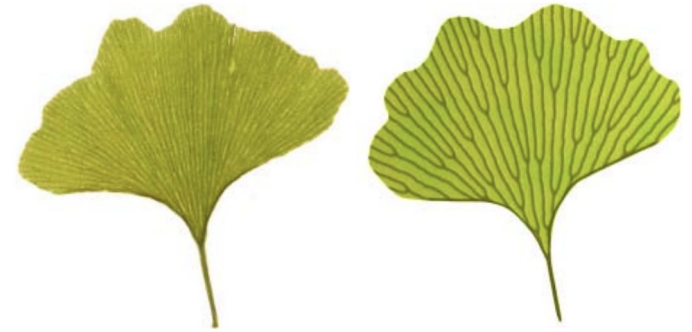
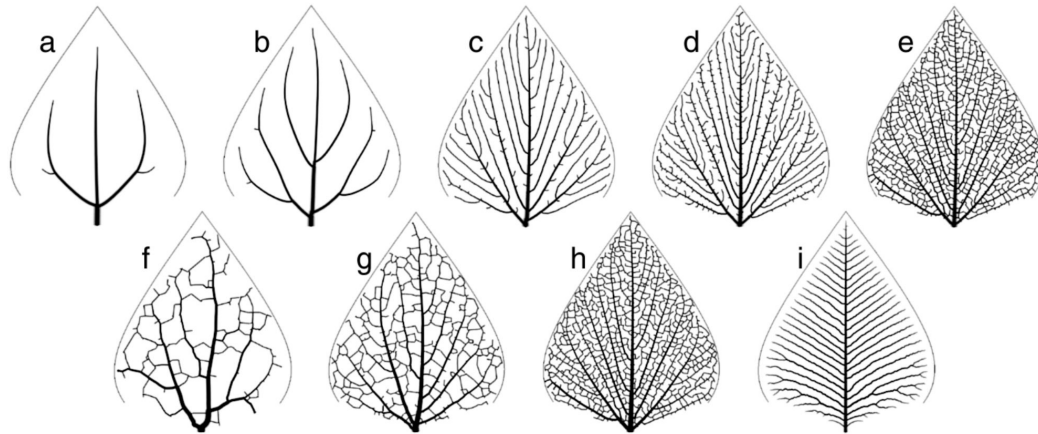
# Today

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- Worksheet: Shadow Techniques
- Readings for Today
- What is Color?
  - Human Perception
  - Color Blindness & Metamerism
- Color Spaces
  - Linear Color Spaces: LMS, RGB, XYZ, HSV
- Projection in Spatially Augmented Reality
  - Human Perception & Perceptual Color Space:  $L^*a^*b^*$
- Readings for Next Time

# Readings for Today

- “Modeling and visualization of leaf venation patterns”, Runions, Fuhrer, Lane, Federl, Roggan-Lagan, & Prusinkiewicz, 2007.



**Figure 10:** A photograph (left) and a rendered model of ginkgo venation (right).



**Figure 11:** A photograph(left) and a rendered model of lady's mantle venation (right).

- Interdisciplinary research
- Botanical/biological inspiration
- Leaves are 2D, 2D algorithm makes sense
- Could have been hacked with a texture, refreshing to see effort to correctly model and simulate natural phenomena

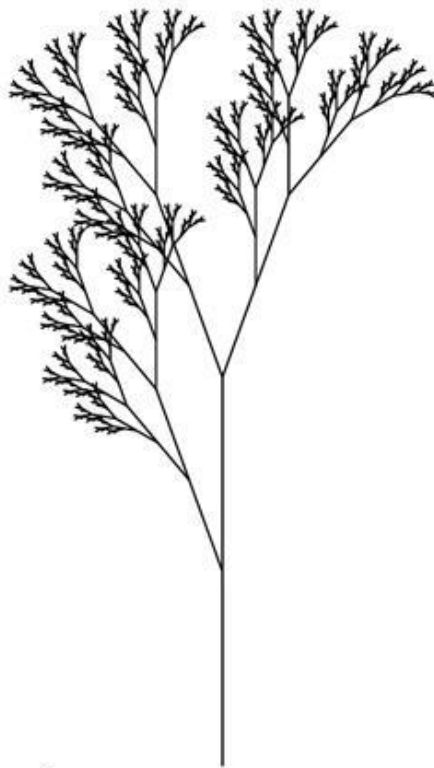
# L-Systems

alphabet: {a,b}  
initiator: a  
production rules:  
a  $\rightarrow$  b  
b  $\rightarrow$  ba

generations:

a  
b  
ba  
bab  
babba  
babbabab  
babbababbabba  
babbababbabbababbabab

Prusinkiewicz & Lindenmayer,  
*The Algorithmic Beauty of Plants*, 1990  
<http://algorithmicbotany.org/>



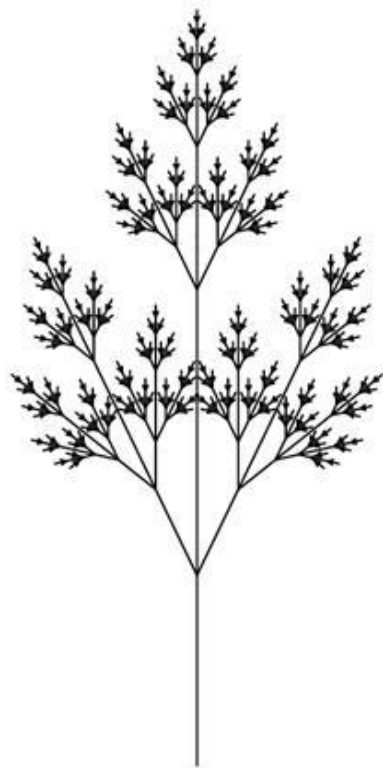
**d**

$n=7, \delta=20^\circ$

X

$X \rightarrow F[+X]F[-X]+X$

$F \rightarrow FF$



**e**

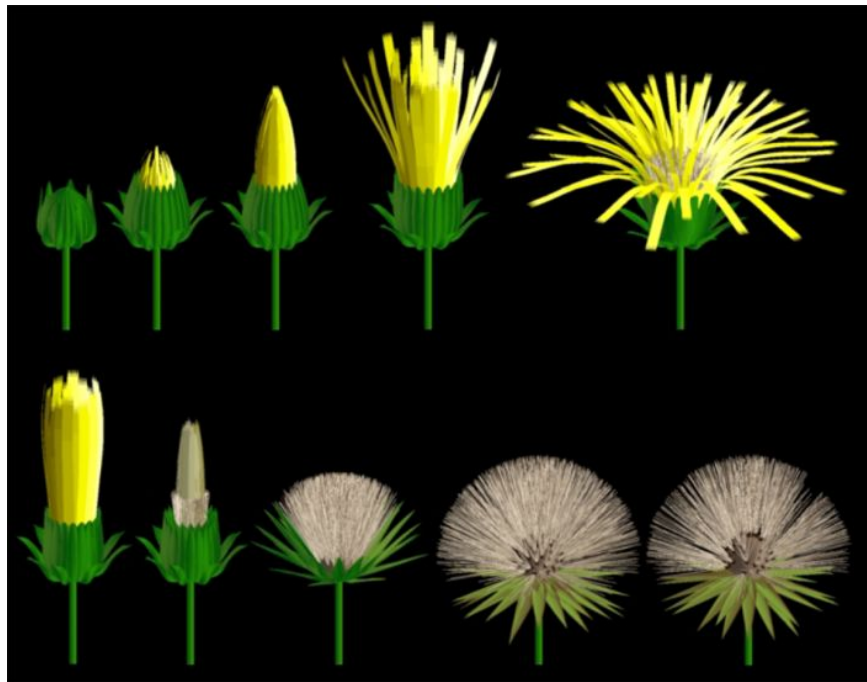
$n=7, \delta=25.7^\circ$

X

$X \rightarrow F[+X]F[-X]FX$

$F \rightarrow FF$

# L-Systems



*Animation of Plant Development*  
Prusinkiewicz et al.,  
SIGGRAPH 1993



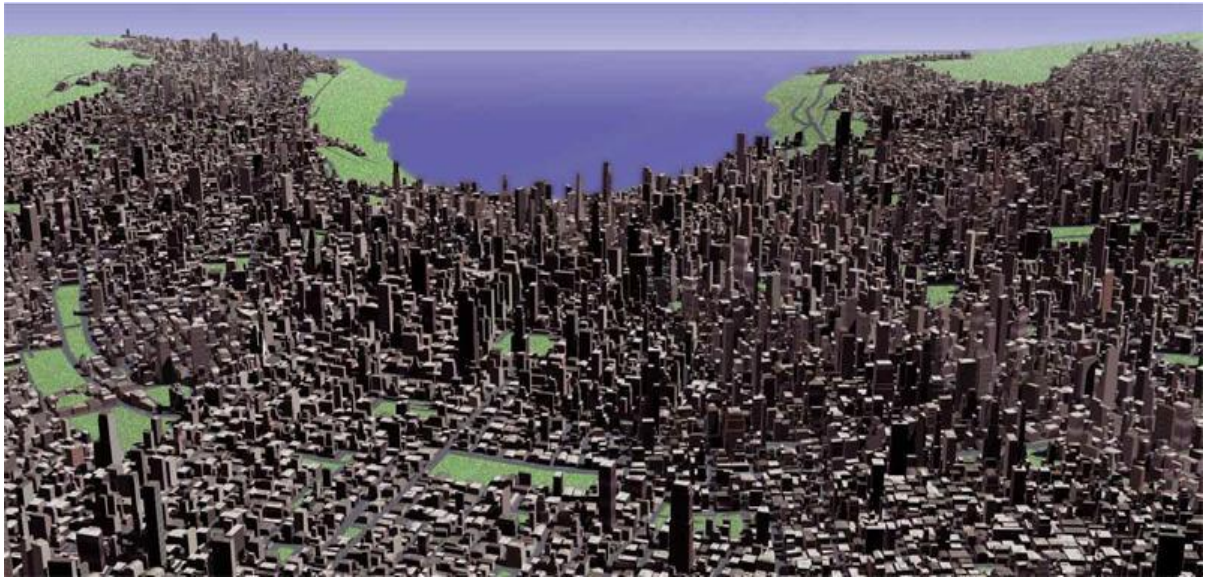
Prusinkiewicz & Lindenmayer,  
*The Algorithmic Beauty of Plants*, 1990  
<http://algorithmicbotany.org/>

“Synthetic Topiary”,  
Prusinkiewicz, James, and Mech,  
SIGGRAPH 1994



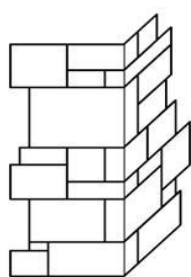
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- “Procedural Modeling of Cities”,  
Parish & Müller,  
SIGGRAPH 2001

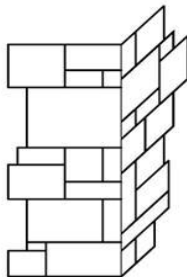


- Details not perfect close up, but good at the scale of a whole city
- City as a living being, e.g., vessels for circulation
- Coarse model, that is refined/sculpted
- Are there limitations on what can be modeled?
  - Older, walkable cities vs. newer, car-focused cities
  - Ancient civilizations, futuristic cities, or alien cities?
- Impressive large, flexible, system
- Thoughtful consideration of runtime and memory efficiency
- Balance between user control and auto-generated content
- Impressive variety from a modest set of rules

# Cellular Texturing

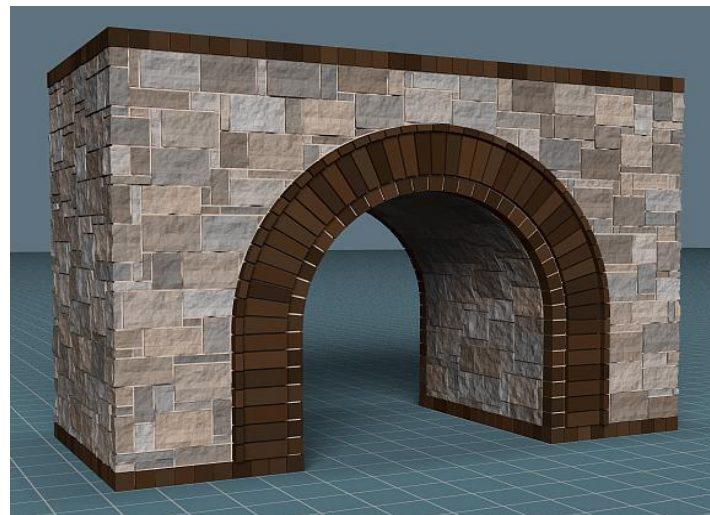
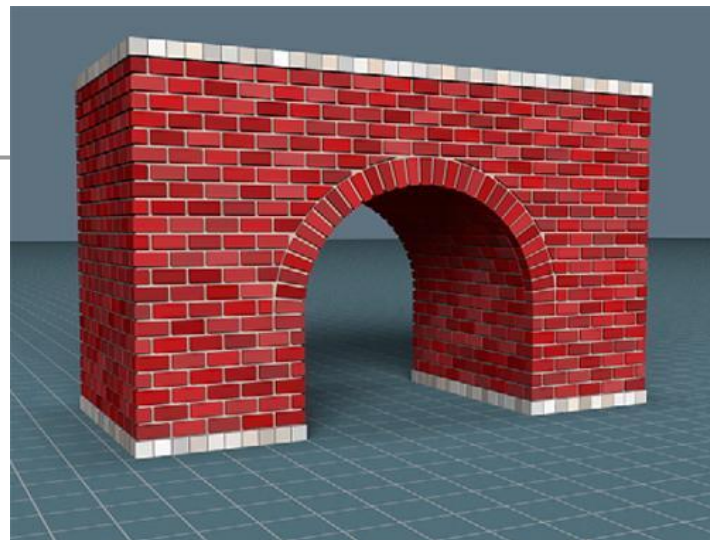
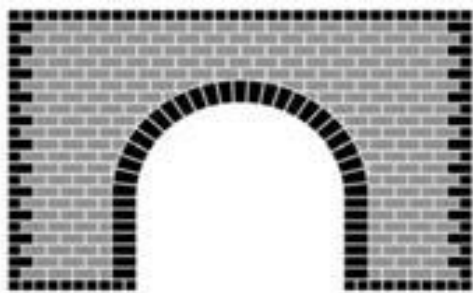
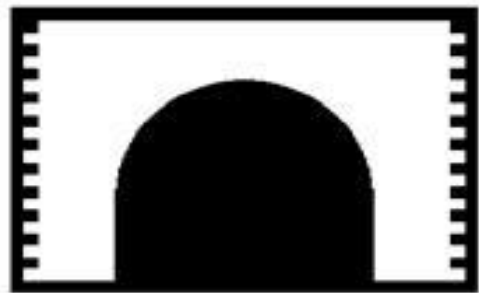


Correct



Incorrect

“Feature-Based  
Cellular Texturing for  
Architectural Models”,  
Legakis, Dorsey, & Gortler,  
SIGGRAPH 2001

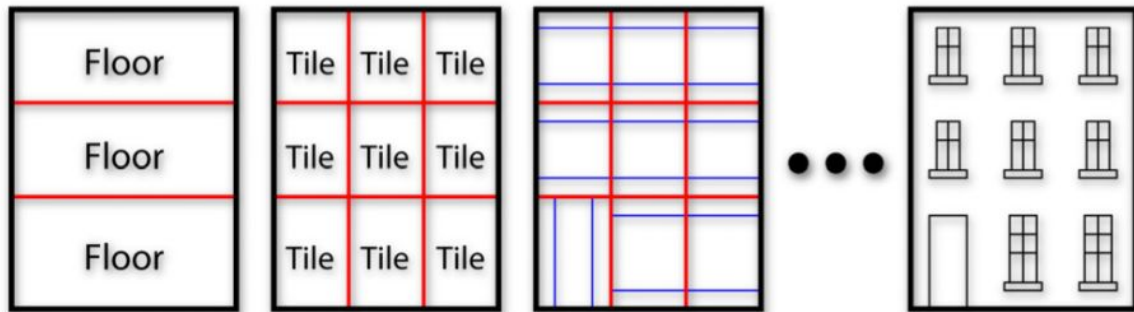


“Procedural Modeling of Buildings”,  
Mueller, Wonka, Haegler,  
Ulmer & Van Gool,  
SIGGRAPH 2006

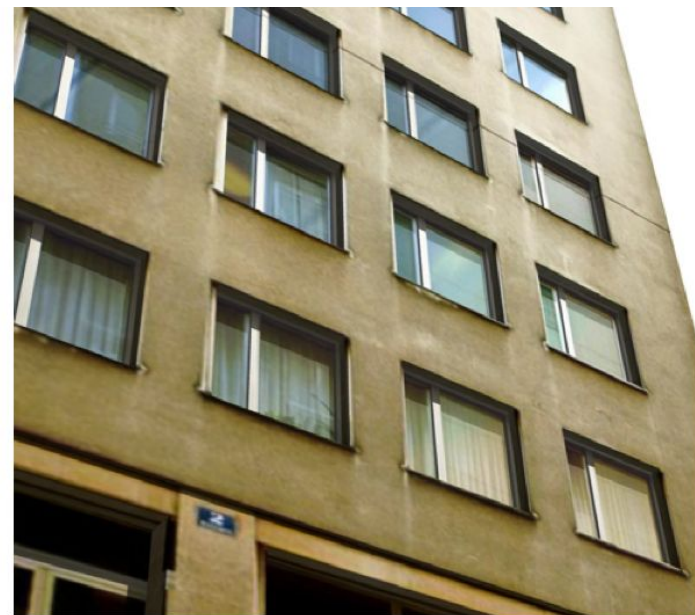


# “Image-based Procedural Modeling of Facades”

Mueller, Zeng,  
Wonka, & Van Gool,  
SIGGRAPH 2007



**Input Photograph**



**Reconstructed 3D Geometry**

# Procedural Modeling Advantages

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- Small representation
- Generate detail as needed (“infinite”? resolution)
- Great for natural mathematical patterns and man-made engineering and design
- Trivial to make many duplicate objects with small variations

# Procedural Modeling Applications

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- Entertainment – Gaming
- Education – Studying botanical variation
- Archeological reconstruction
- Realism for Training
- Predicting the future (how will things grow over time)
- Urban planning (preparing for traffic)
- Accommodate for that growth/change

# Questions about Procedural Modeling

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- Number of rules necessary?
- Cost in human designer time of creating procedural model?
- Re-useability of procedural model?
- Validation
- Can you build a procedural model that produces a specific target?

