CSCI 4560/6560 Computational Geometry

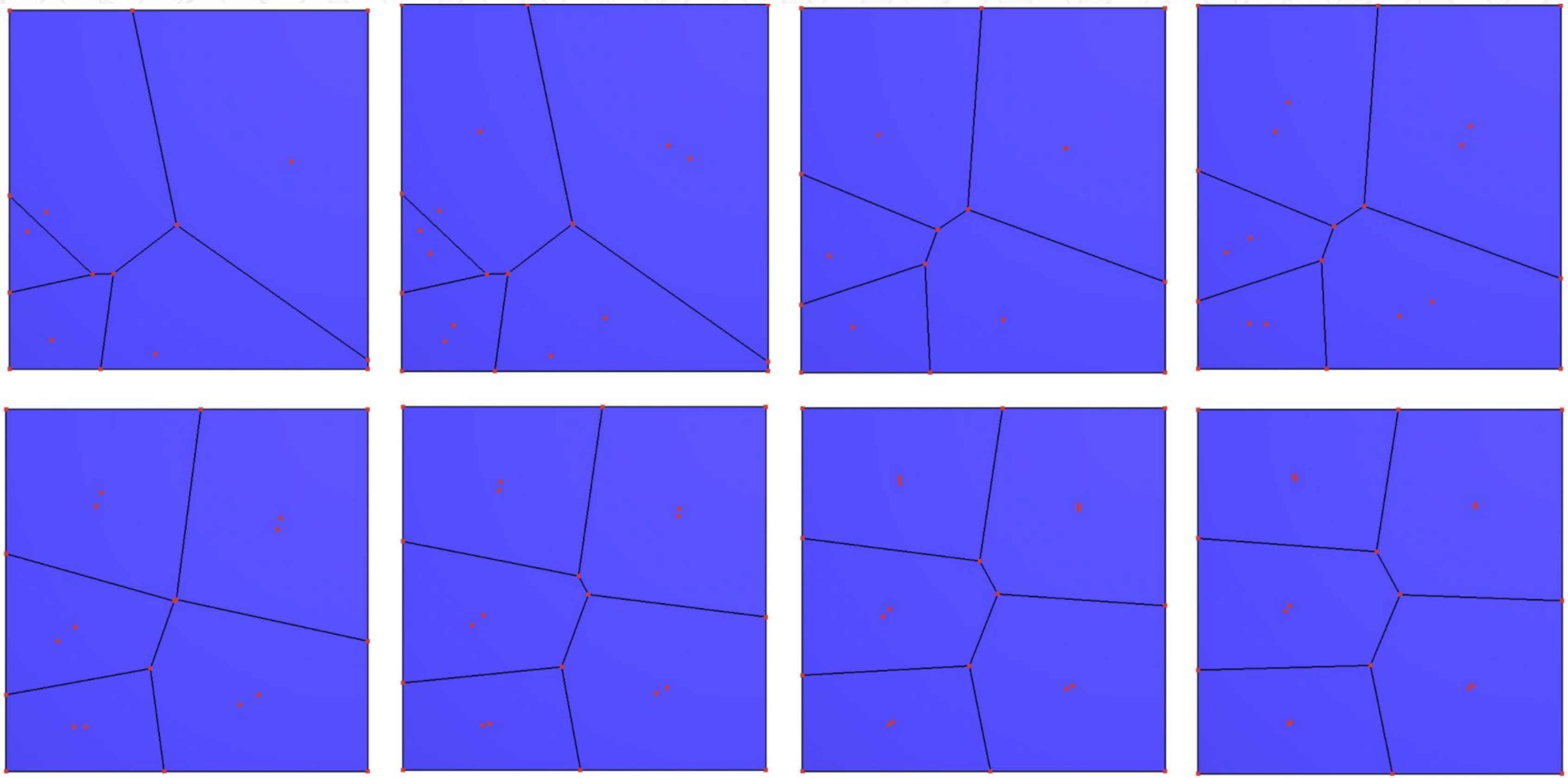
https://www.cs.rpi.edu/~cutler/classes/computationalgeometry/S22/

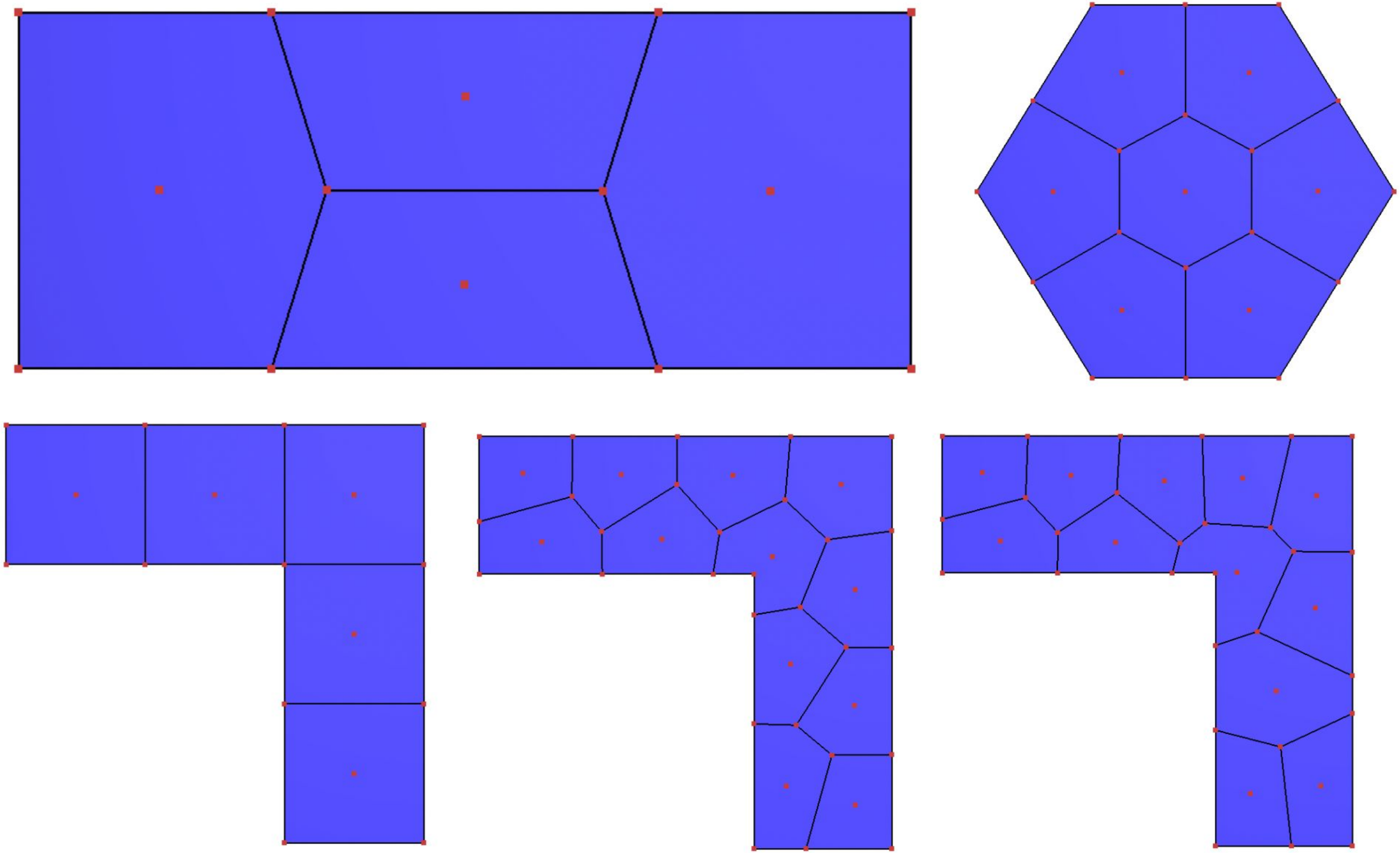
Lecture 18:
Isocontours & Level Sets

Outline for Today

- Homework 5 Questions?
- Last Time: Quad Trees
- Explicit vs. Implicit Surface Representations
- Signed Distance Field
- Level Sets (Surface → Signed Distance)
- Fast Marching Method
- Medical Imaging
- Marching Cubes (Signed Distance → Surface)
- Marching Tetrahedra
- Next Time: ?

Homework 5 Questions?





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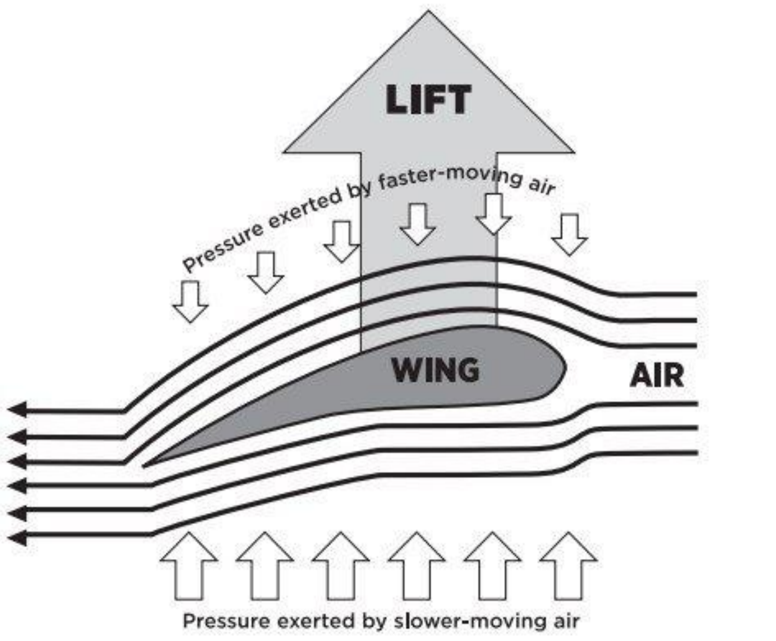
Motivation: Finite Element Modeling (FEM) & Computational Fluid Dynamics