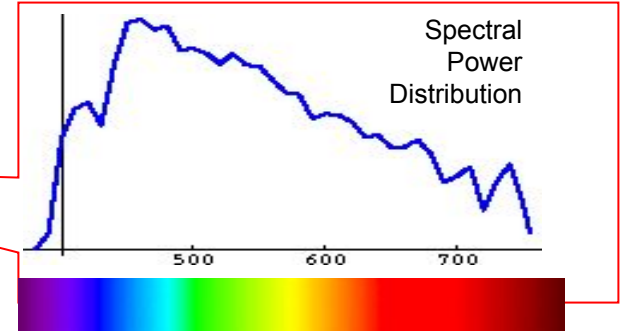
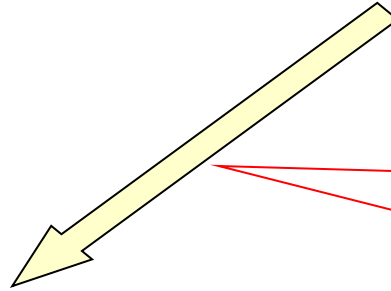
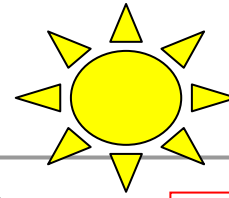
*From a photo of a specific rare wood grain,  
can you create a compact procedural model that  
creates texture that looks like it came from a  
different location of the same/similar tree?*

# Today

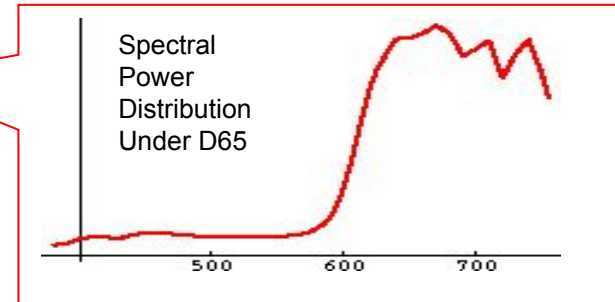
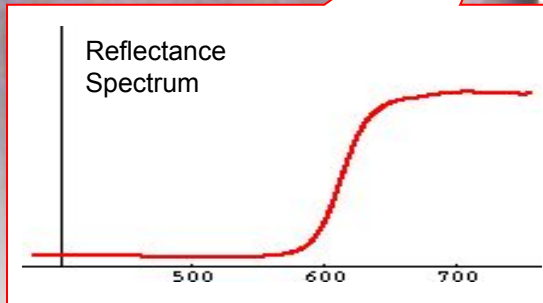
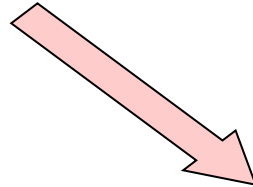
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# What Color is this Apple?

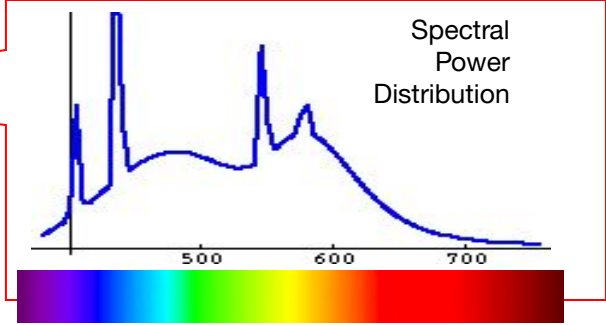
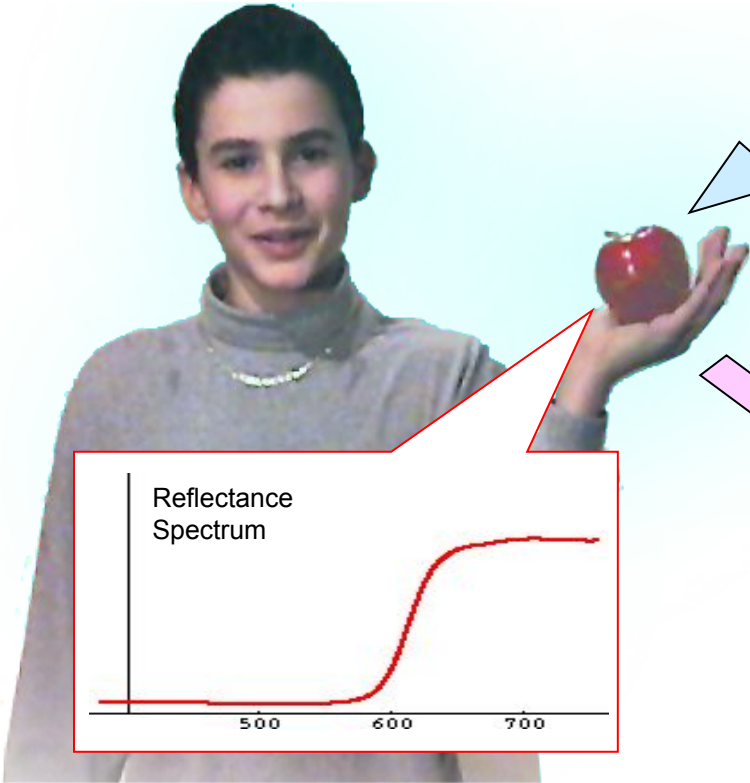


Illuminant D65  
(CIE standard for natural daylight)

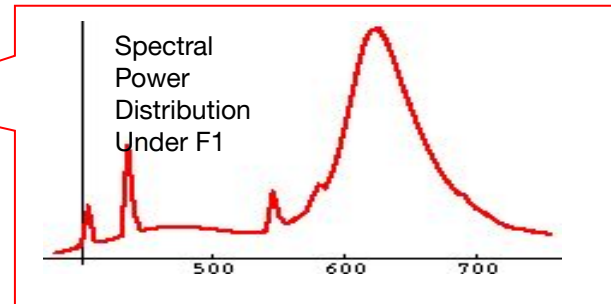
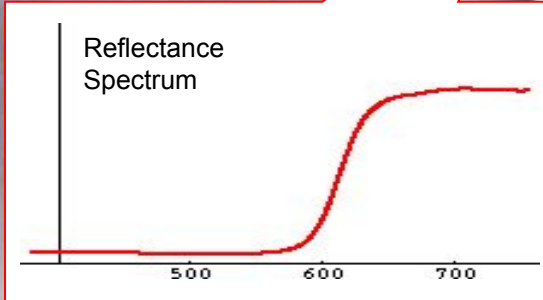


# What Color is this Apple?

Neon Lamp



Illuminant F1  
(one of the CIE standards  
for fluorescent lighting)

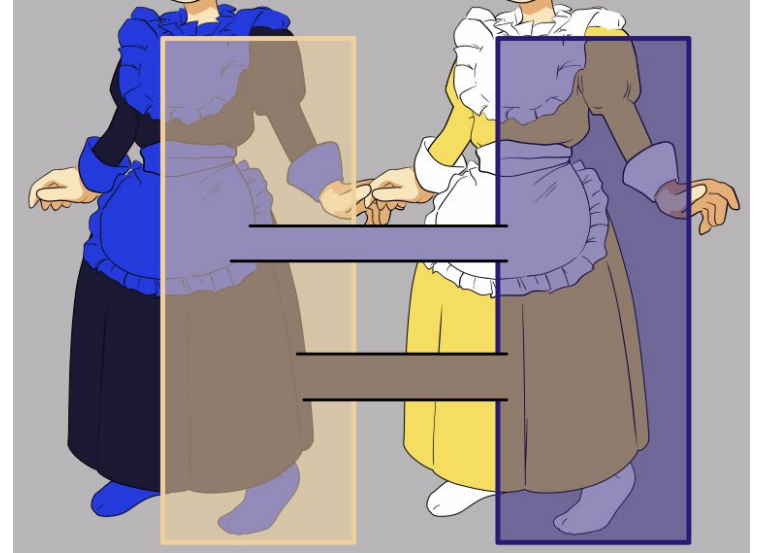


# What Color is this Dress?

[https://en.wikipedia.org/wiki/The\\_dress](https://en.wikipedia.org/wiki/The_dress)



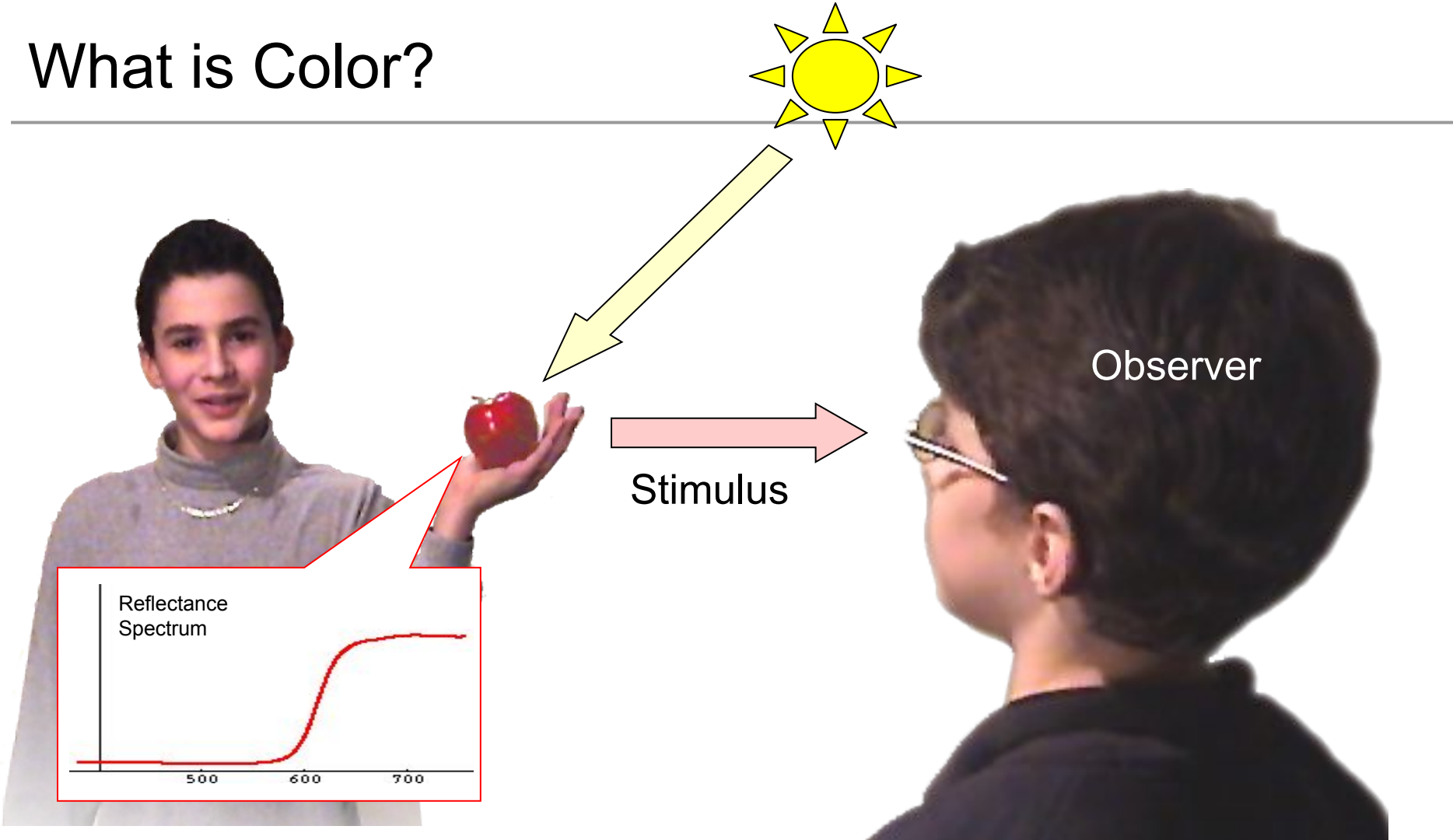
What does the viewer infer about the scene illumination?



Blue & Black under yellow-tinted illumination?

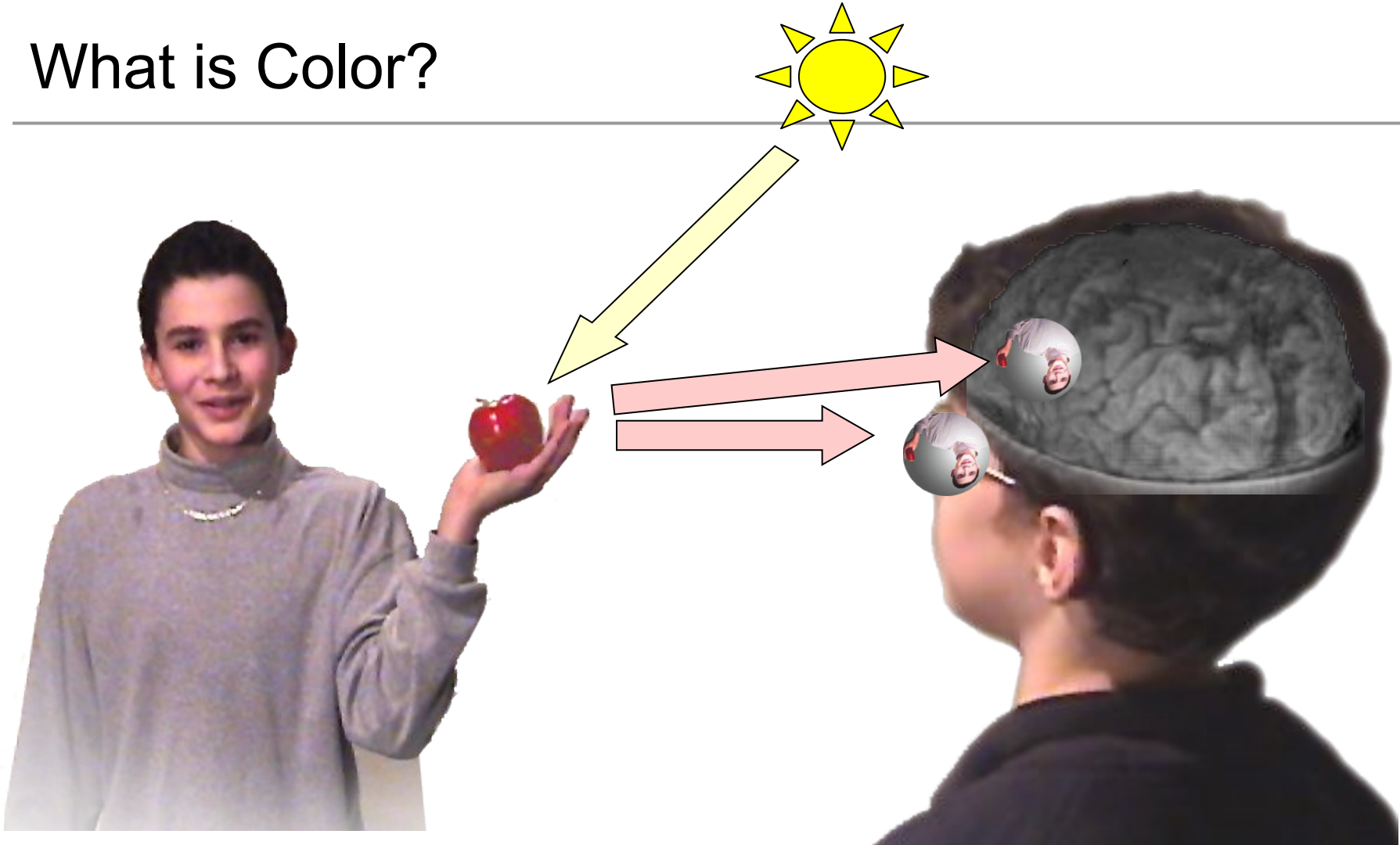
White & Gold under blue tinted illumination?