https://www.scienceworld.ca/resource/plane-wing-simulator/

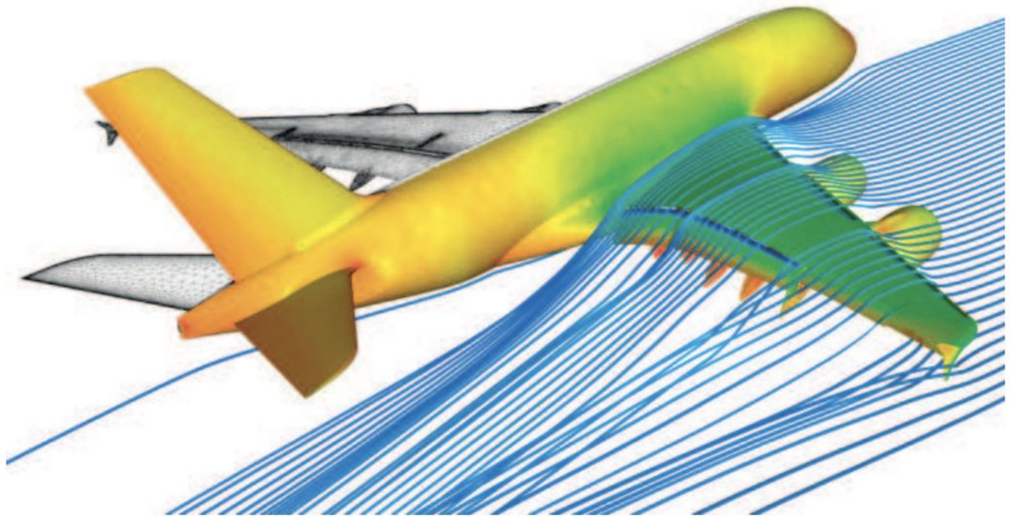
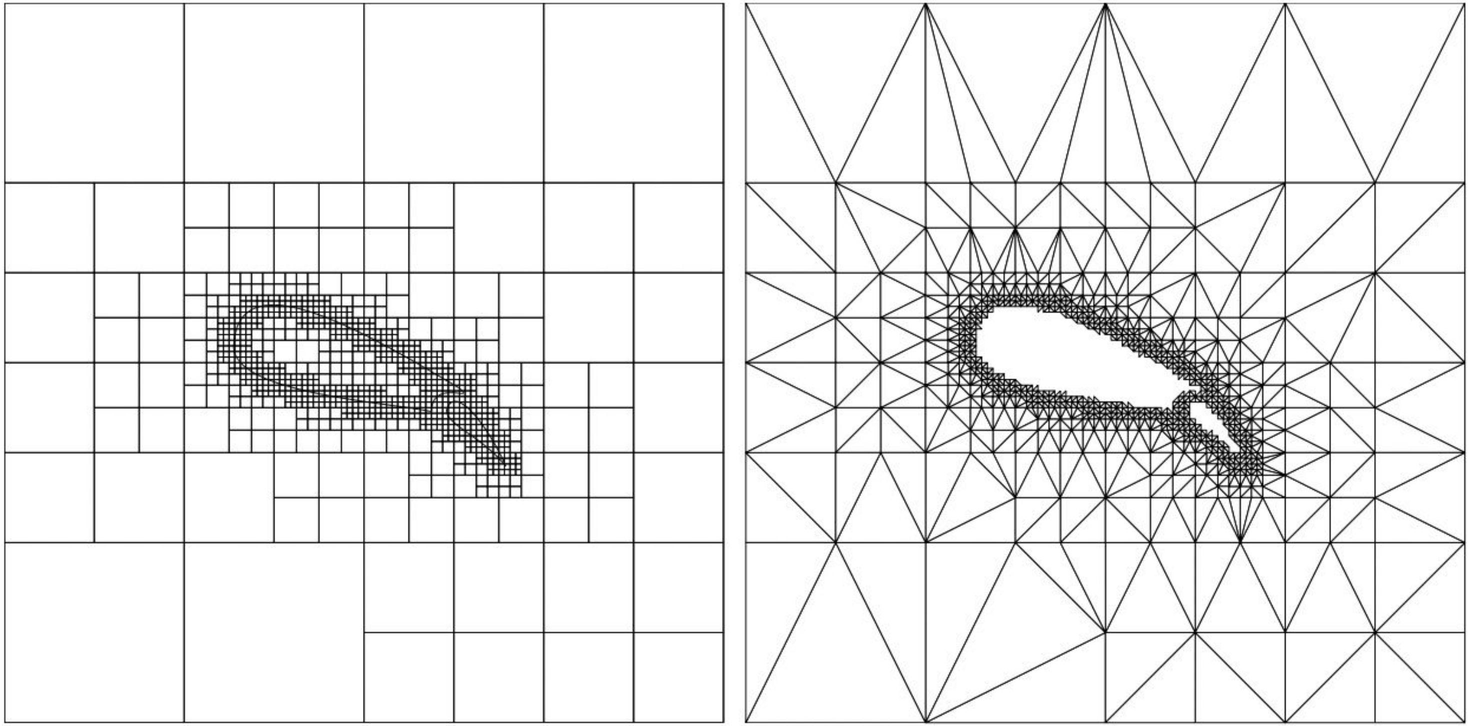


Figure 9: Numerical flow simulation for the Airbus A380 (picture credit: Airbus. Copyright: Dr. Klaus Becker, Senior Manager Aerodynamic Strategies, EGAA, Airbus, Bremen, Germany)

Motivation: Finite Element Modeling (FEM) & Computational Fluid Dynamics (CFD)



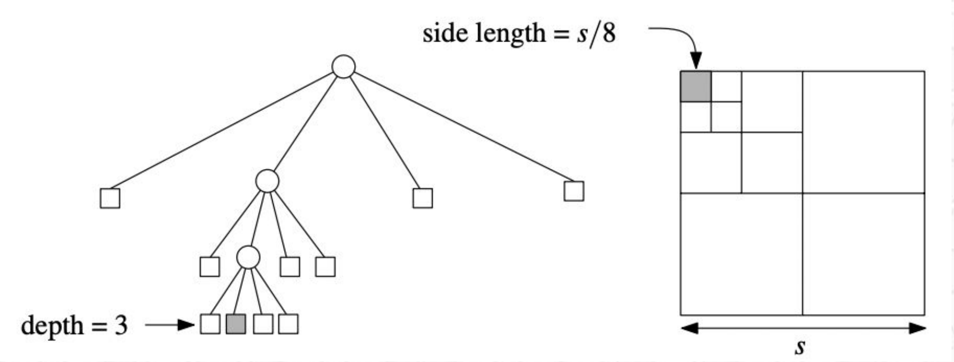
“Delaunay Refinement for Curved Complexes”, Adriano Chaves Lisboa, 2008.

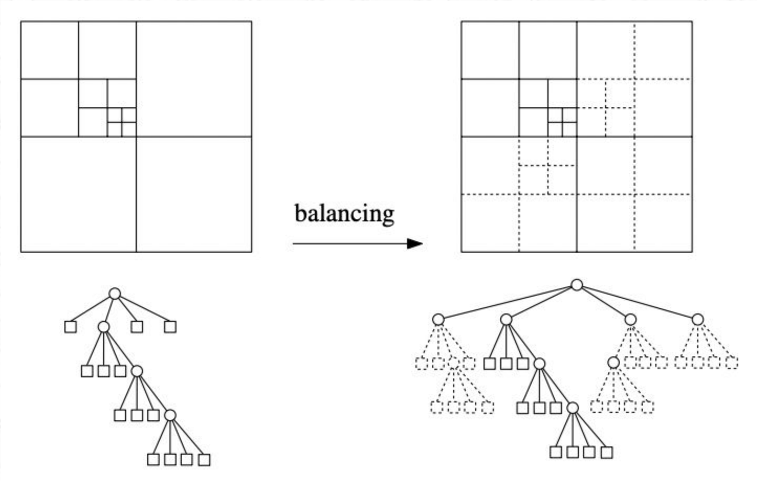
Quad Tree Analysis

- n = # of points
- c = smallest distance between any two points
- s = side length of initial square

- d = depth = $\log(s/c) + 3/2$
- m = # of nodes in unbalanced tree $= O((d+1)n)$
- *time to construct* = $O((d+1)n)$

- *# of nodes in balanced tree* = $O(m)$
- *Time to balance a tree* = $O((d+1)m)$



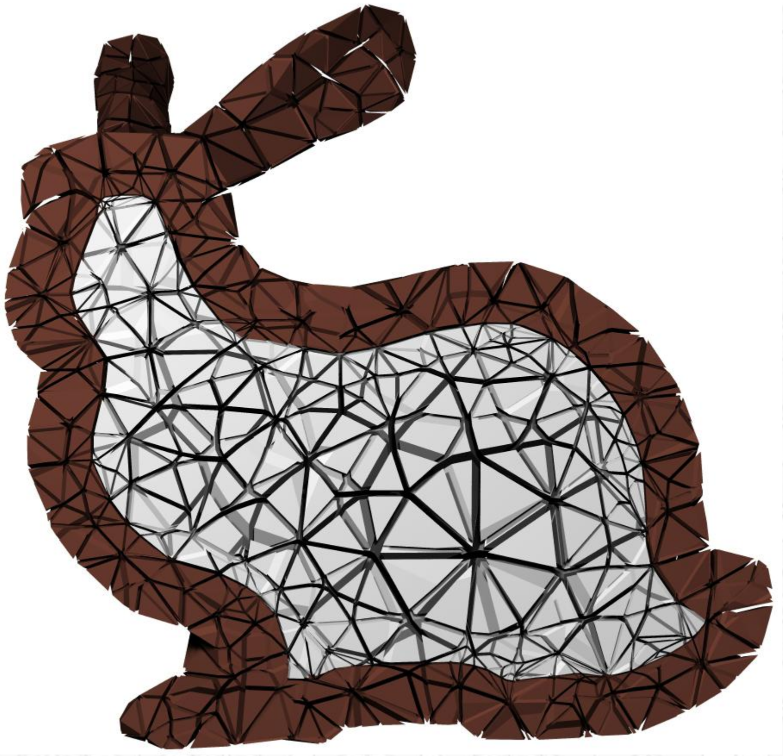


Computational Geometry Algorithms and Applications,
de Berg, Cheong, van Kreveld and Overmars, Chapter 14

3D Mesh Simplification

"Simplification and Improvement of Tetrahedral Models for Simulation"
Cutler, Dorsey, and McMillan
SGP 2004

1,050K tetras
(133K faces)



10K tetras
(3K faces)

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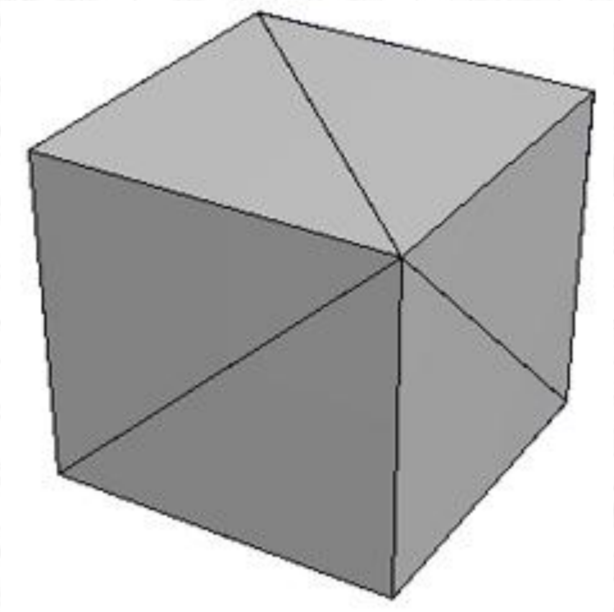
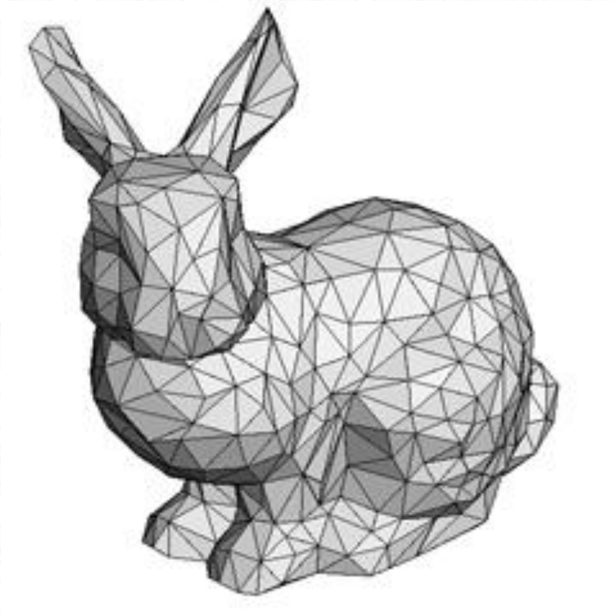
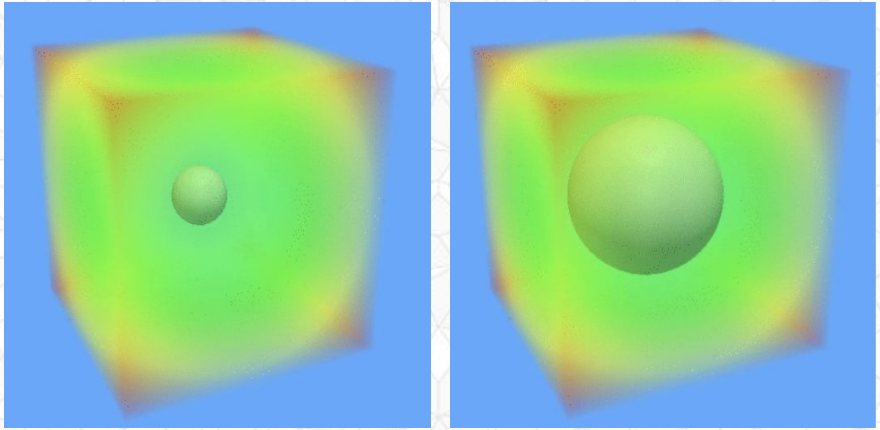
Explicit Surface Mesh Representation

- Often we focus on modeling surfaces with polygon or triangle meshes separating “inside” from “outside”





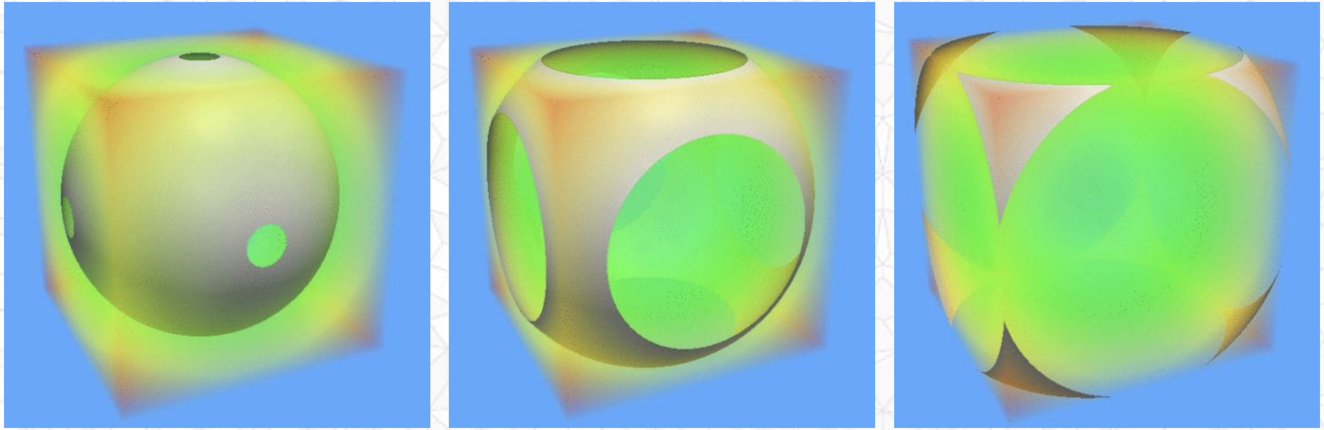
Implicit Surfaces



- Alternately, some objects are easily represented by an equation:
- E.g., a sphere:

  $H(x,y,z) = x^2 + y^2 + z^2 - r^2$

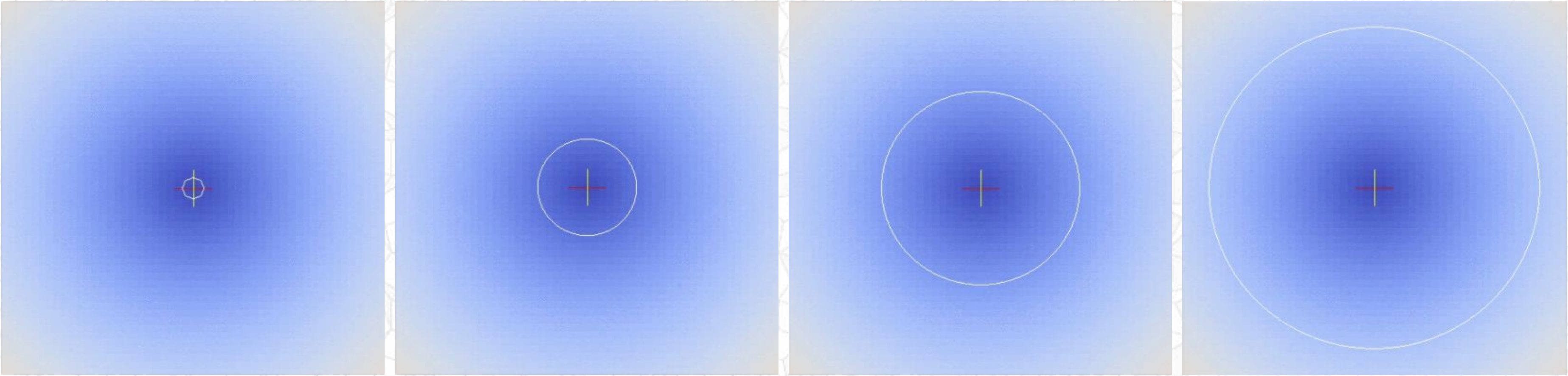
- If $H(x,y,z) = 0$, on surface
- If $H(x,y,z) > 0$, outside surface
- If $H(x,y,z) < 0$, inside surface



Isocontours / Isosurfaces

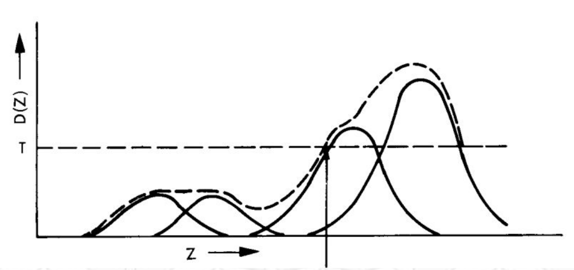
- "iso-" (from Greek word meaning 'equal')
- Everywhere that the data equals a specified value
- E.g., different radii for a circle or sphere centered at the origin