# What is Color?

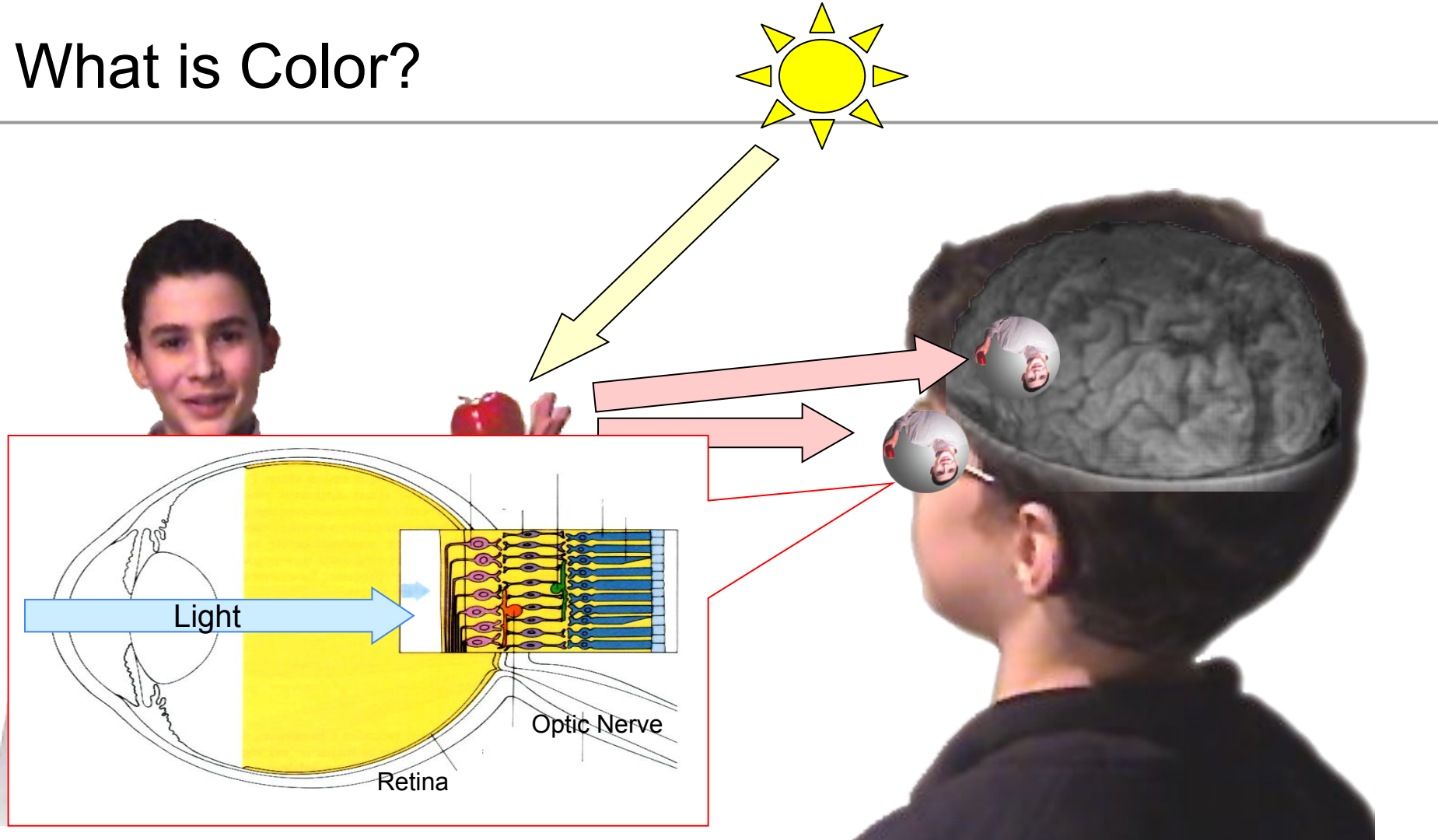


# What is Color?

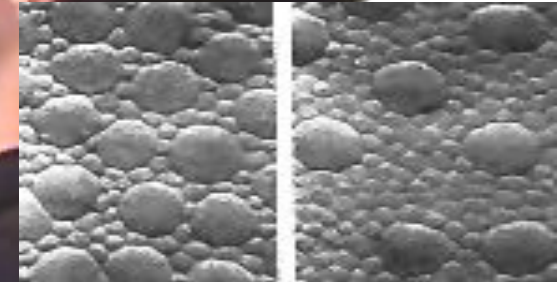
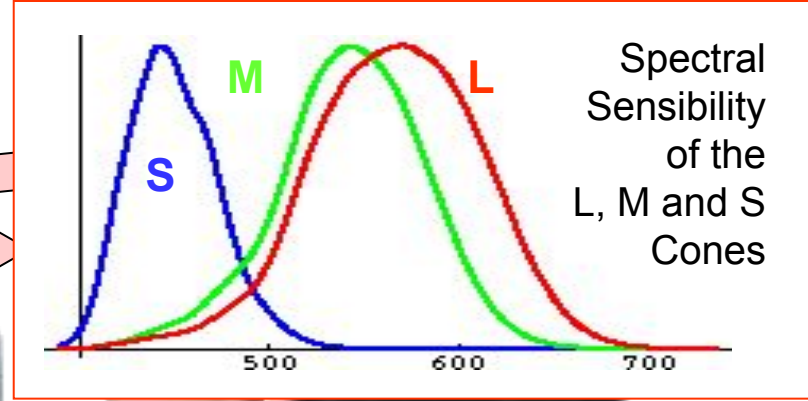
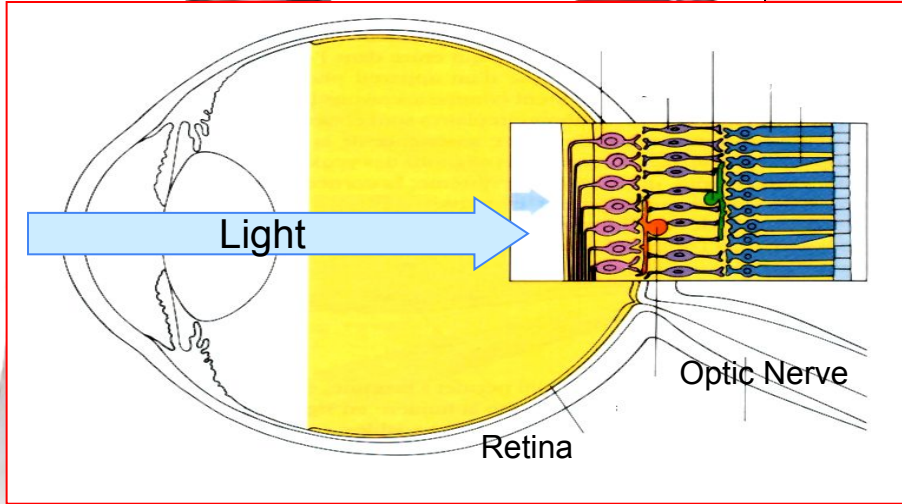
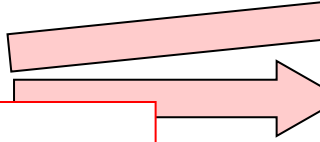
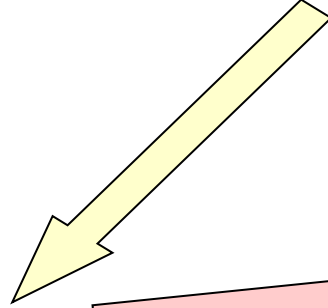
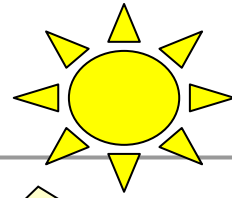
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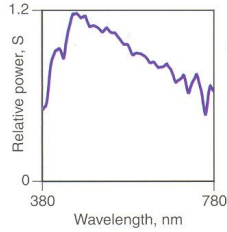
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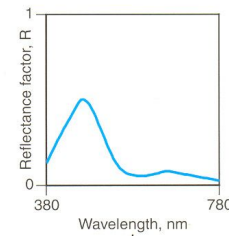
# What is Color?



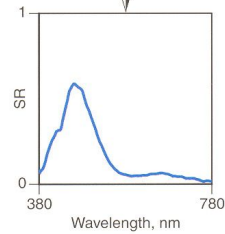
# Incoming Light



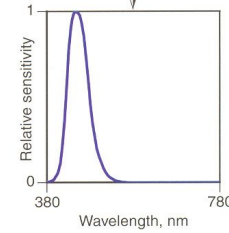
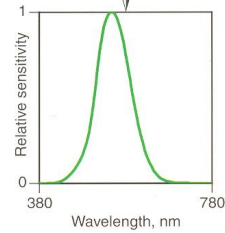
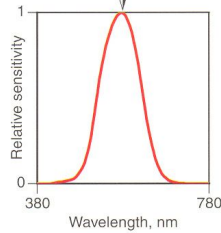
# Material Reflectance



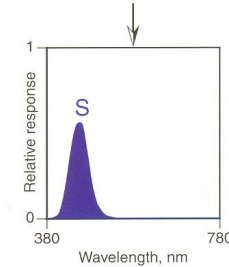
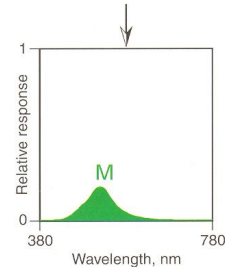
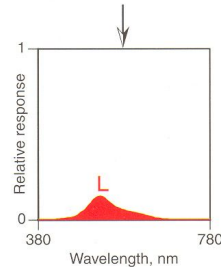
## Stimulus



## Cone Sensitivity

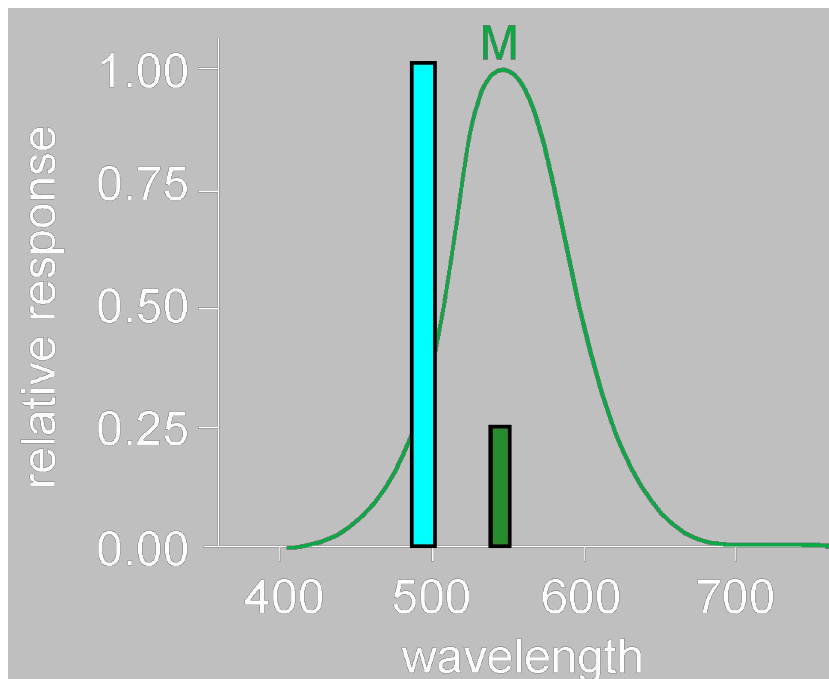


## Cone Responses



# Cones do not “See” Colors

- Different wavelength, different intensity
- *May have same response to a single cone*

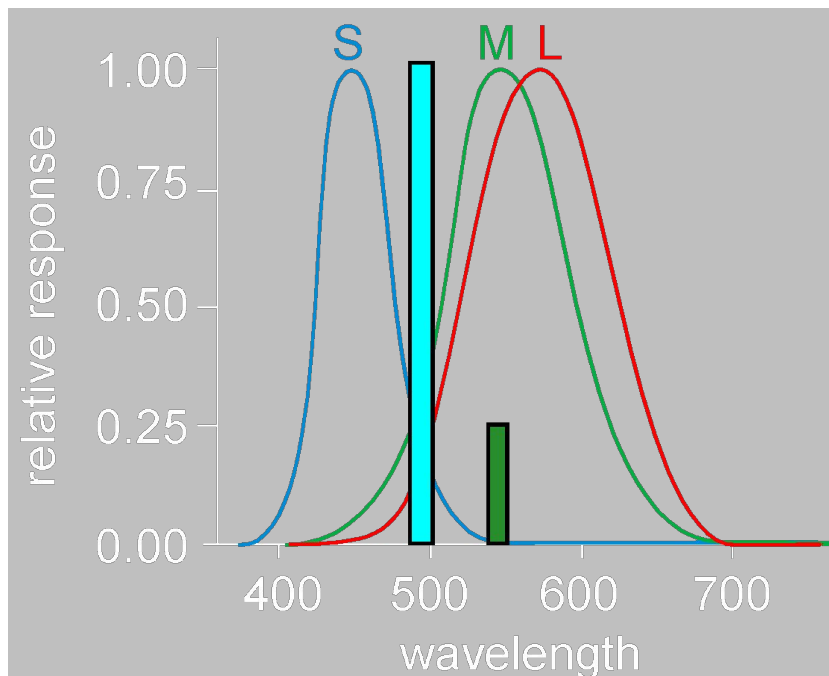


Dim green  
Cone M/G: 0.25

Bright cyan  
Cone M/G: 0.25

# Response Comparison

- Different wavelength, different intensity
- *Will have different responses for different cones*