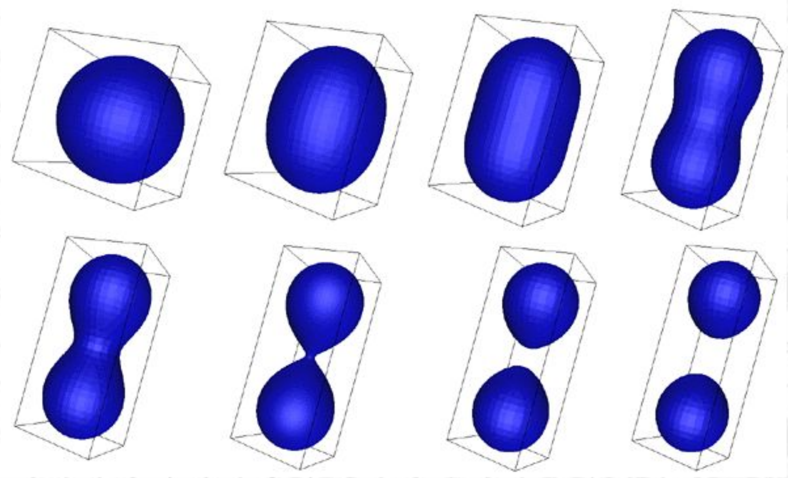
$$H(x,y,z) = x^2 + y^2 + z^2 - r^2$$



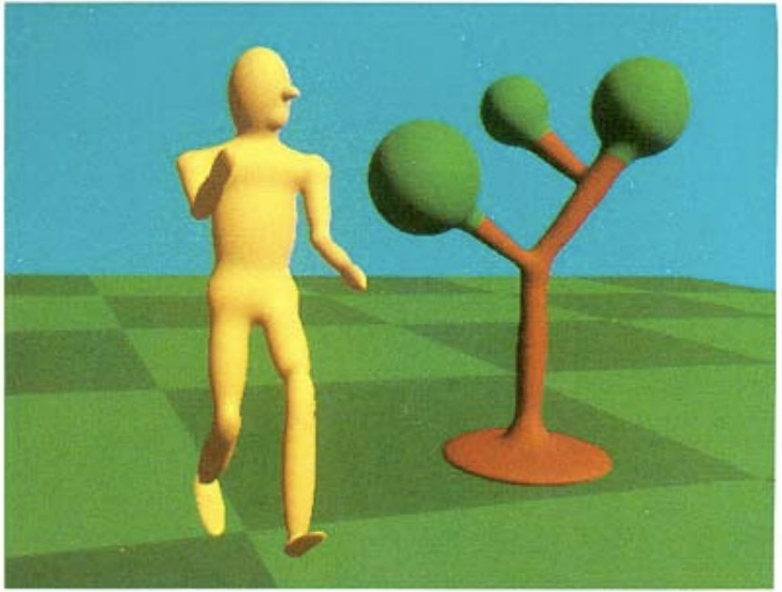
Implicit Surfaces: Blobby Surfaces / Metaballs

- Compact representation to model soft, round objects





http://paulbourke.net/geometry/implicitsurf/index.html

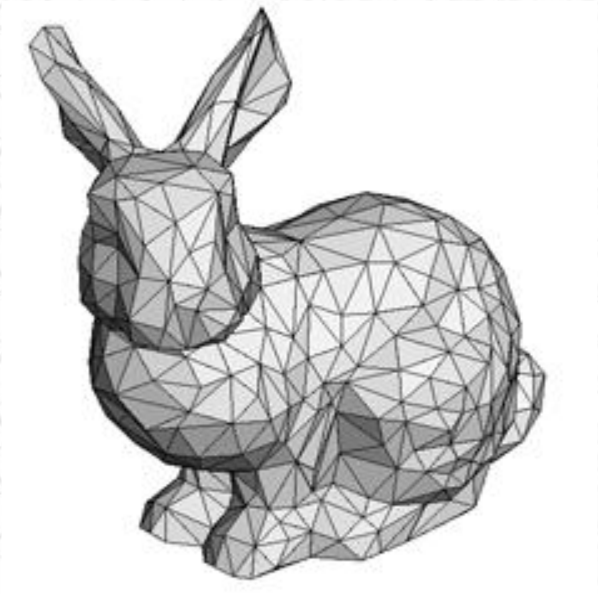


"A Generalization of Algebraic Surface Drawing", Blinn, 1982.

Explicit vs. Implicit Surface Representations

- Some objects can accurately represented either implicitly or explicitly
- Can we convert the bunny mesh into an implicit equation?

  Why might we want to do this?

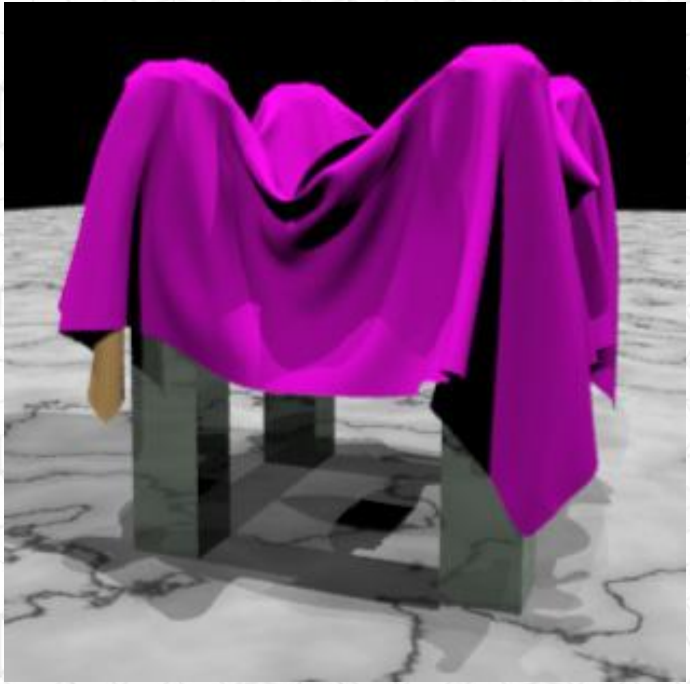


$\longrightarrow$ $H(x,y,z) = ?$

“Robust Treatment of Collisions, Contact and Friction for Cloth Animation”
Bridson, Fedkiw, & Anderson, SIGGRAPH 2002
“Simulation of Clothing with Folds and Wrinkles”,
Bridson, Marino, & Fedkiw, SCA 2003

Motivation: Collision Detection

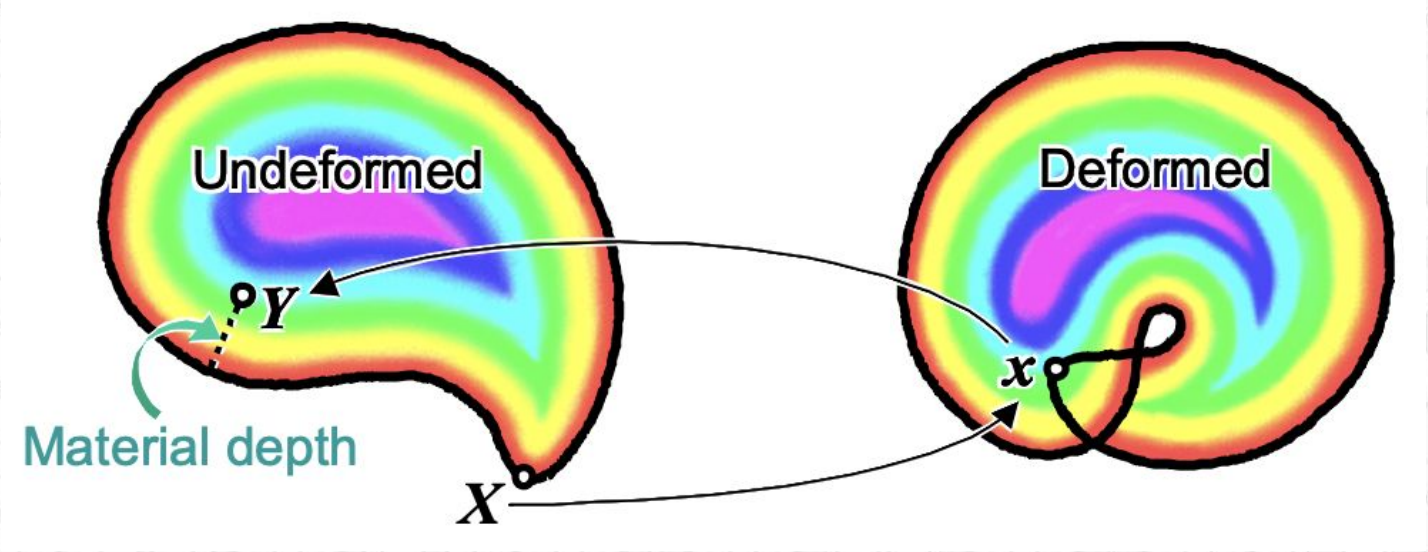
- Detecting Intersections between rigid (or deformable!) objects