Dim green

Cone L/R: 0.20

Cone M/G: 0.25

Cone S/B: 0.01

Bright cyan

Cone L/R: 0.20

Cone M/G: 0.25

Cone S/B: 0.25

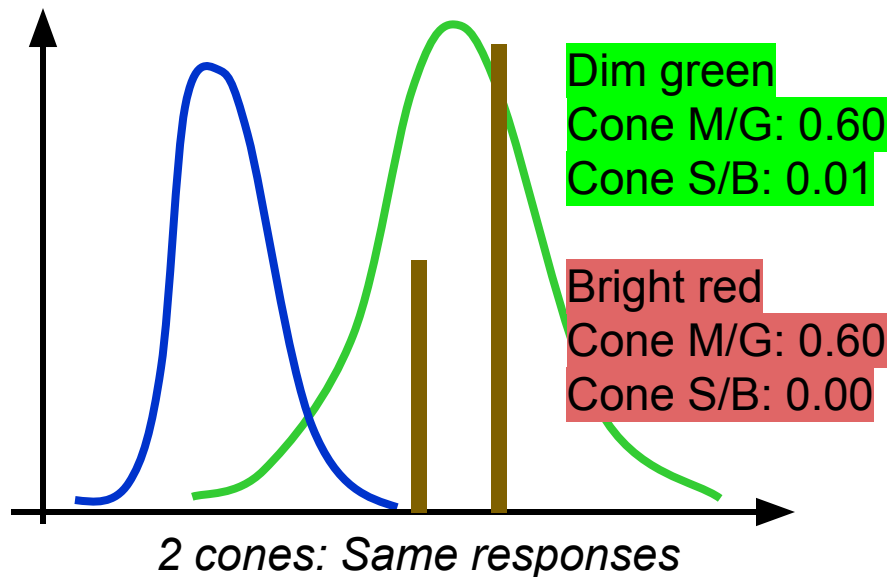
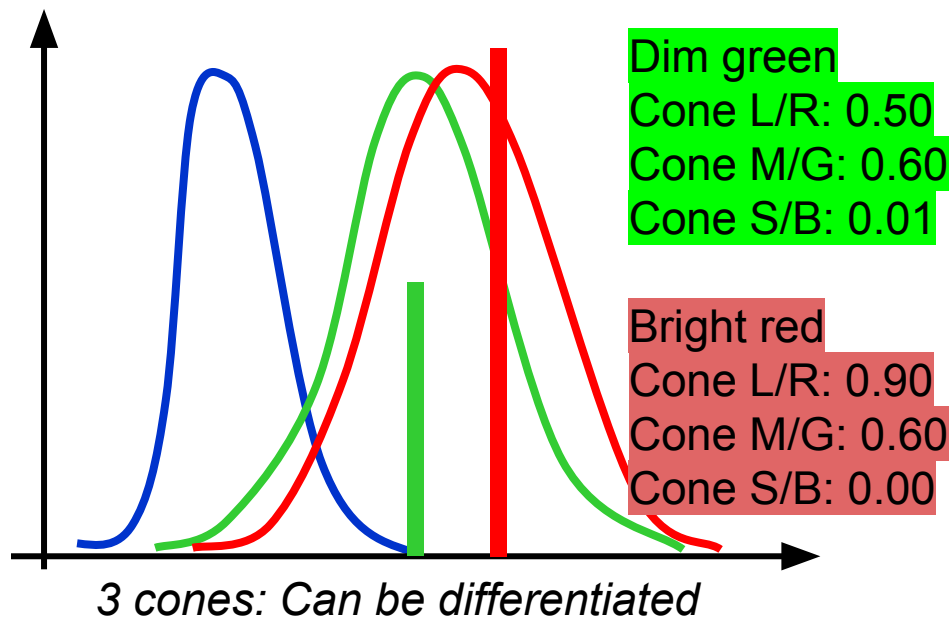
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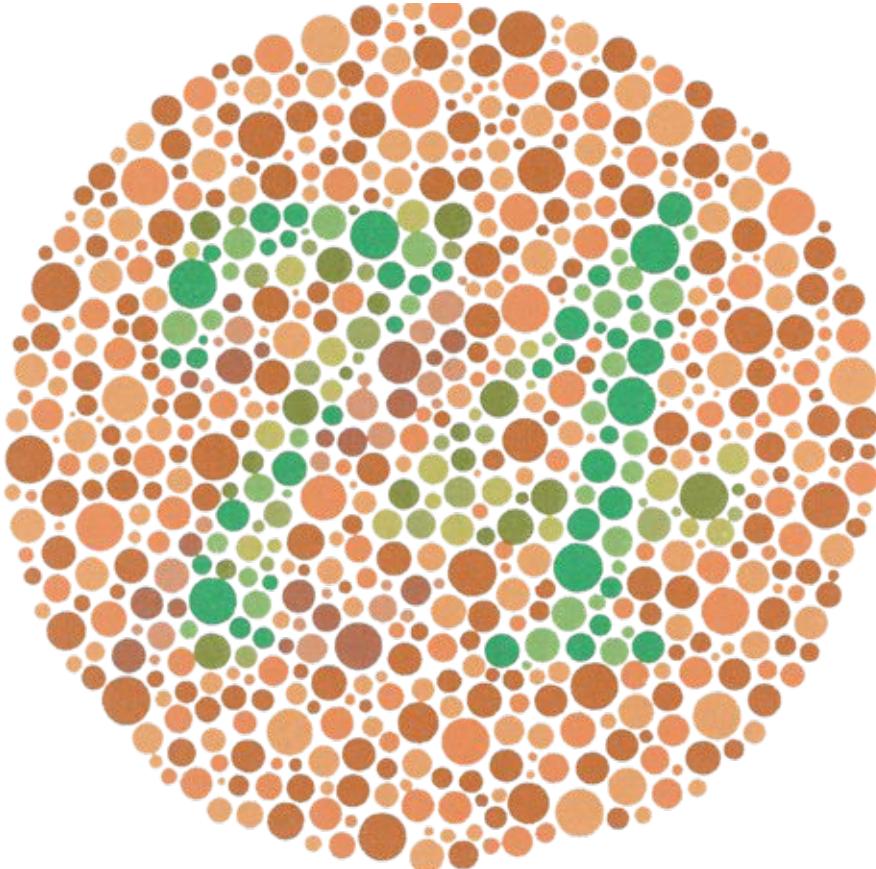
# Color Blindness

- Classical case: 1 type of cone is missing (e.g. red)
- Response is projected onto lower-dim space (2D)
- *Makes it impossible to distinguish some spectra*



# Ishihara Color Blindness Test

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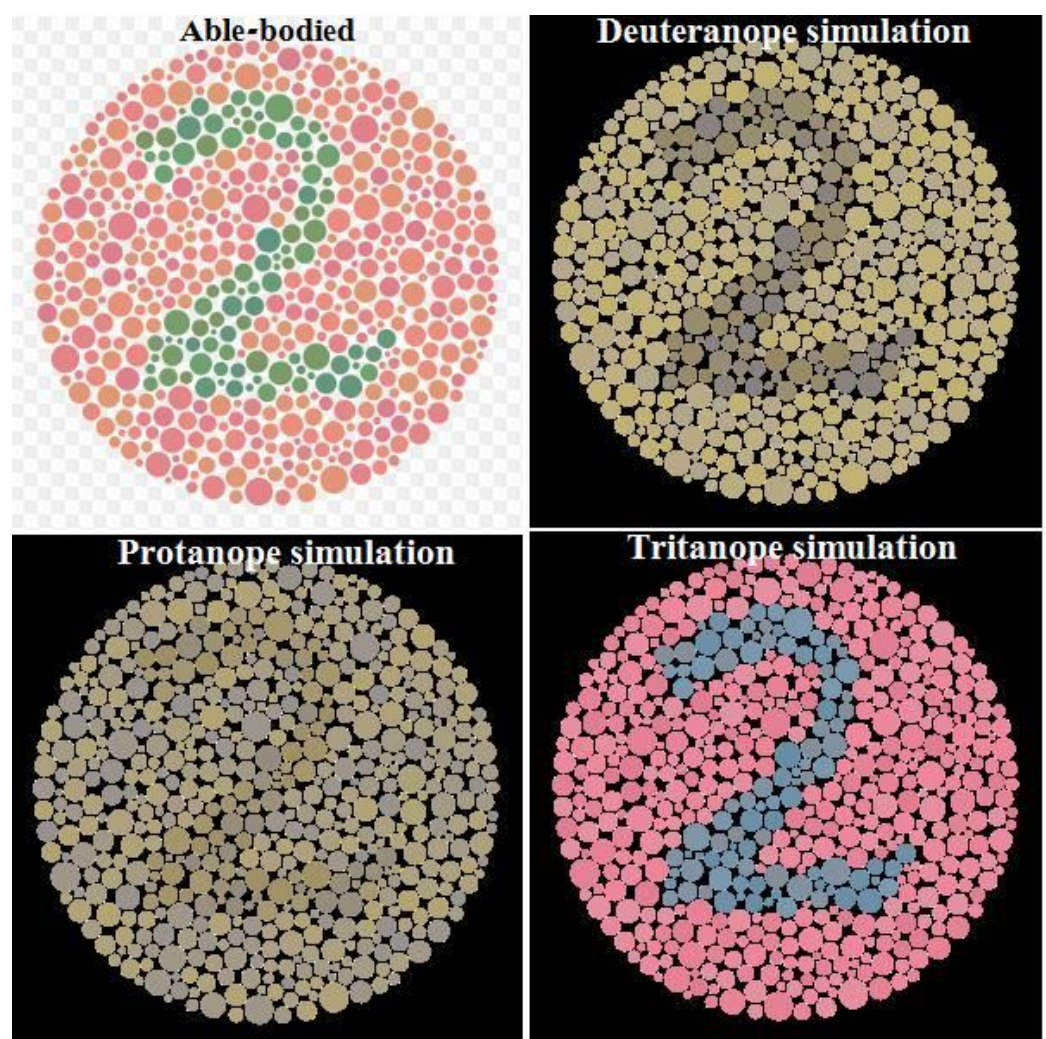


*As we have already discussed...*  
Illumination is very important to proper color perception.

This test must be conducted with a calibrated sample and controlled lighting for an accurate diagnoses.

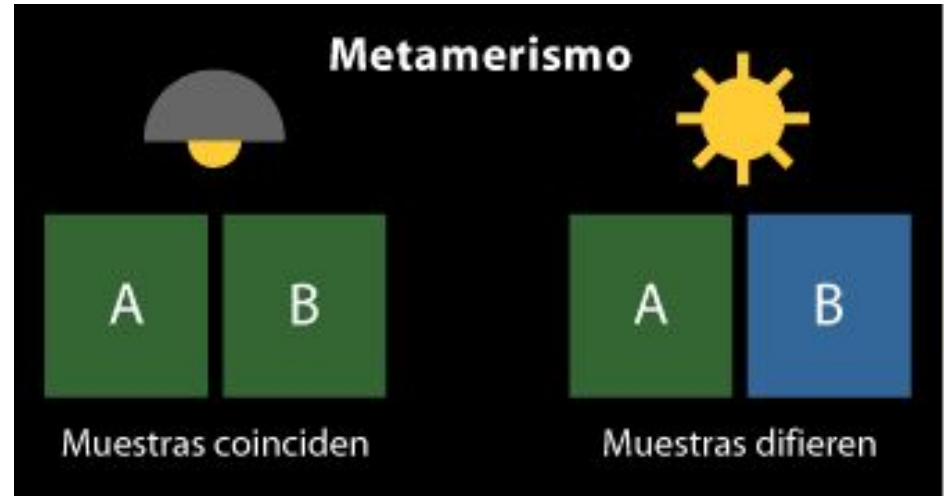
- Deuteranopia:  
missing  
medium / green cone
- Protanopia:  
missing  
long / red cone
- Tritanopia: (rare)  
missing  
short / blue cone

[http://en.wikipedia.org/wiki/  
File:Ishihara\\_compare\\_1.jpg](http://en.wikipedia.org/wiki/File:Ishihara_compare_1.jpg)



# Metamerism: Apparent Matching

- When two materials look the same under one lighting condition (a coincidence), but look different under another.
- E.g. the shirt & pants matched in the store lighting, but not outside!
- Different spectral distribution of input light yield different visual stimuli
- *We all experience some color blindness*



<http://gusgsm.com/metamerismo>

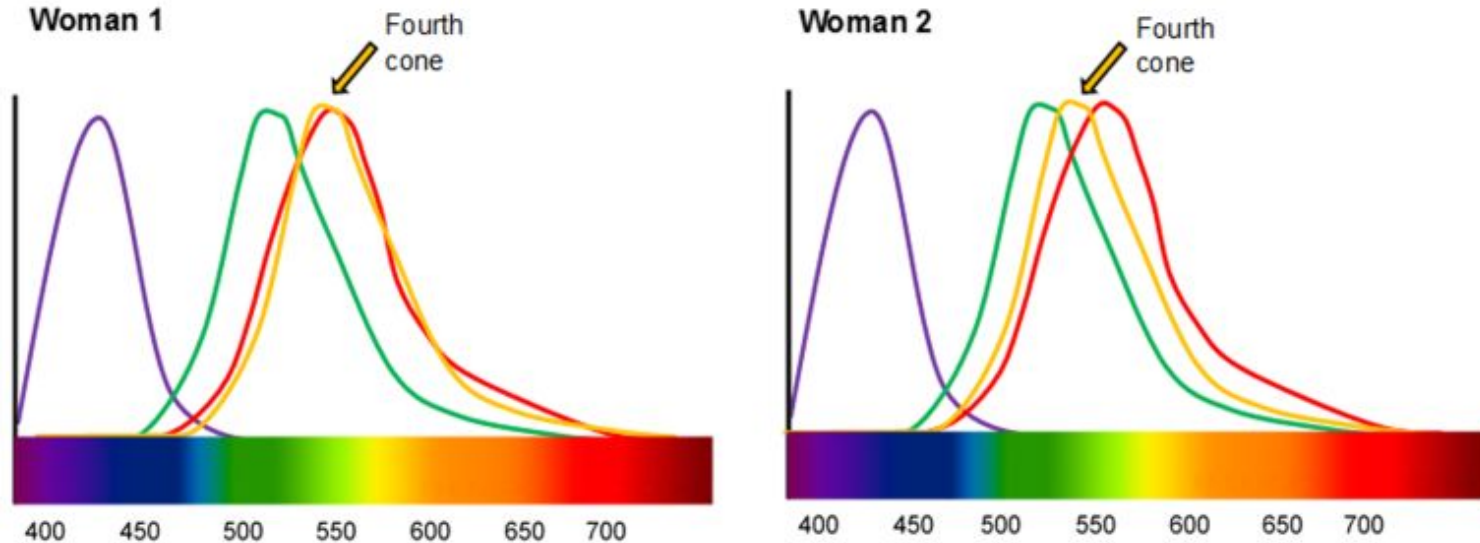
# Tetrachromacy: Some People have 4 Cones!?!?

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*Typically a slight or moderate mutation of the red or green cone.*

*May be detectable by a vision test. Less likely to experience metamerism.*

*But cannot see wavelengths not visible to other humans. Not superhuman vision!*



<https://theneurosphere.com/2015/12/17/the-mystery-of-tetrachromacy-if-12-of-women-have-four-cone-types-in-their-eyes-why-do-so-few-of-them-actually-see-more-colours/>

# Glasses to “Correct” Colorblindness?

- *“Enchroma does not endorse use of the glasses to pass occupational screening tests such as the Ishihara test.”*
- Enchroma (and other similar products) is not a cure for color blindness.
- Does not repair missing cones.
- Does not make the eyes more sensitive.
- Filters (selectively darkens) input stimulus.
- Reaction videos are mostly/entirely staged for viral internet marketing.