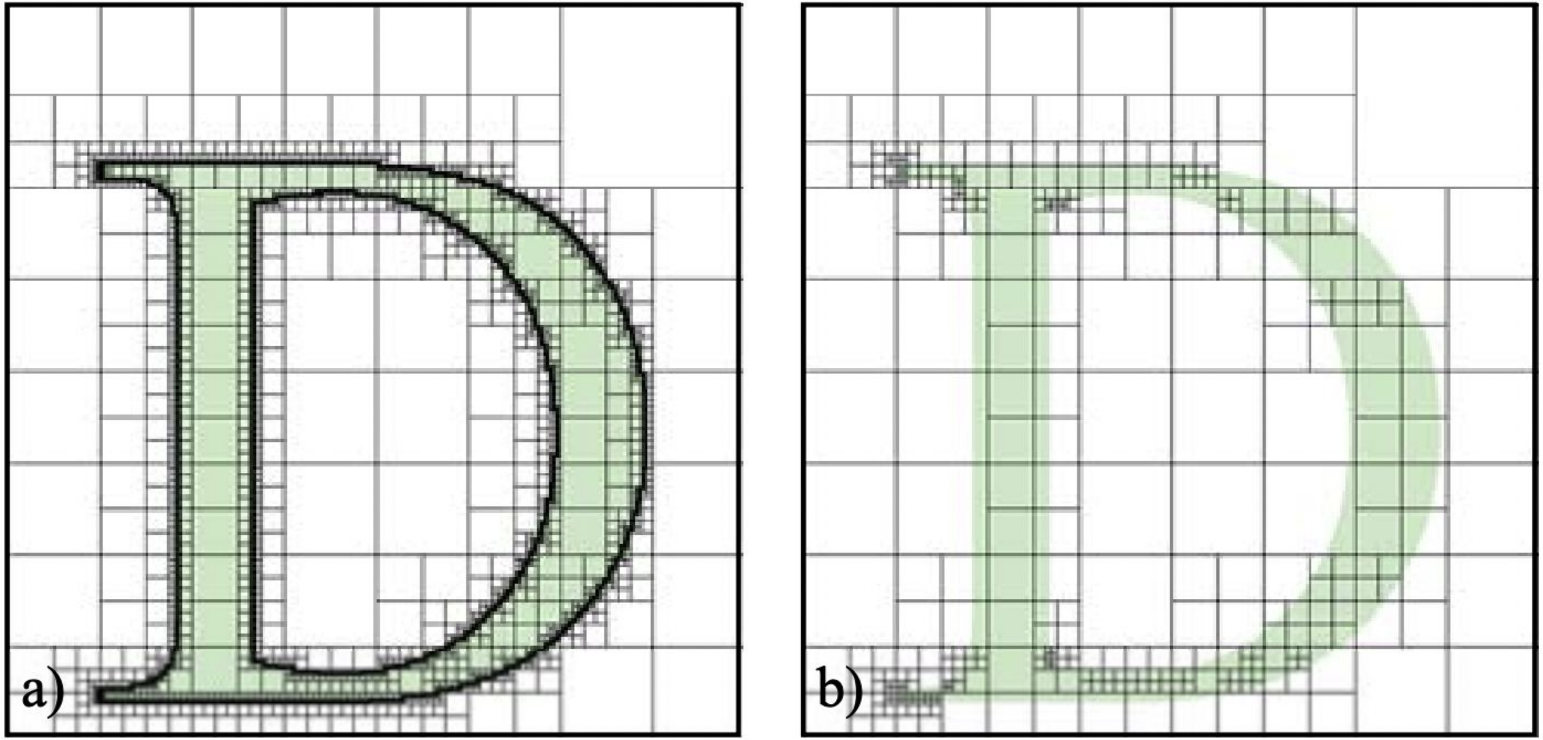
Motivation: Collision Detection

- Detect the intersection
- Depth of intersection penetration
- Gradient & normal of closest surface – Determine penalty force to resolve collision

“An Implicit Finite Element Method for Elastic Solids in Contact”, Hirota, Fisher, State, Lee, & Fuchs, SCA 2001

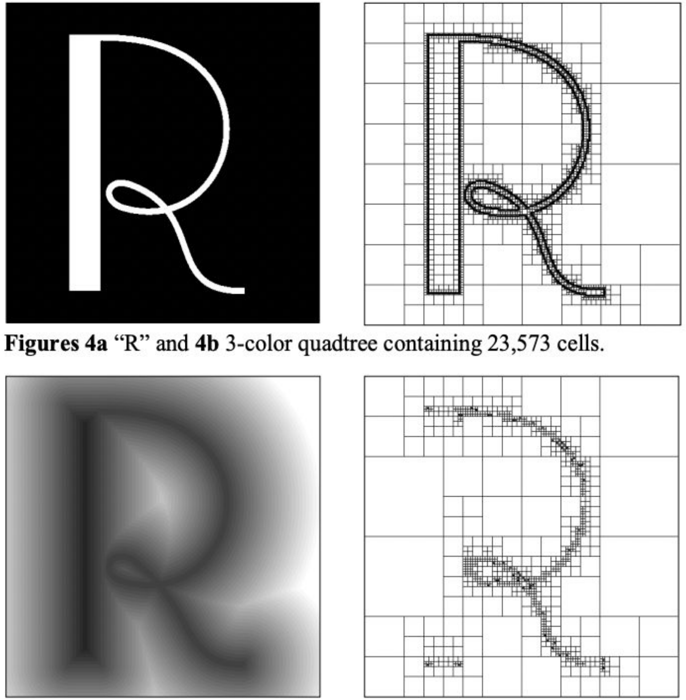


Motivation: Alternate Surface Representation



“Adaptively Sampled Distance Fields: A General Representation of Shape for Computer Graphics”, Frisken, Perry, Rockwood, and Jones, SIGGRAPH 2001
“Designing with Distance Fields”, Frisken and Perry, 2006

Motivation: Surface Sculpting



Figures 4a "R" and **4b** 3-color quadtree containing 23,573 cells.

Figures 4c Distance field of "R" and **4d** ADF containing 1713 cells.

"Adaptively Sampled Distance Fields: A General Representation of Shape for Computer Graphics",
Frisken, Perry, Rockwood, and Jones, SIGGRAPH 2001
"Designing with Distance Fields", Frisken and Perry, 2006

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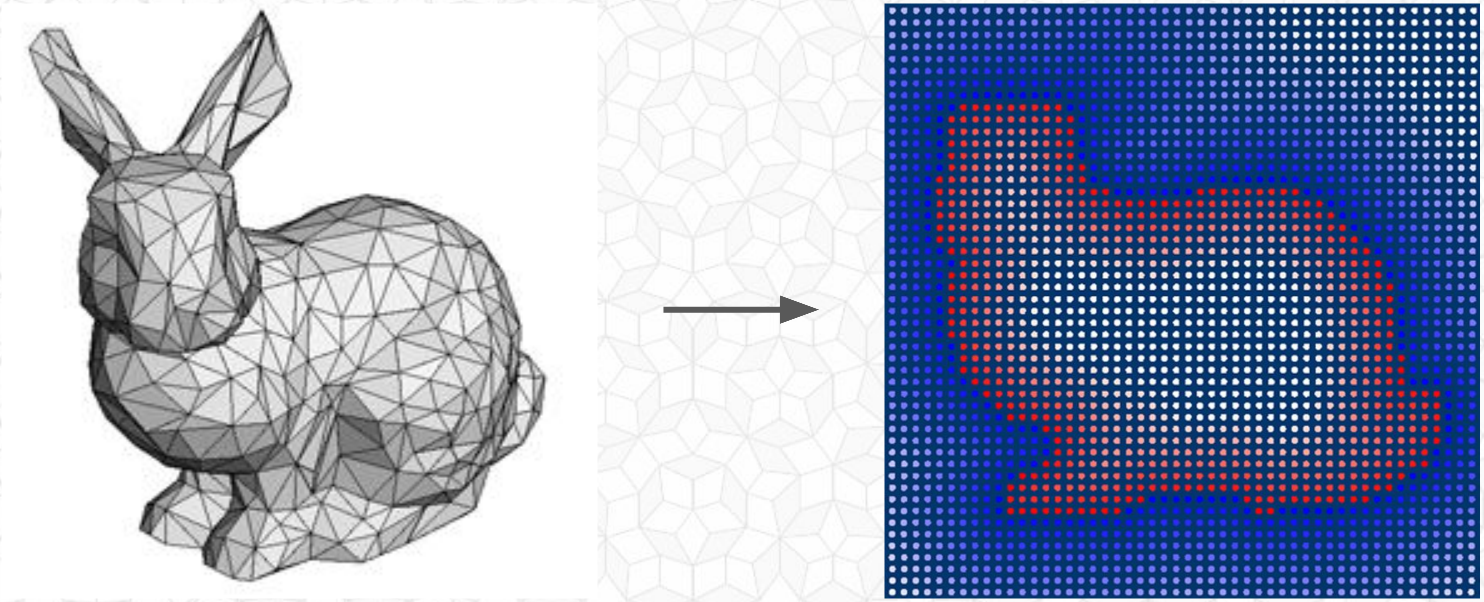
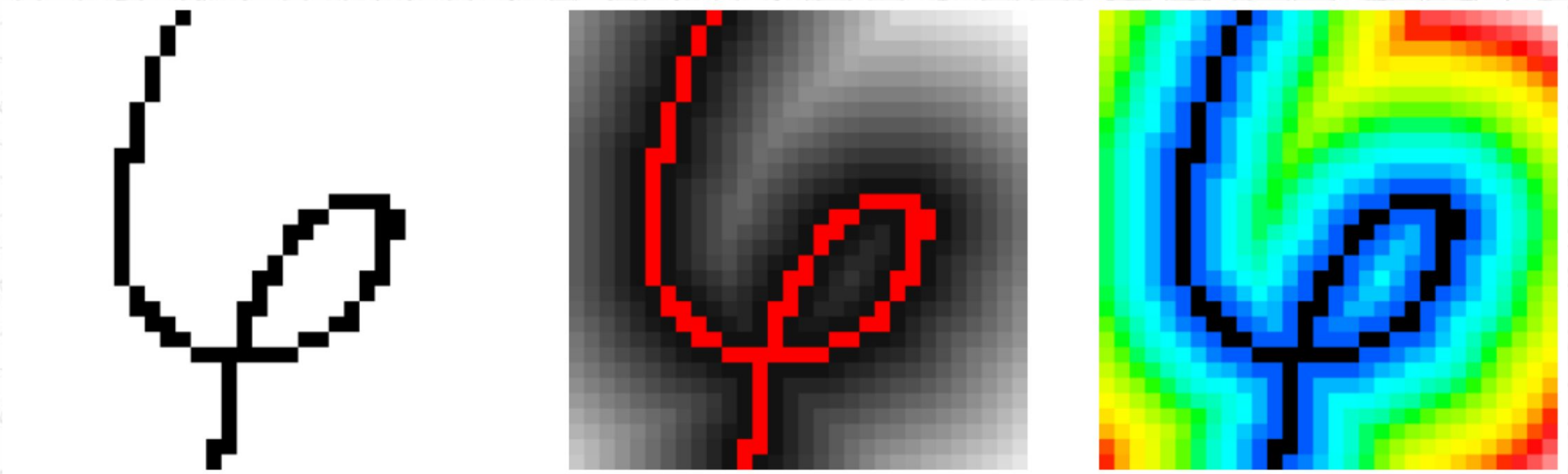
Explicit vs. Implicit Surface Representations

- We may not be able to construct a compact mathematical function…
- But can we convert the bunny mesh into a signed distance field?



Computing a Signed Distance Field

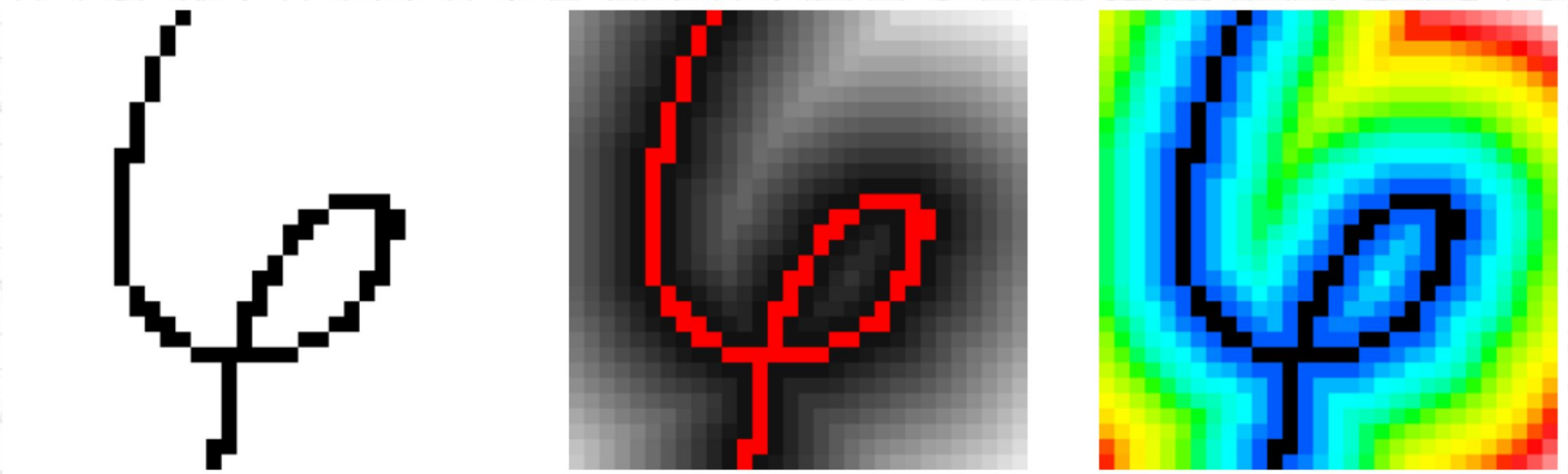
- Given a shape/surface
- Cost to compute shortest distance to original shape for each point (on a grid) in the volume?



Computing a Signed Distance Field

- Given a shape/surface
- Cost to compute shortest distance to original shape for each point (on a grid) in the volume?

Naive: O(# of volume grid samples * # of surface elements) = O(w²h²)



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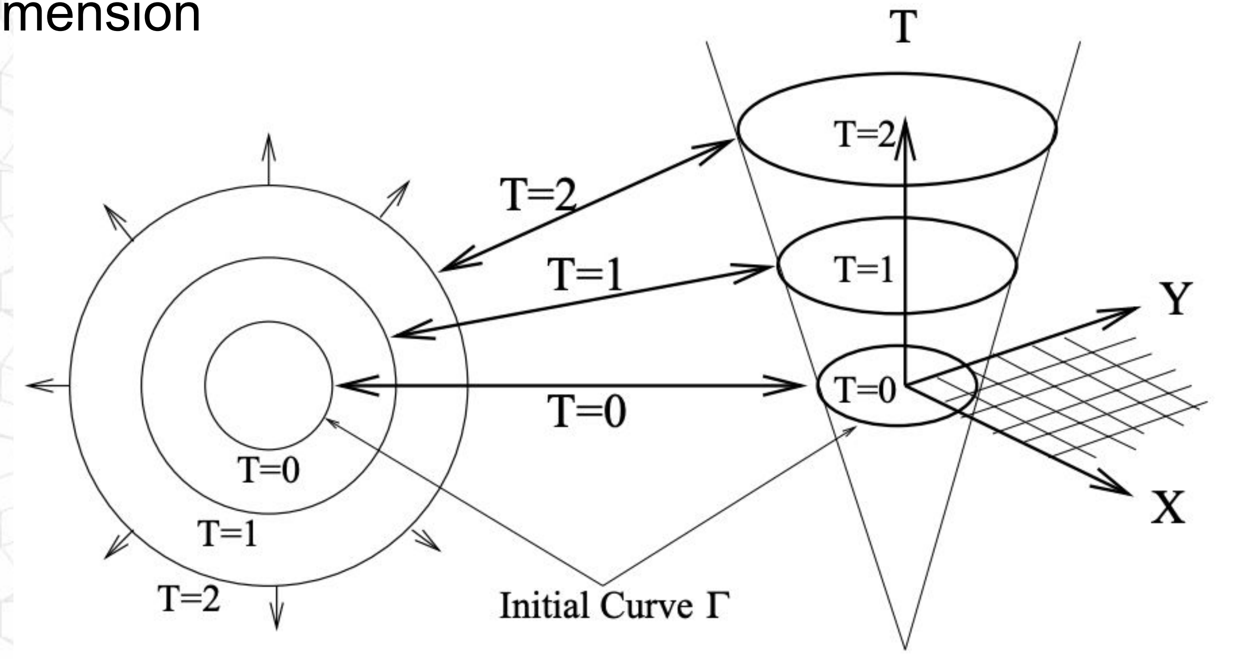
Level Sets

- For a 2D problem… we can visualize level sets with time (T) as the 3rd dimension

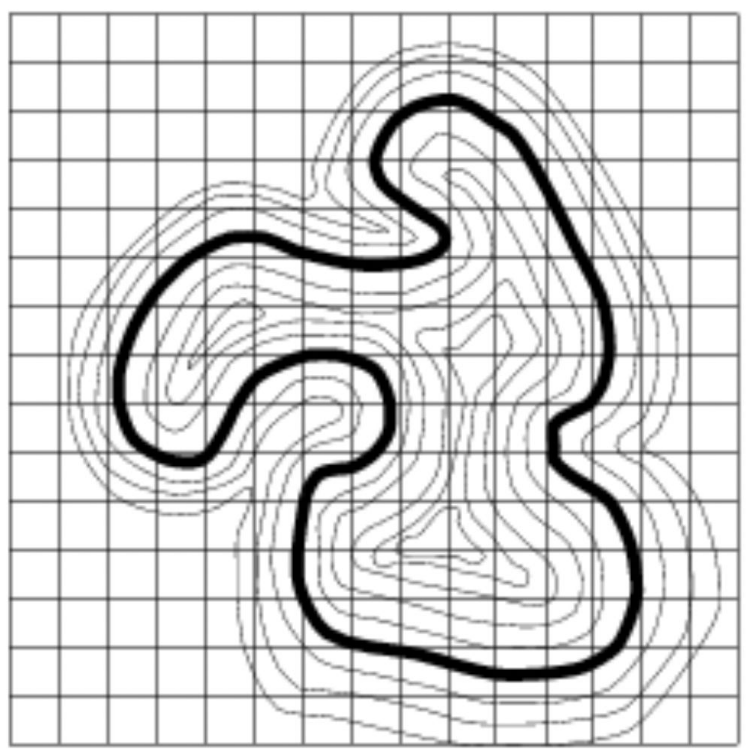


Level Set Methods and Fast Marching Methods, Sethian, 1999

Level Sets - Topology / Connectivity Changes!

- Depending on the application, we may want to grow/advance the surface in the outward direction
- Or we may want to shrink the surface in the inward direction

- Sharp corners will round
- Smooth areas may pinch at sharp point

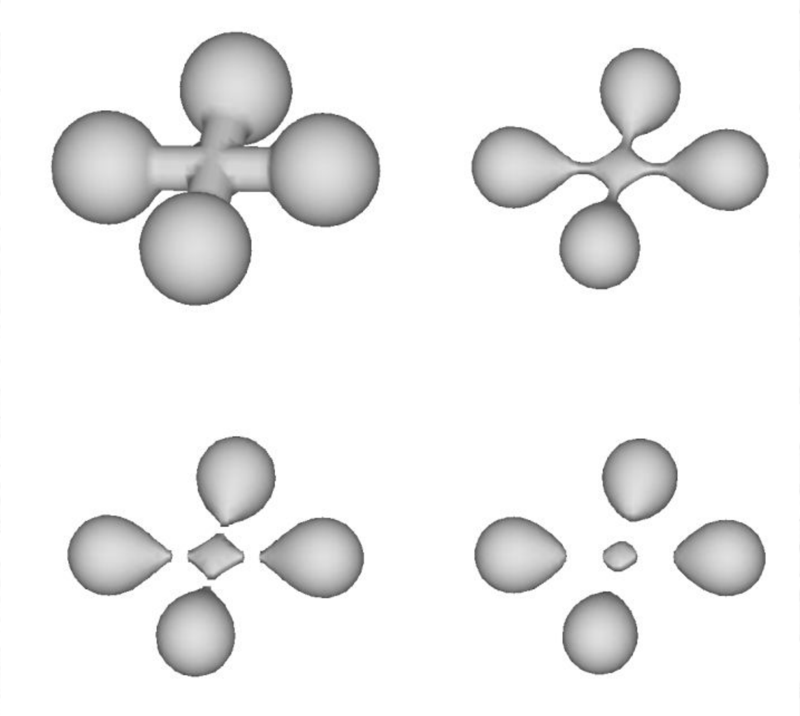


Level Set Methods and Fast Marching Methods, Sethian, 1999

Level Sets - Topology / Connectivity Changes!

- As we trace the level sets the topology of the surface may change!