*Debunked by Jonathon, a.k.a., MegaLag*

<https://www.youtube.com/watch?v=Ppobi8VhWwo&t=0s>



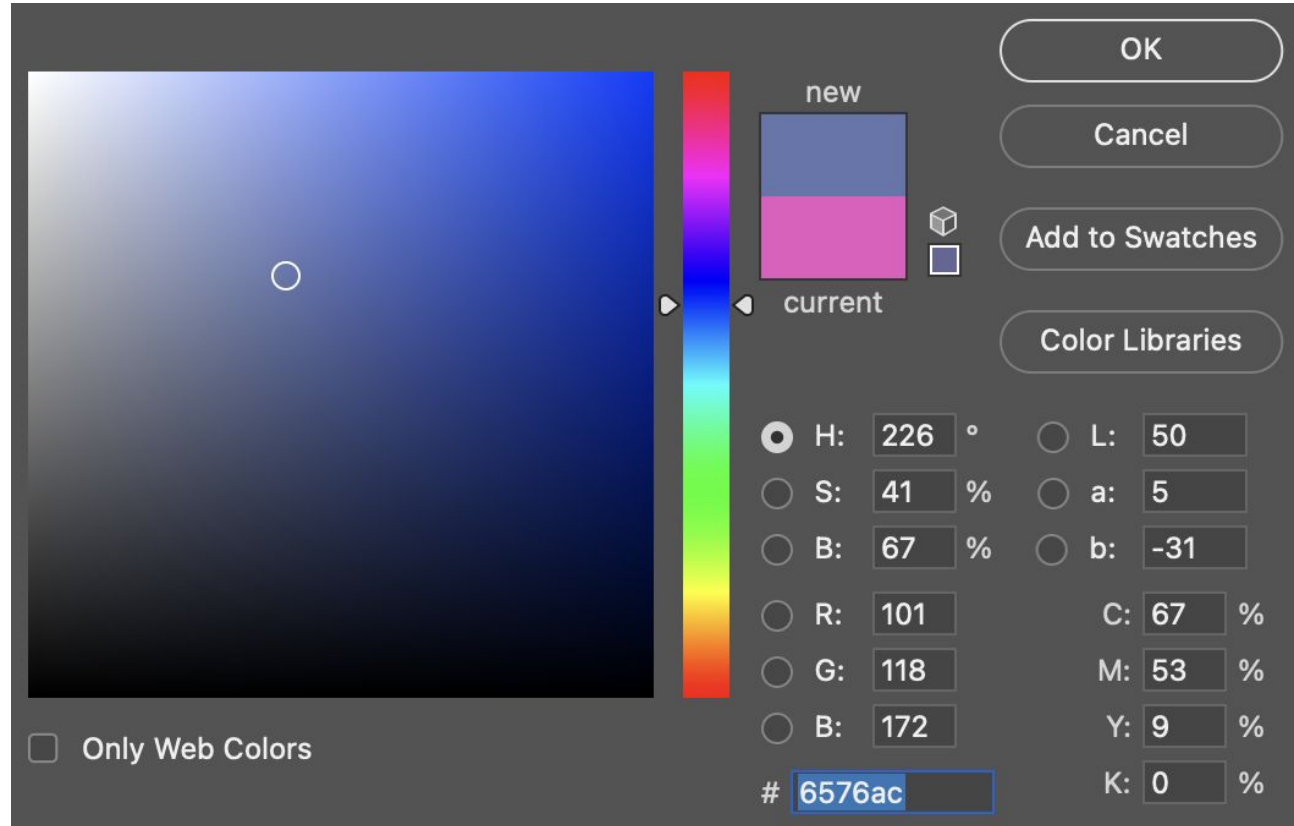
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# Color Picker in Photoshop

- What are all the different choices?



# Standard Color Spaces

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- Colorimetry: Science of color measurement
- Quantitative measurements of colors are crucial in many industries
  - Television, computers, print, paint, luminaires
- Naive digital work uses a vague notion of RGB
  - Unfortunately, RGB is not precisely defined, and depending on your monitor, you might get something different
- We need a principled color space...

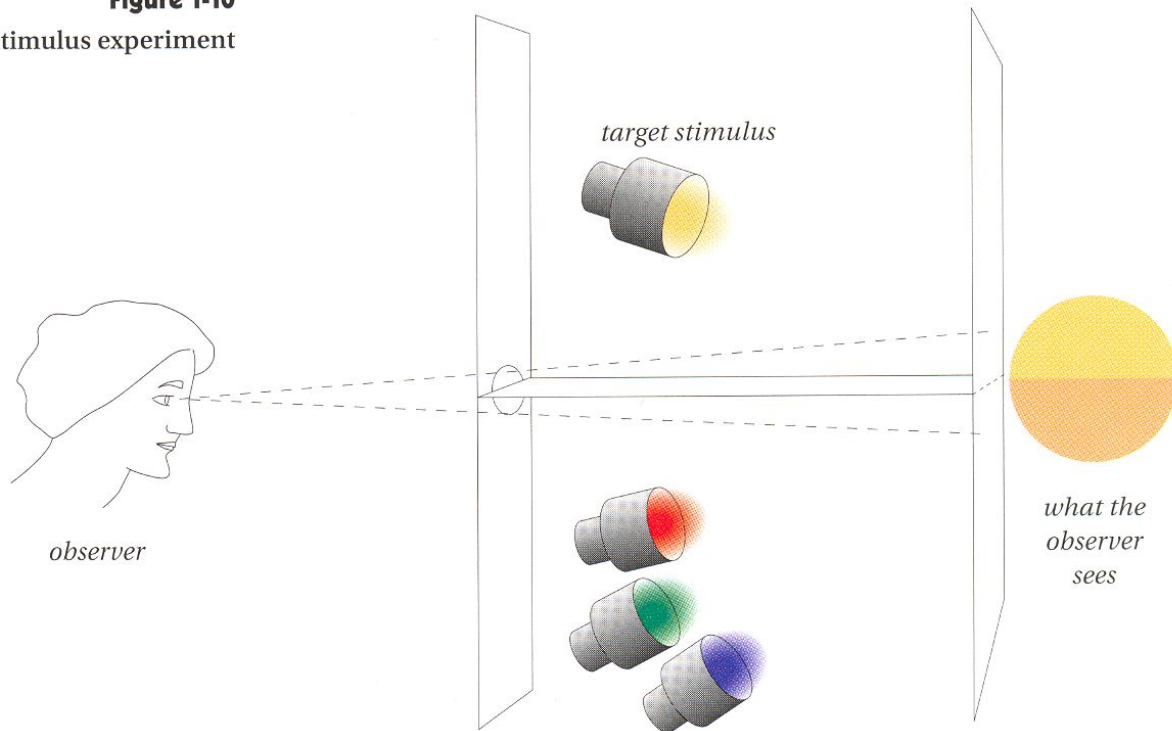
# CIE Color Matching Experiments

Commission  
Internationale de  
l'éclairage  
(CIE)

a.k.a.

International  
Commission on  
Illumination

**Figure 1-10**  
Tristimulus experiment



*The observer adjusts the intensities of the red, green, and blue lamps until they match the target stimulus on the split screen.*

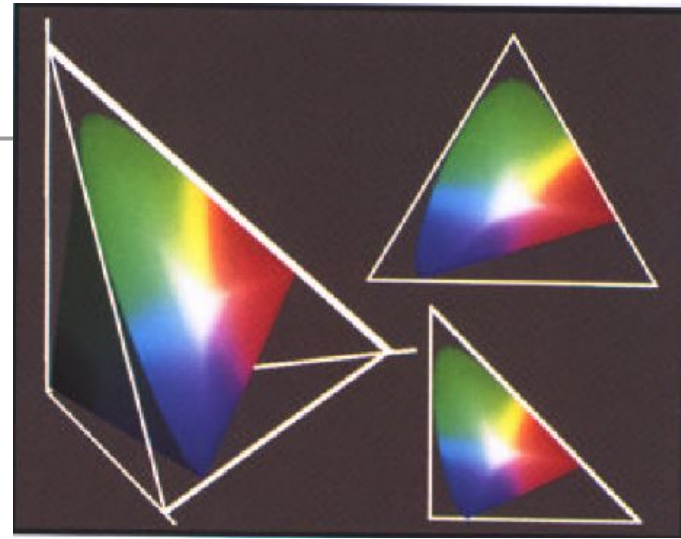
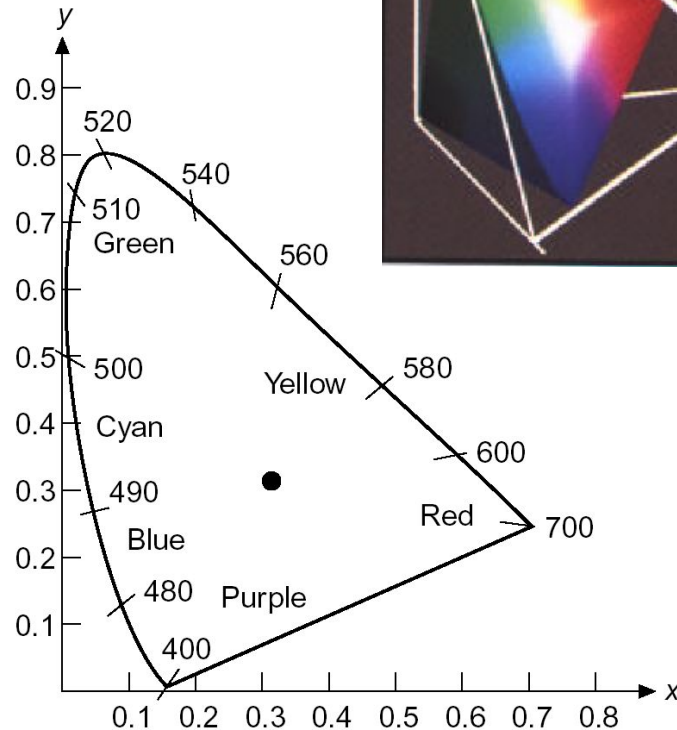
# CIE XYZ Color Space

- Can think of X, Y, & Z as 3D coordinates
- Linear transform to/from typical LMS or RGB

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = \begin{pmatrix} 3.24 & -1.54 & -0.50 \\ -0.97 & 1.88 & 0.04 \\ 0.06 & -0.20 & 1.06 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 0.41 & 0.36 & 0.18 \\ 0.21 & 0.72 & 0.07 \\ 0.02 & 0.12 & 0.95 \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

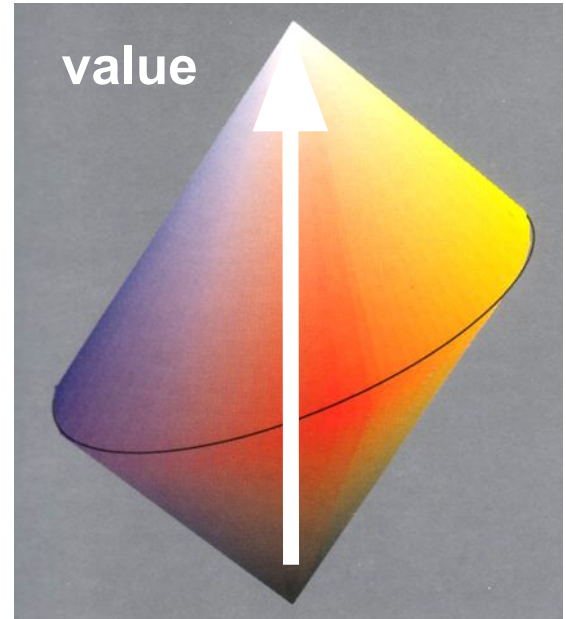
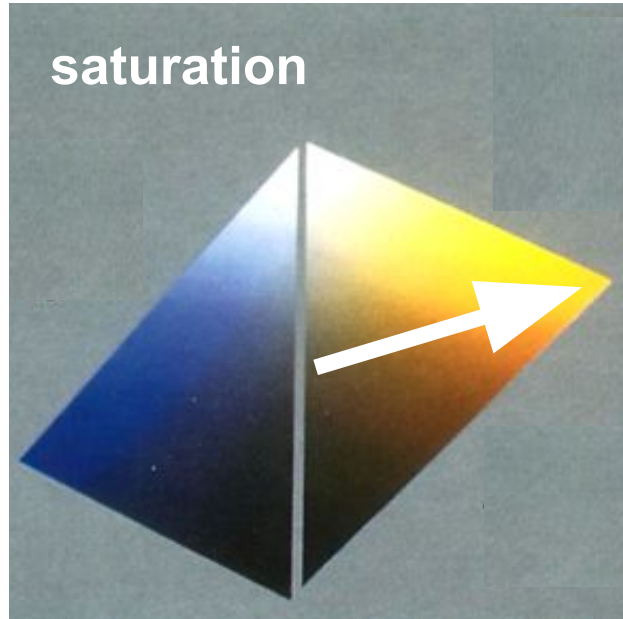
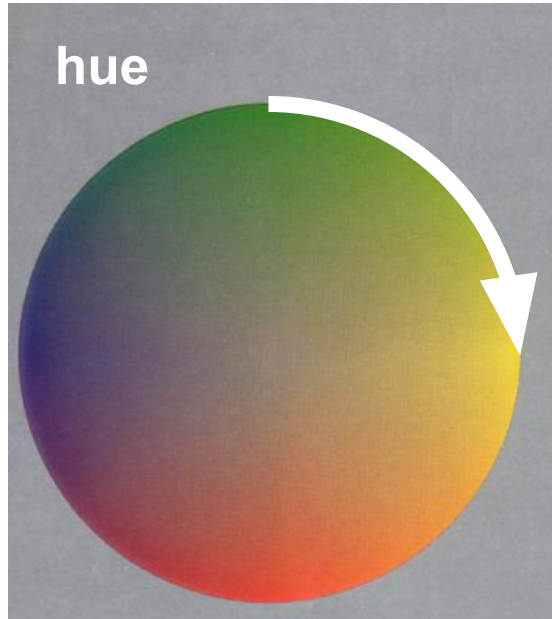
- Note that many points in XYZ do not correspond to visible colors!



# Hue Saturation Value (HSV)

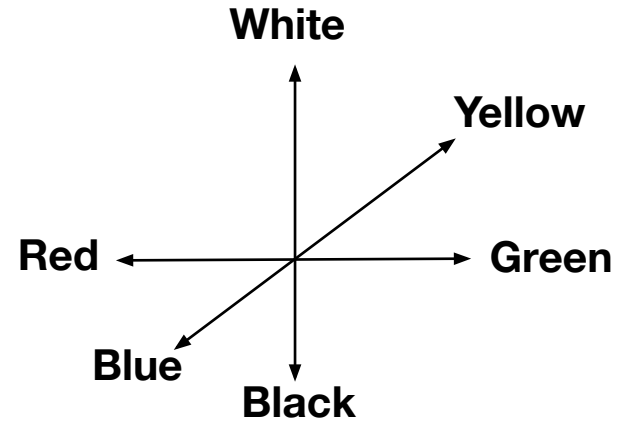
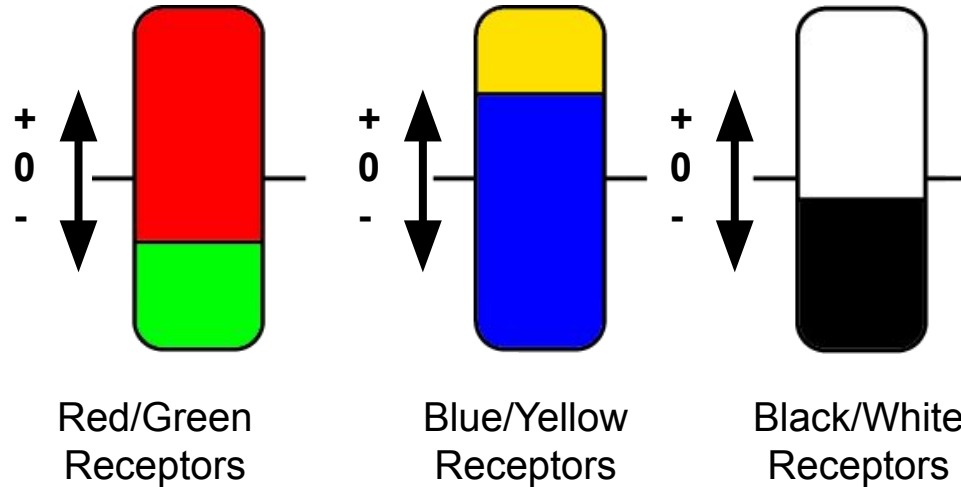
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- Hue: dominant color (red, orange, etc)
- Saturation: from gray to vivid color (a.k.a. Chroma)
- Value: from black to white (a.k.a. Brightness, similar to Lightness)



# Hering 1874: Opponent Colors

- Hypothesis of 3 types of receptors:  
Red/Green, Blue/Yellow, Black/White
- Explains well several visual phenomena

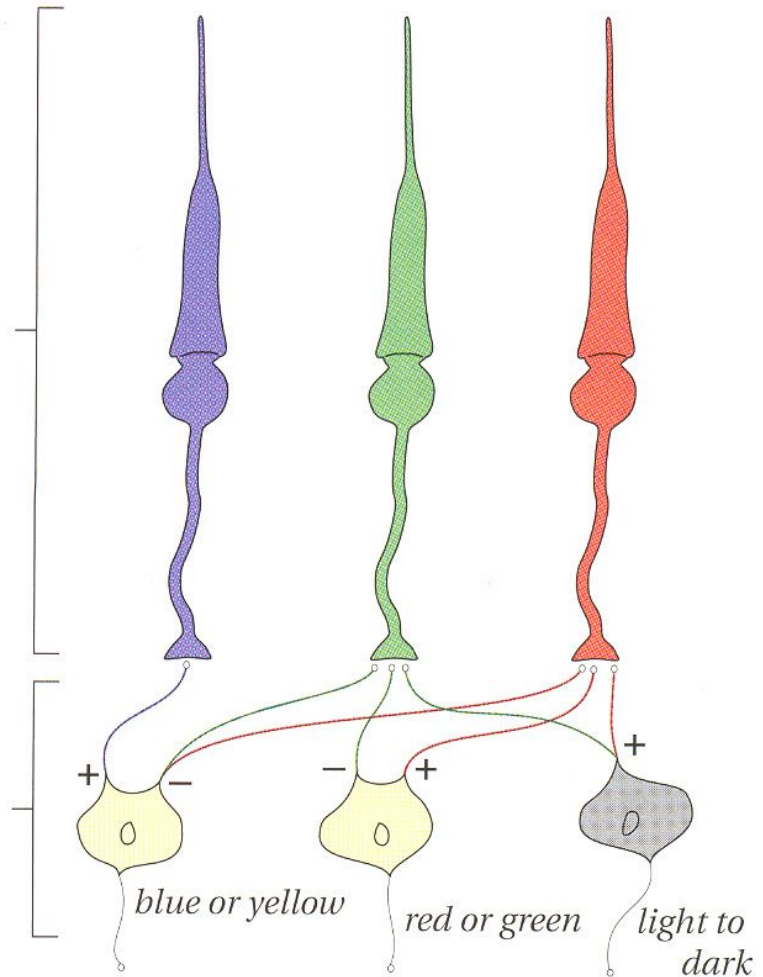


# Color Opponents “Wiring”

- Sums for brightness
- Differences for color opponents
- It's just a 3x3 matrix to convert HSV from/to LMS, RGB, or XYZ

*First zone (or stage):  
layer of retina with  
three independent  
types of cones*

*Second zone (or stage):  
signals from cones  
either excite or inhibit  
second layer of  
neurons, producing  
opponent signals*

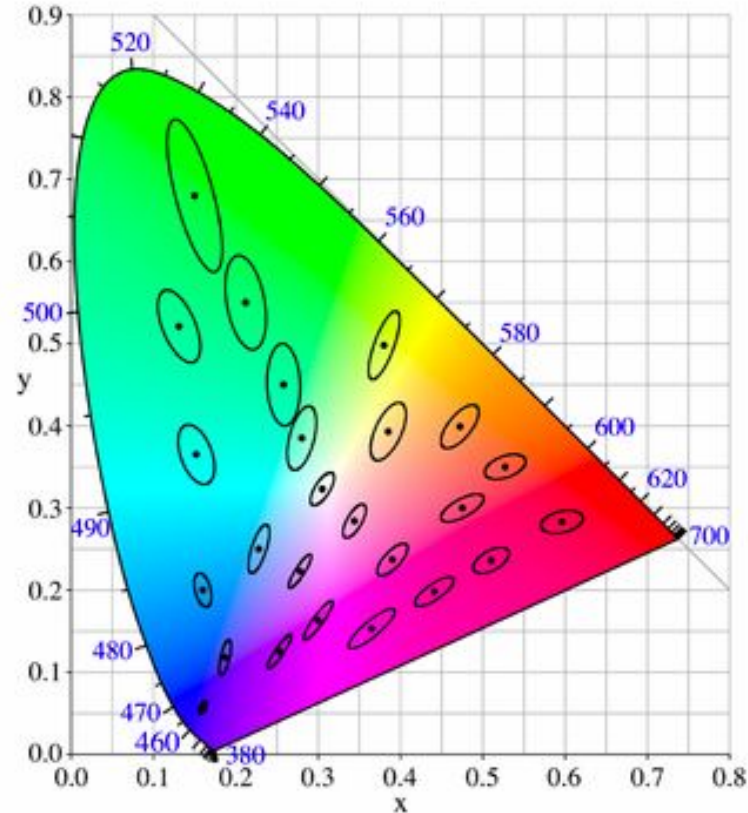


# Linear Color Spaces: RGB/XYZ/YPbPr

- Can convert between these spaces with a 3x3 matrix multiplication, e.g.:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.4124 & 0.3576 & 0.1805 \\ 0.2126 & 0.7152 & 0.0722 \\ 0.0193 & 0.1192 & 0.9505 \end{bmatrix} \begin{bmatrix} R_{\text{linear}} \\ G_{\text{linear}} \\ B_{\text{linear}} \end{bmatrix}$$

- However, equal steps in linear color spaces do not correspond to equal differences for human perception
- MacAdam ellipses visualize the lack of perceptual uniformity [MacAdam 1942]



# Today

---

- Worksheet: Shadow Techniques
- Readings for Today
- What is Color?
  - Human Perception
  - Color Blindness & Metamerism
- Color Spaces
  - Linear Color Spaces: LMS, RGB, XYZ, HSV
- Projection in Spatially Augmented Reality
  - Human Perception & Perceptual Color Space:  $L^*a^*b^*$
- Readings for Next Time

# Spatially Augmented Reality (SAR) Projection

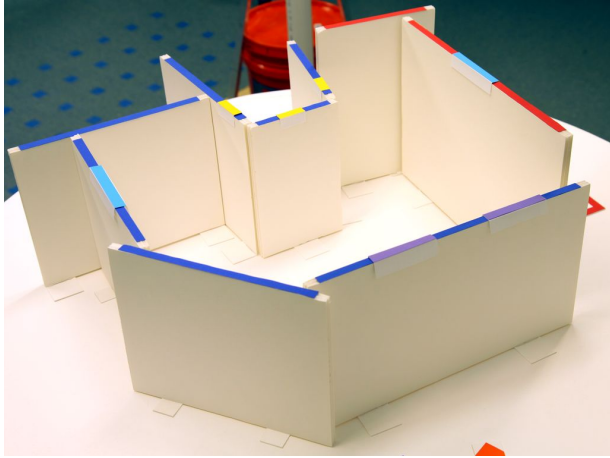


*camera detects  
design geometry*

*6 projectors  
augment design*

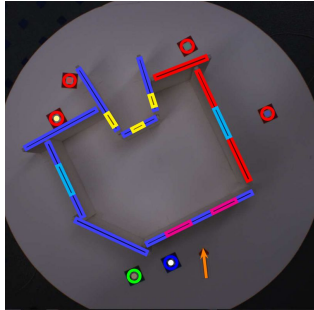
*design sketched with  
foam-core walls*

# Tangible Interface for Architectural Design

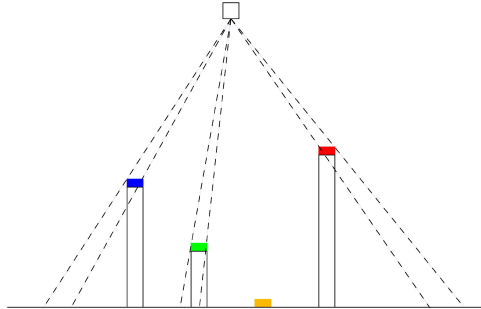


*Exterior & interior wall  
Tokens for:*

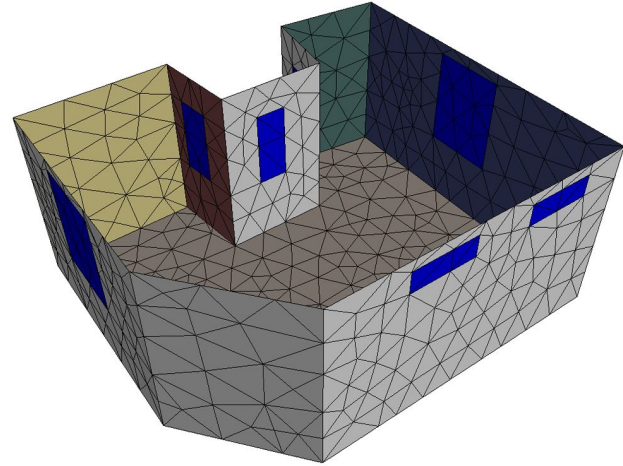
- *Windows*
- *Wall/floor colors*
- *North arrow*



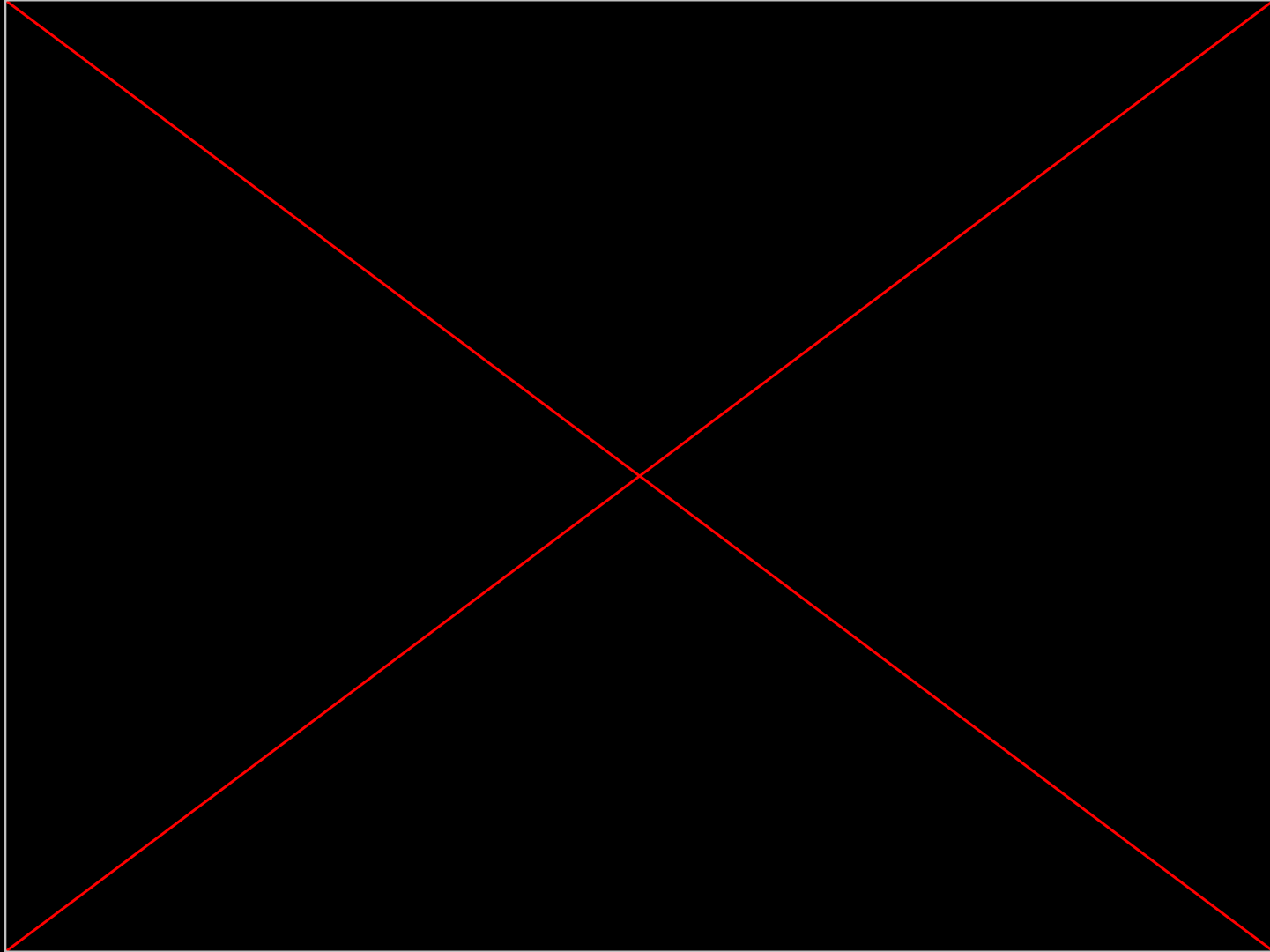
*Overhead camera*



*Projection geometry*



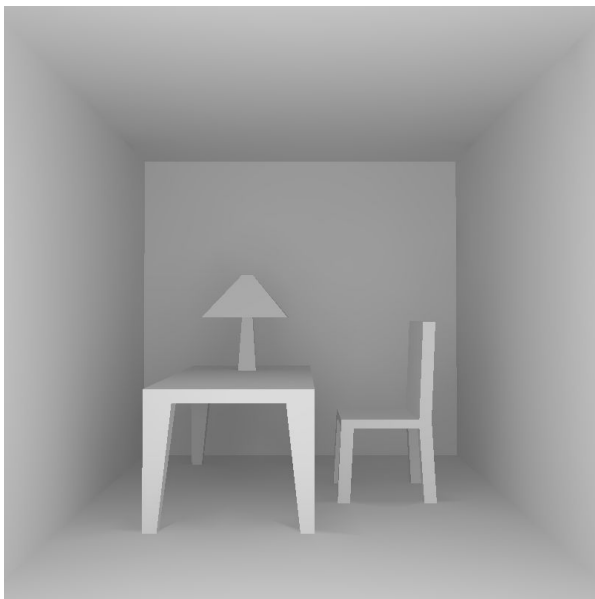
*Inferred design*



# Motivation

---

*Can we do a better job reproducing the desired appearance?*



*geometry & materials*



*desired appearance*



*uncompensated projection*

# Related Work: Radiometric Compensation

- Minimize artifacts caused by light modulation with local surface [Bimber et al. 2005; Nayar et al. 2003; Grundhöffer & Bimber 2008]
- Does not consider global light inter-reflection



Grundhöffer & Bimber 2008

# Our Problem Statement

---

- Known scene geometry
- Known surface reflectances, *all ideal diffuse*
- Fixed, calibrated projectors
- Given:
  - Desired target surface appearance (texture)  
for each physical surface
- Solve for:
  - Projection texture for each physical surface that most faithfully reproduces the desired appearance

# Related Work: Reverse Radiosity

- Forward lighting with radiosity

$$B = (I - F)^{-1} E$$

*values for rendering*

*form factor matrix*

*direct light*

- Inverse lighting with radiosity:

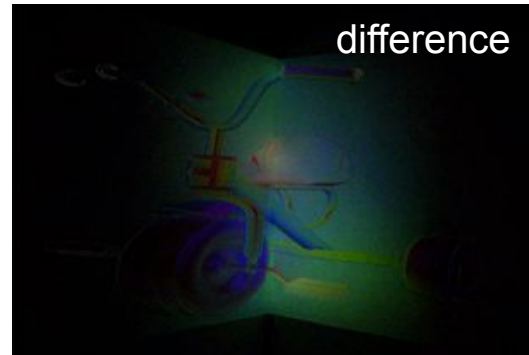
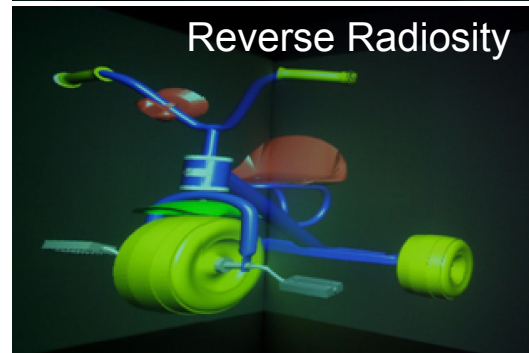
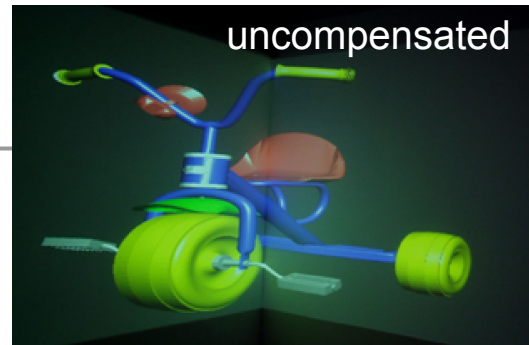
Reverse Radiosity (RR)

- [Bimber et al. 2006]