- The surface may become disconnected
- Disconnected pieces may merge

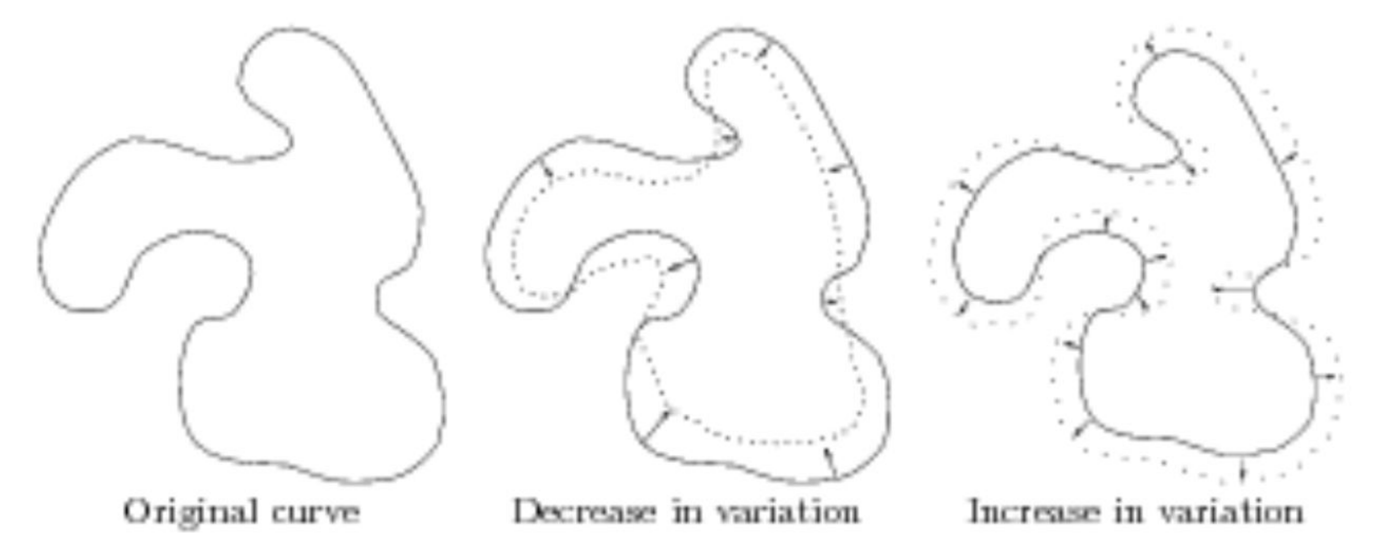


Level Set Methods and Fast Marching Methods, Sethian, 1999

Level Sets - Speed & Direction of Propagation

Depending on the application

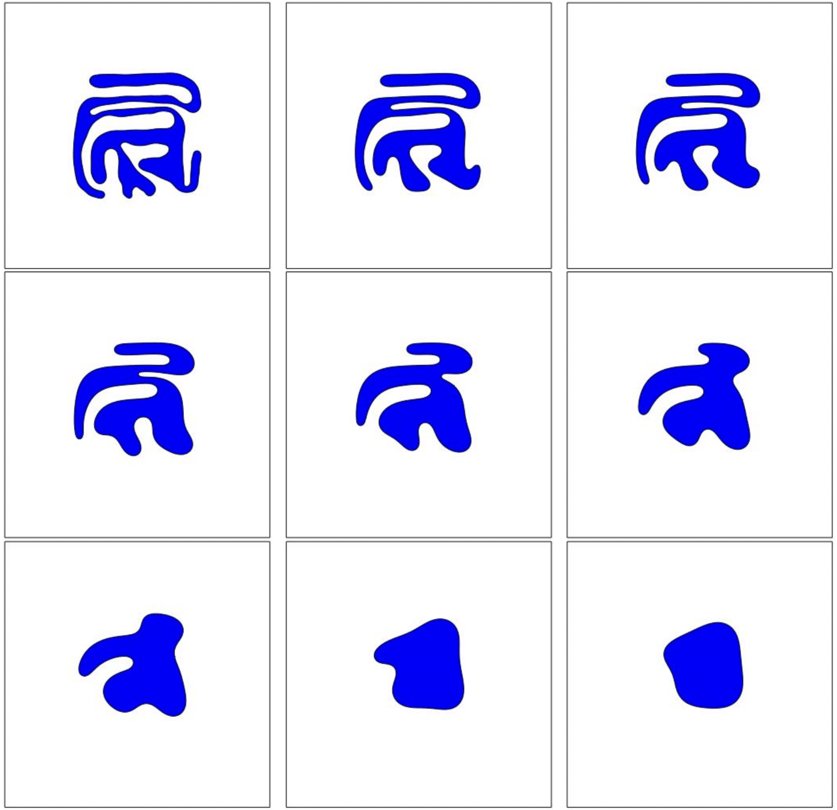
- Speed may not be uniform or constant
- Direction of propagation may be inward and/or outward in different places along the curve/surface!
- And may change over time.



Level Set Methods and Fast Marching Methods, Sethian, 1999

Level Sets - Topology / Connectivity Changes!

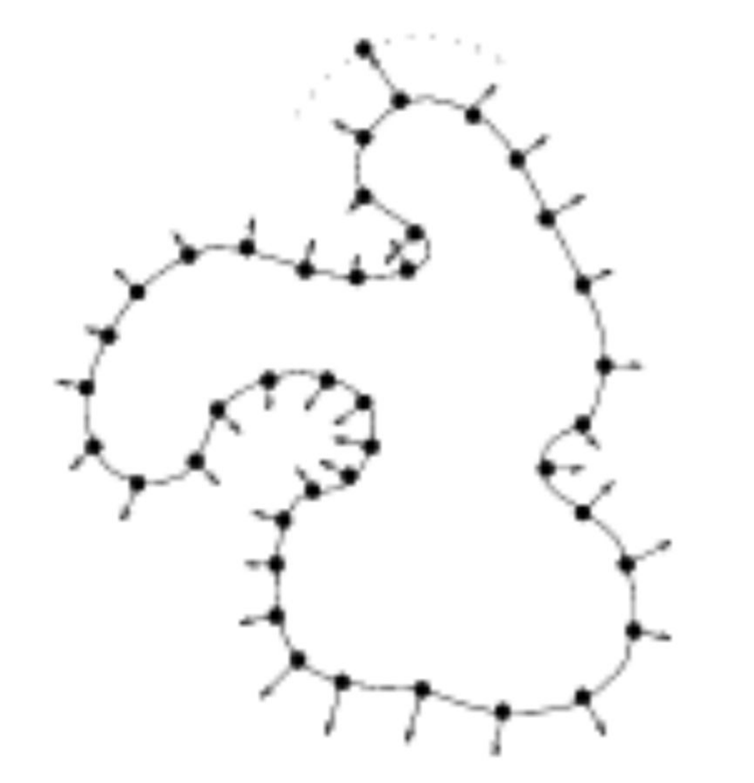
- Locally grow/expand where the curvature is concave
- Locally shrink where the curvature is convex
- All complex curves will collapse to a point!



Level Set Methods and Fast Marching Methods, Sethian, 1999

Computing Level Sets / Signed Distance Field

- Marker & string method:
  Copy the mesh &
  move the vertices…



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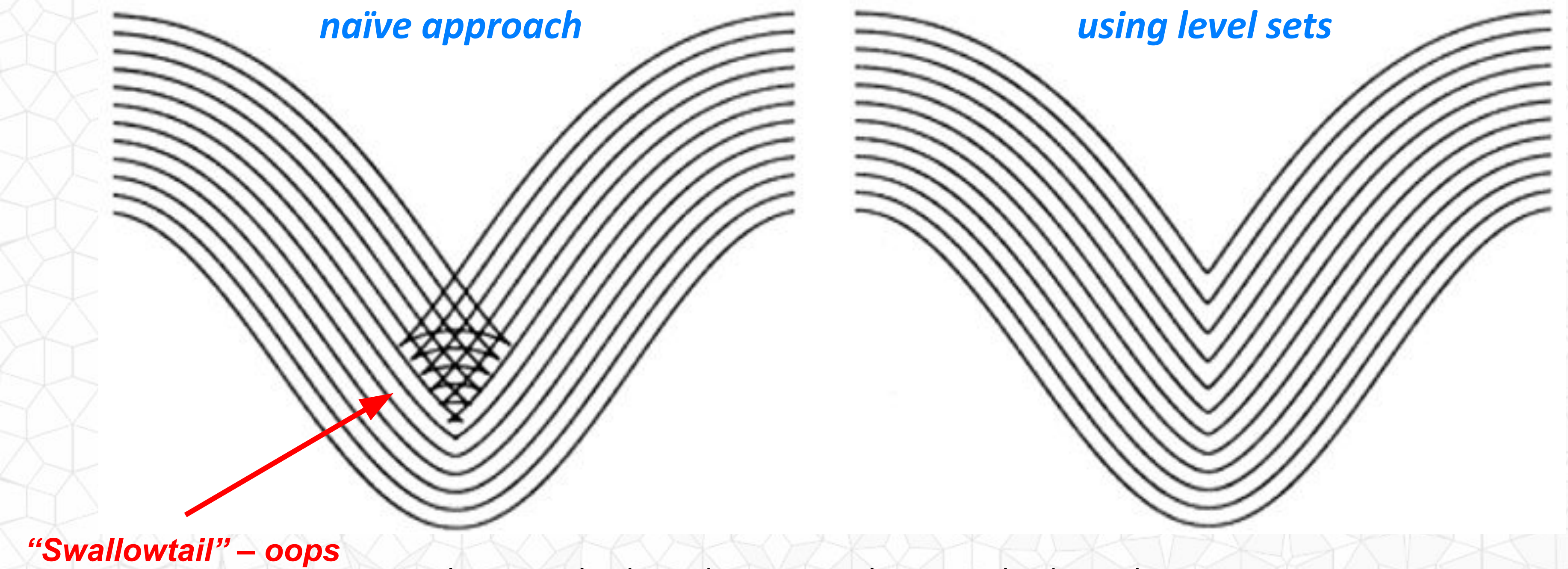

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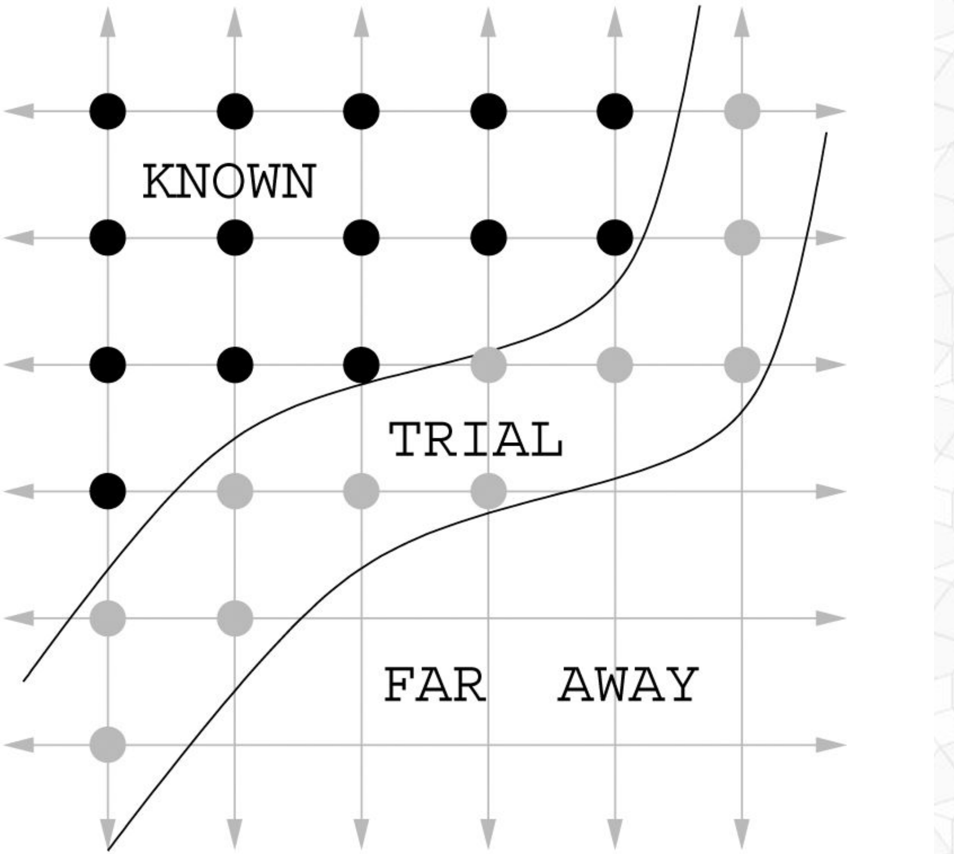
Fast Marching Method