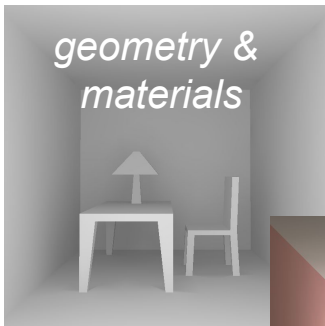
$$E = (I - F)B$$

*projection values*

*desired appearance*



geometry &  
materials



desired  
appearance



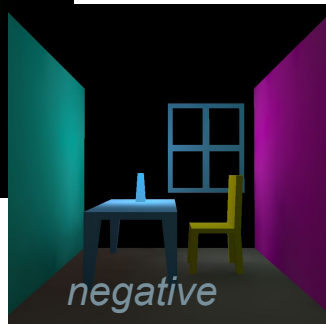
uncompensated



Reverse Radiosity

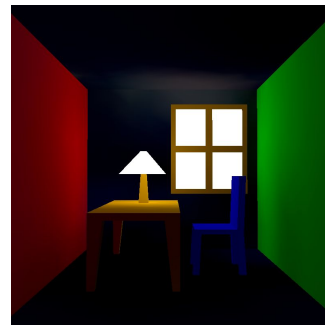
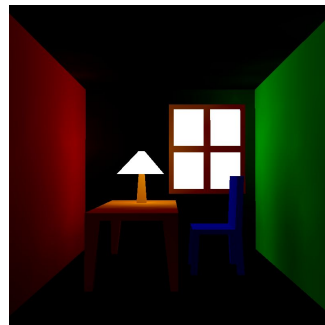


positive



negative

Sheng et al. 2010 Sheng et al. 2011  
YPbPr  $L^*a^*b^*$



# L\*a\*b\*: a Perceptual Color Space

---

- Designed to match human color perception data

$$\begin{array}{l} \text{intensity} \\ \text{red-green} \\ \text{yellow-blue} \end{array} \begin{bmatrix} L^* \\ a^* \\ b^* \end{bmatrix} = \begin{bmatrix} 116h\left(\frac{Y}{Y_n}\right) - 16 \\ 500 \left( h\left(\frac{X}{X_n}\right) - h\left(\frac{Y}{Y_n}\right) \right) \\ 200 \left( h\left(\frac{Y}{Y_n}\right) - h\left(\frac{Z}{Z_n}\right) \right) \end{bmatrix}$$

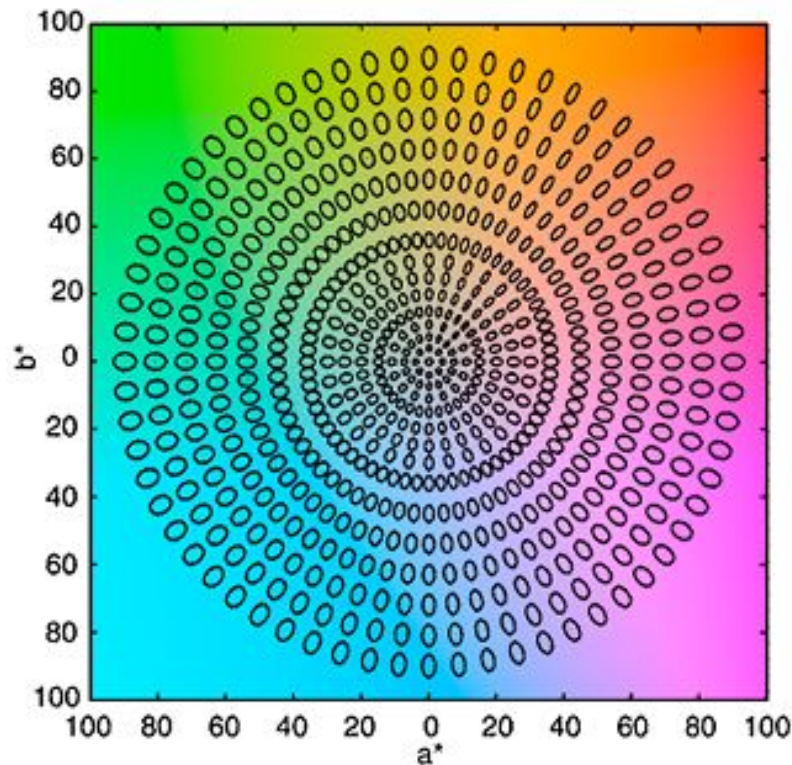
$$h(t) = \begin{cases} t^{\frac{1}{3}} & t > (6/29)^3 \\ \frac{1}{3} \left( \frac{29}{6} \right)^2 t + \frac{4}{29} & \text{Otherwise} \end{cases}$$

- L\*a\*b\* is *nonlinear*, a challenge for optimization

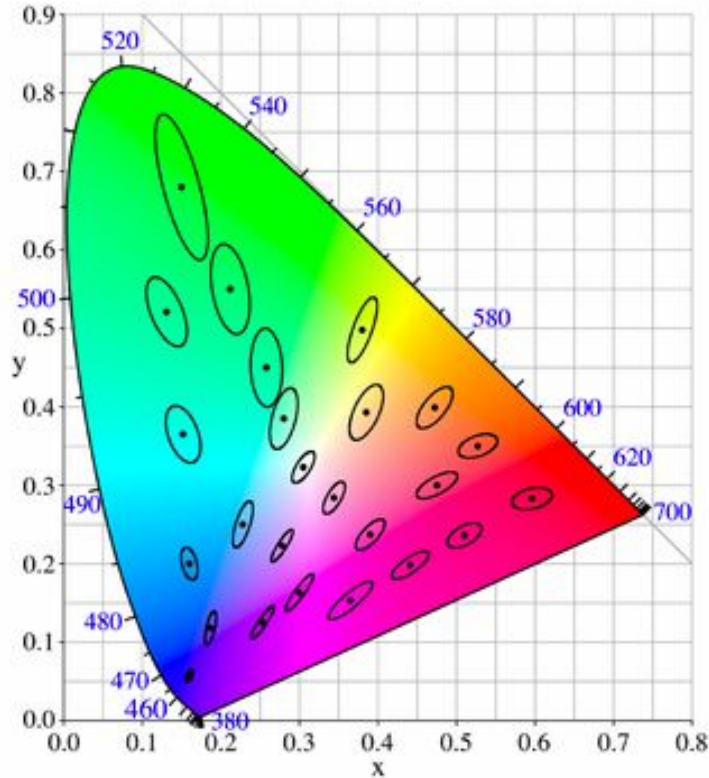
# Quantitative Perceptual Comparison

$$\Delta E = \sqrt{(L_1 - L_2)^2 + (a_1 - a_2)^2 + (b_1 - b_2)^2}$$

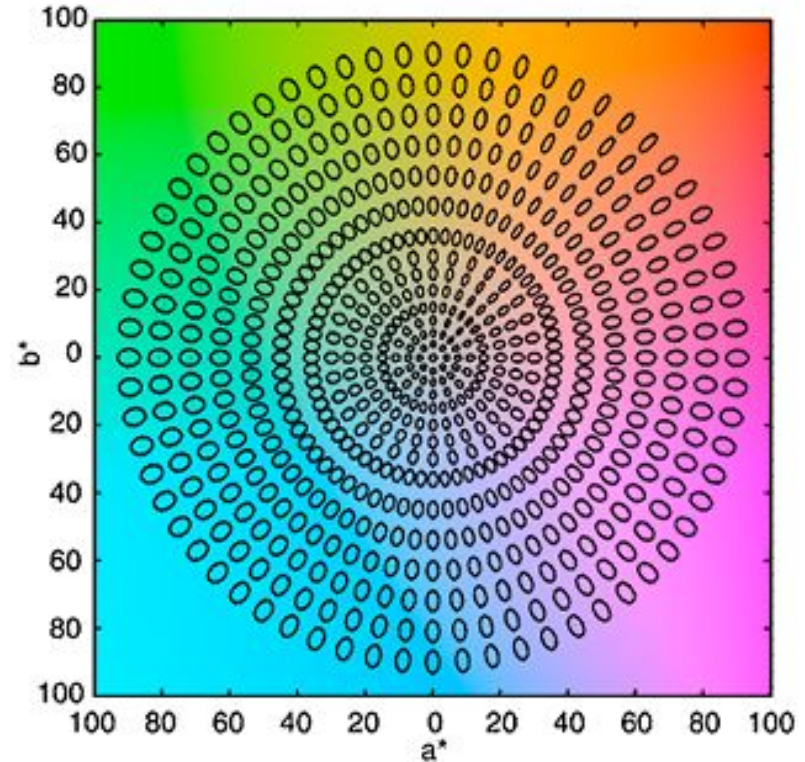
- Where 2.3  $\Delta E = \text{JND}$   
“just noticeable difference”
- The MacAdams ellipses  
are more equal size circles  
in  $L^*a^*b^*$



# MacAdams Ellipses: XYZ vs. L\*a\*b\*



[http://en.wikipedia.org/wiki/File:CIExy1931\\_MacAdam.png](http://en.wikipedia.org/wiki/File:CIExy1931_MacAdam.png)



[http://w3.kcu.ac.jp/~fujiwara/infosci/ellipses\\_lab.png](http://w3.kcu.ac.jp/~fujiwara/infosci/ellipses_lab.png)

# Our Optimization Formulation

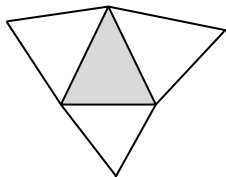
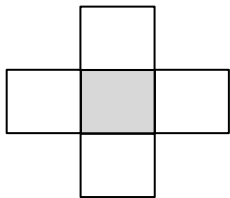
- Absolute Error:

$$\phi_{abs} = \frac{\sum_i A_i [(L_i - L'_i)^2 + (a_i - a'_i)^2 + (b_i - b'_i)^2]}{A_{avg}}$$

*desired appearance* *projection result*

- Spatial Error:

$$\phi_{spt} = \sum_{(i,j) \in nbd} [(L_i - L_j) - (L'_i - L'_j)]^2 + [(a_i - a_j) - (a'_i - a'_j)]^2 + [(b_i - b_j) - (b'_i - b'_j)]^2$$

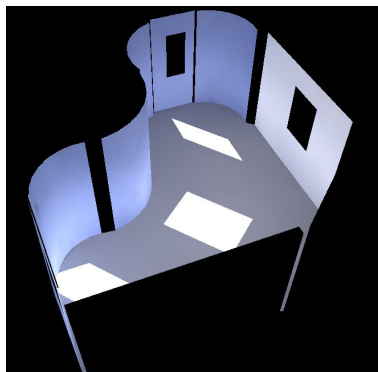


*gradient in  
desired appearance*

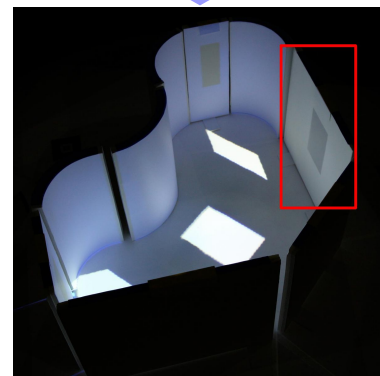
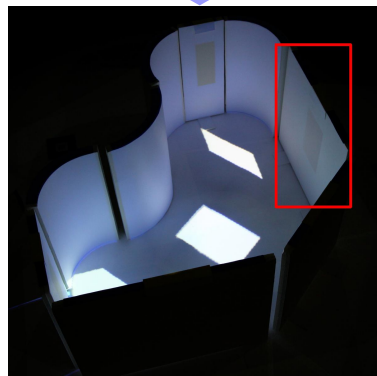
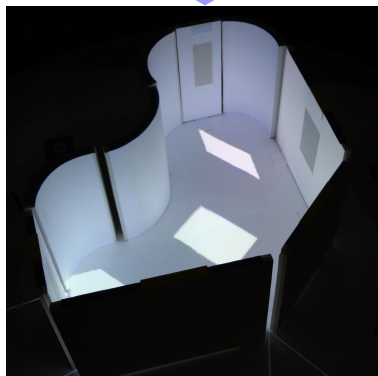
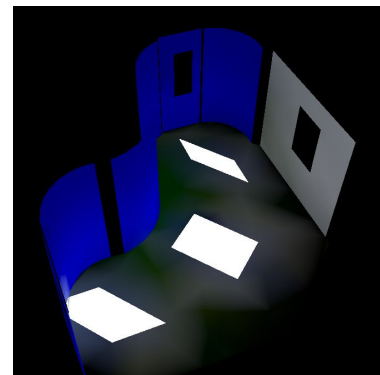
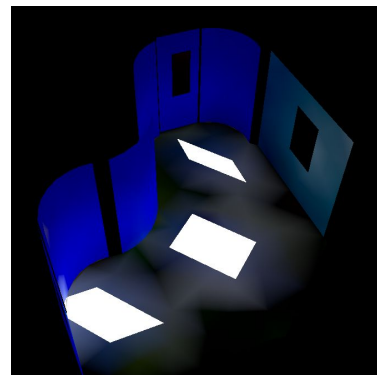
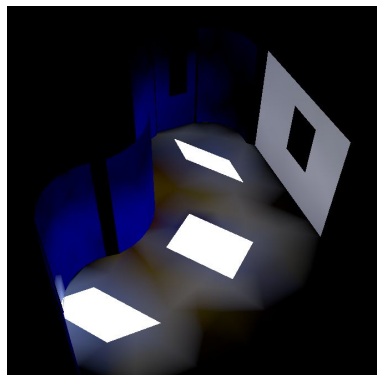
*gradient in  
projection result*

- Complete Objective Function:  $\phi = \alpha \phi_{abs} + (1 - \alpha) \phi_{spt}$  *We use  $\alpha = 0.9$*
- Box constraints:  
minimum & maximum brightness of projector system *No negative light!*

*Desired*



*Calculated projection imagery*



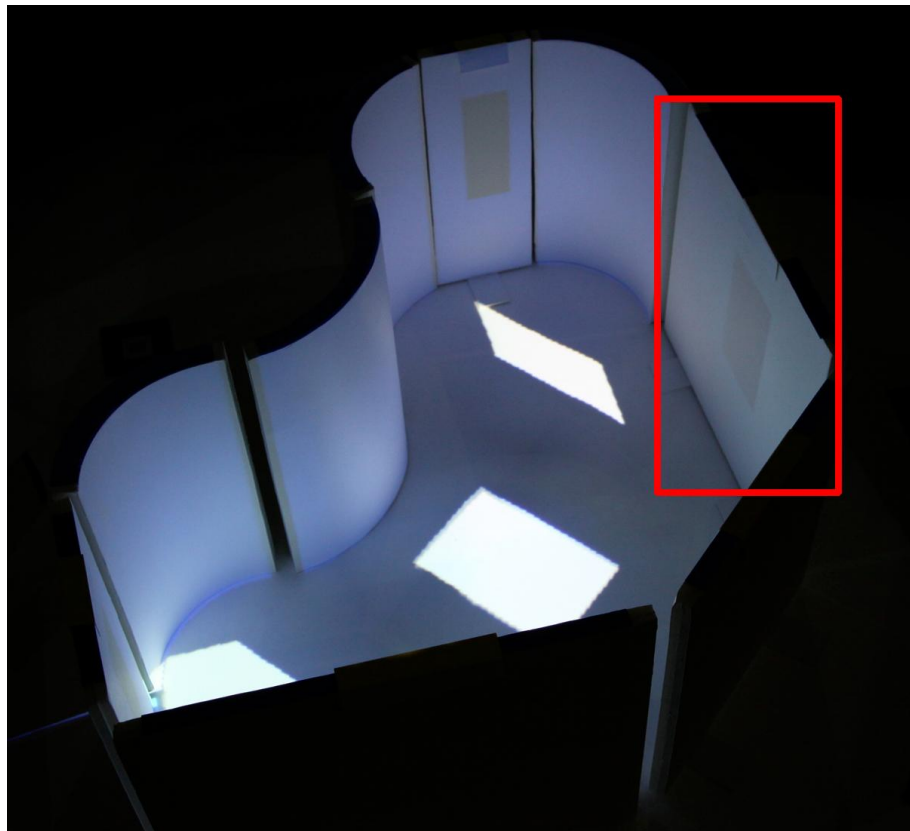
*Uncompensated*

*Reverse Radiosity*

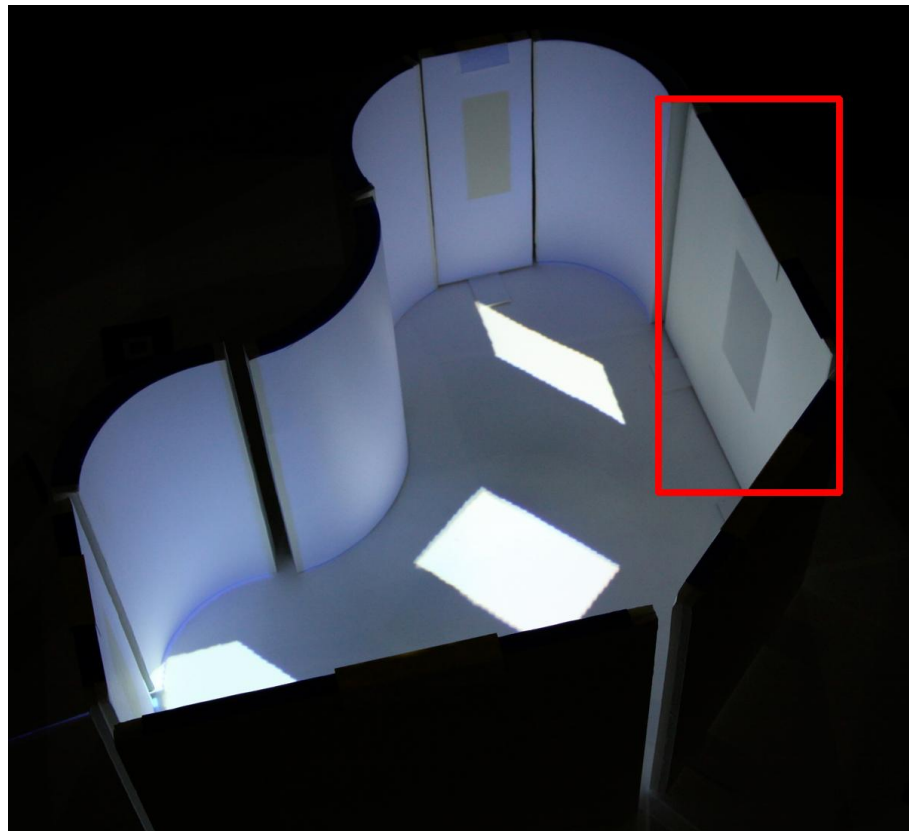
*YPbPr*

*L\*A\*B\**

*Photographs*

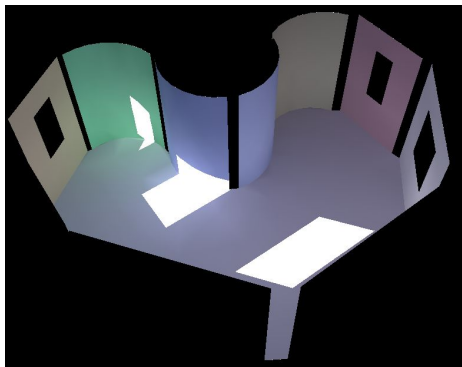


*Sheng et al. 2010*  
*Optimized in YPbPr space*

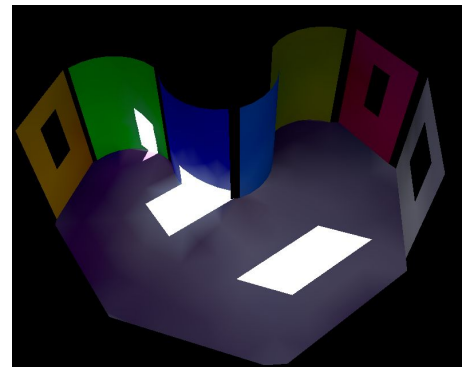
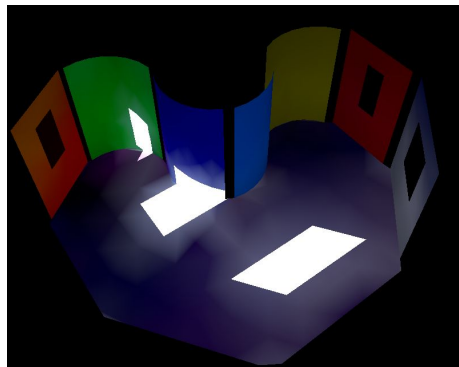
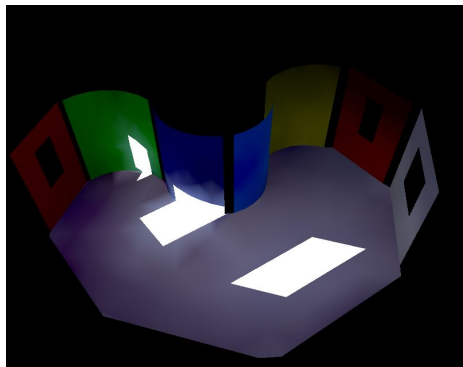


*Sheng et al. 2011*  
*Optimized in L\*A\*B space*

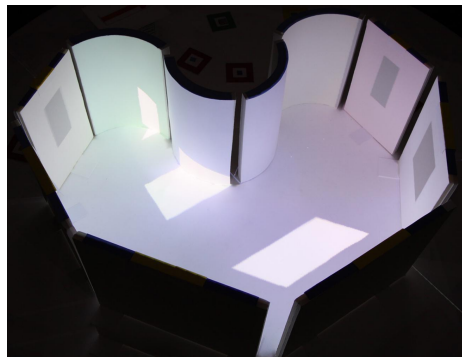
*Desired*



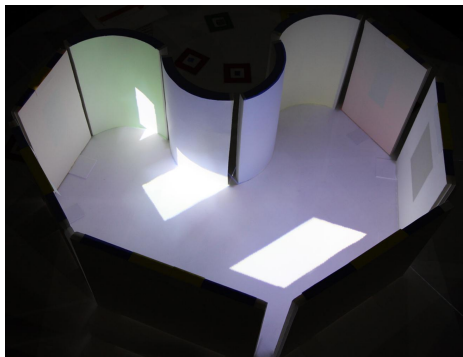
*Calculated projection imagery*



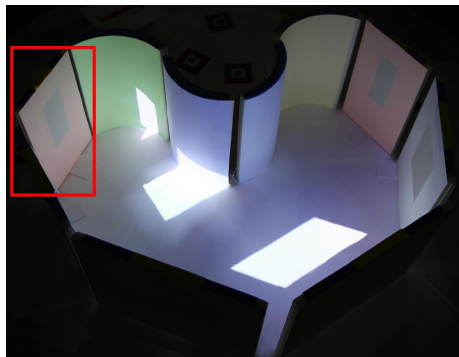
*Uncompensated*



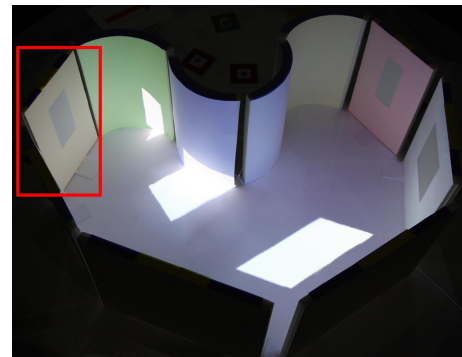
*Reverse Radiosity*



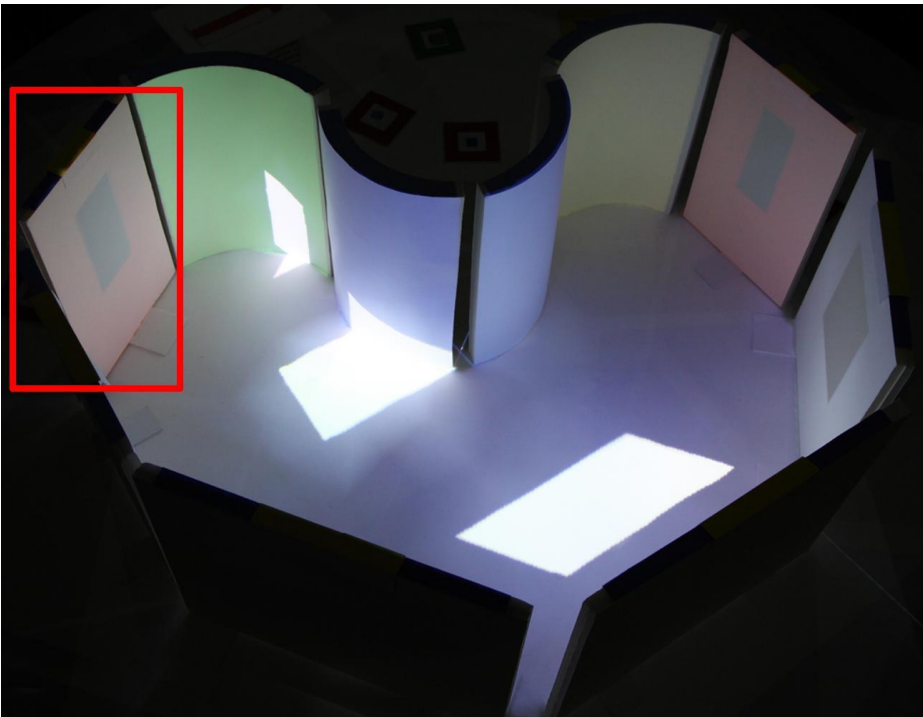
*YPbPr*



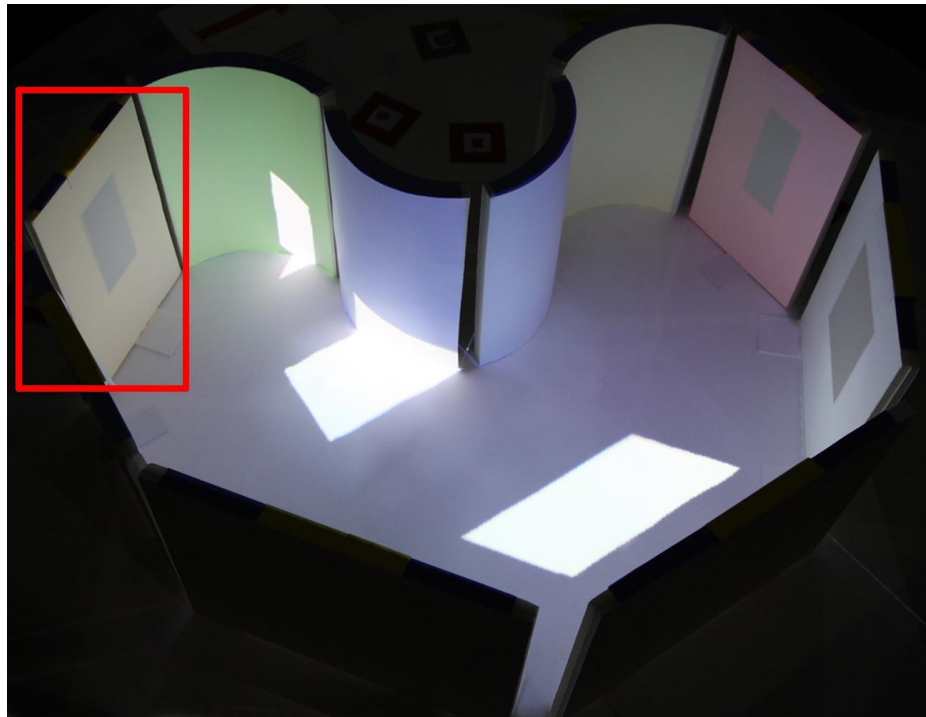
*L\*A\*B\**



*Photographs*



*Sheng et al. 2010*  
*Optimized in YPbPr space*



*Sheng et al. 2011*  
*Optimized in L\*A\*B space*

“Perceptual Global Illumination Cancellation in Complex Projection Environments”  
Yu Sheng, Barbara Cutler, Chao Chen, and Joshua Nasman  
Eurographics Symposium on Rendering (EGSR), June 2011.

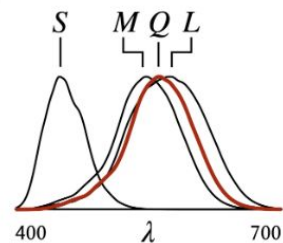
# Today

---

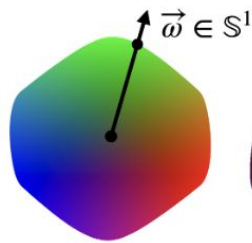
- Worksheet: Shadow Techniques
- Readings for Today
- What is Color?
  - Human Perception
  - Color Blindness & Metamerism
- Color Spaces
  - Linear Color Spaces: LMS, RGB, XYZ, HSV
- Projection in Spatially Augmented Reality
  - Human Perception & Perceptual Color Space:  $L^*a^*b^*$
- Readings for Next Time

# Readings for Next Time (*pick one*)

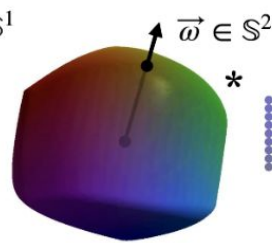
- "Theory of Human Tetrachromatic Color Experience and Printing"  
Lee, Jennings, Srivastava, & Ng,, ACM TOG 2024



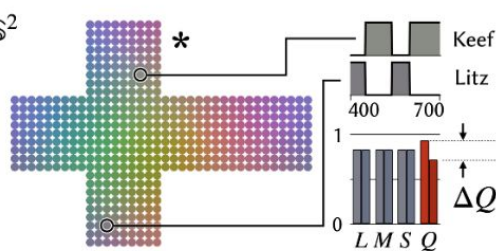
A: Human Tetrachromatic Spectral Response Functions



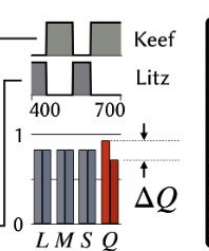
B: Regular Trichromatic Chromaticities + Hues



C: Tetrachromatic Chromaticities + Hues



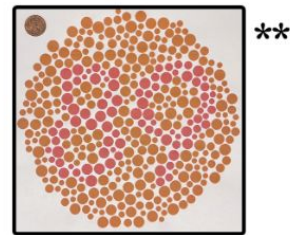
D: Tetrachromatic Hue Sphere, Flattened, and Keef/Litz Hues



E: Keef/Litz Reflectances and  $\Delta Q$



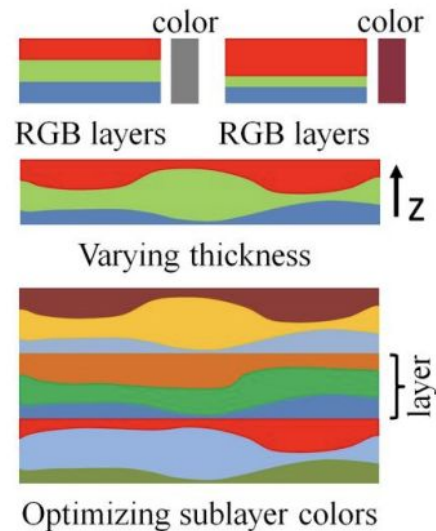
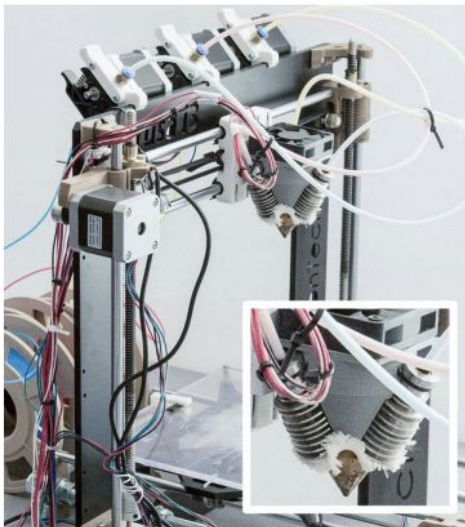
F: Prototyping Platform for Tetrachromatic Printers



G: Photo of Example Tetrachromatic Print; Color Test

# Readings for Next Time (*pick one*)

- "Colored Fused Filament Fabrication",  
Song, Martinez, Bedell, Vennin, & Lefebvre, ACM TOG 2019



# Readings for Next Time (*pick one*)

---

- "Flash Photography Enhancement via Intrinsic Relighting",  
Eisemann & Durand, SIGGRAPH 2004



**no flash**  
*warm ambiance,  
noisy*



**flash**  
*flat lighting*



**combined result**  
*original lighting,  
denoised*