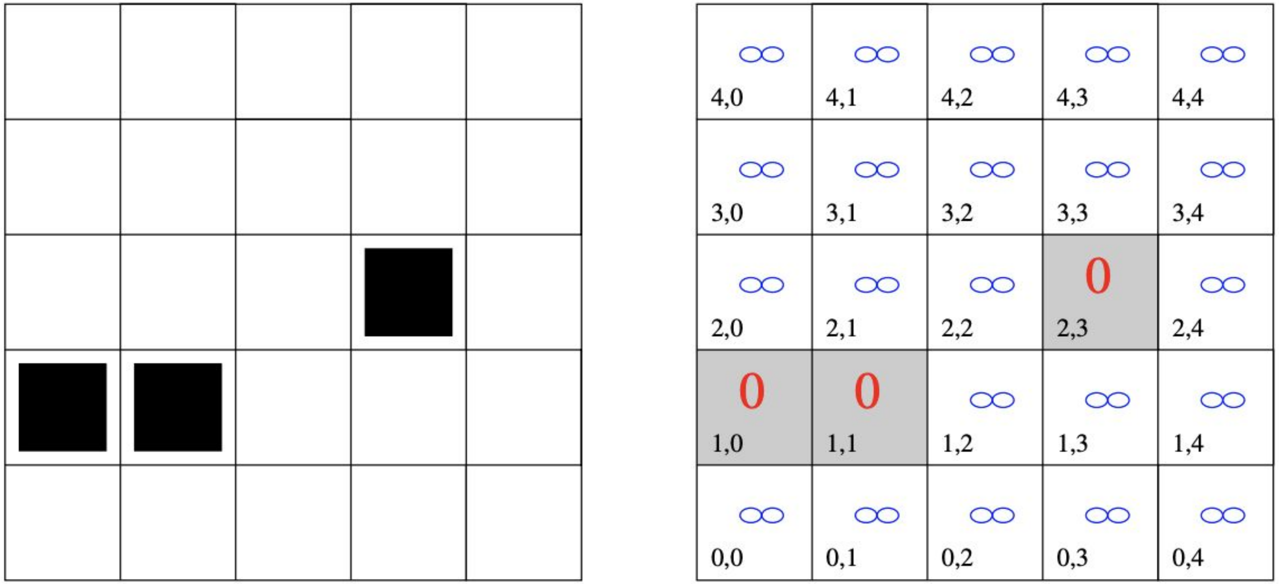
- Efficient method for computing the signed distance field.
- For applications where the front does not *change direction* – it moves outward only (alternately, inward only)

Level Set Methods and Fast Marching Methods, Sethian, 1999



Fast Marching Method Implementation (DS HW!)

Initially, only the surface pixels are "known" to have level set value, a.k.a. distance = 0



input image

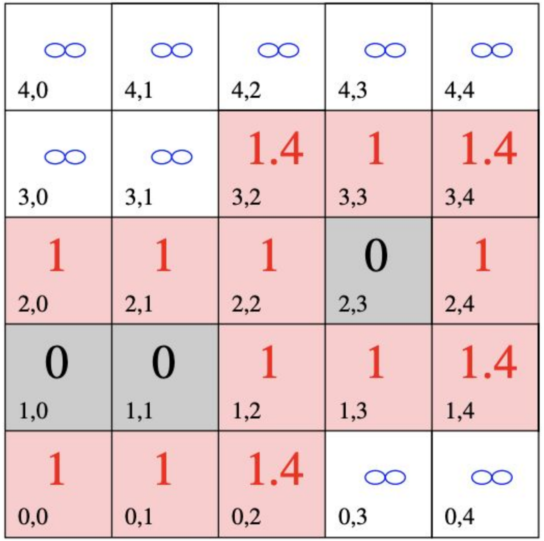
initialization of the signed distance field

https://www.cs.rpi.edu/academics/courses/fall18/csci1200/hw/10_level_sets/hw.pdf

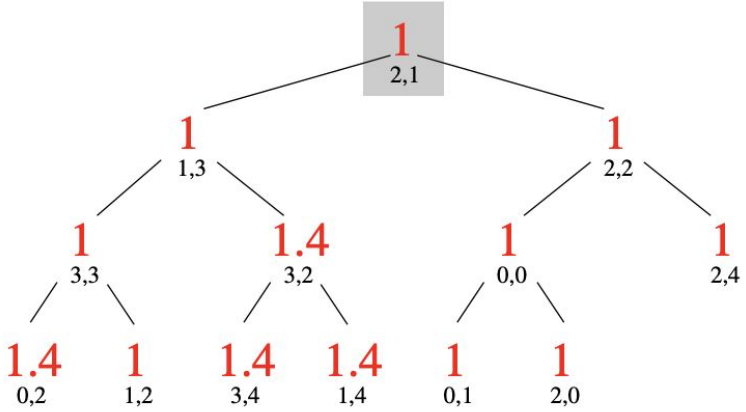
Fast Marching Method Implementation (DS HW!)

We compute the distance of all neighbors of these “known” pixels

Put all these new pixels in a priority queue, ordered by distance



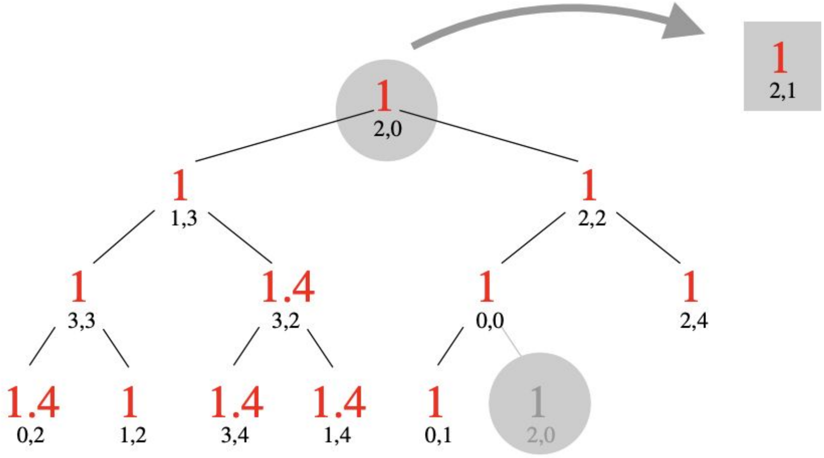
propagating initial values



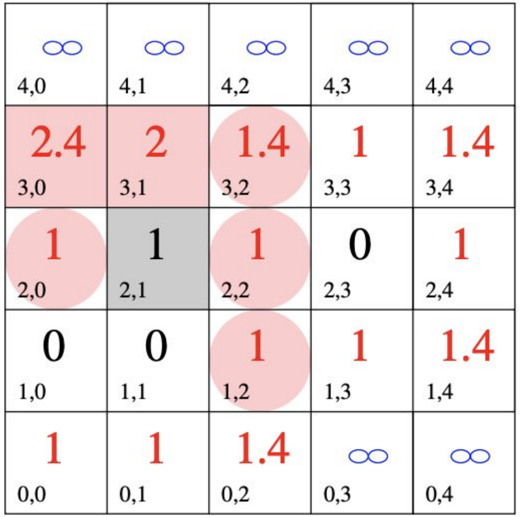
initial priority queue of pixels

Fast Marching Method Implementation (DS HW!)

Grab the top item from the priority queue...



after popping & fixing the top value, grab the last leaf & percolate down



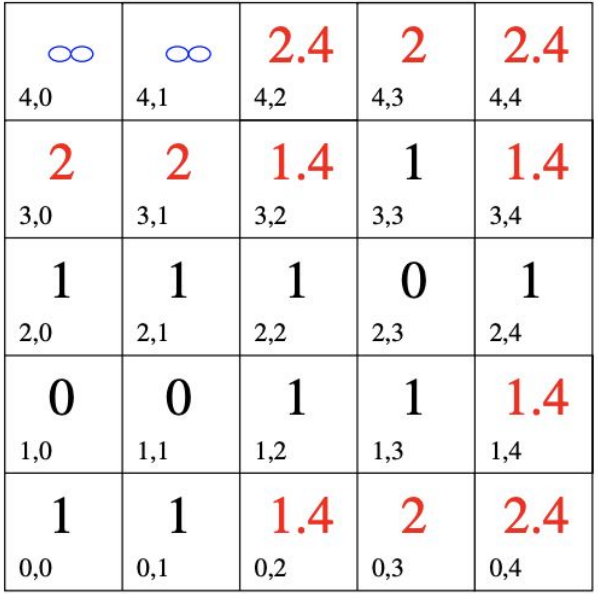
∞ 4,0	∞ 4,1	∞ 4,2	∞ 4,3	∞ 4,4
2.4 3,0	2 3,1	1.4 3,2	1 3,3	1.4 3,4
1 2,0	1 2,1	1 2,2	0 2,3	1 2,4
0 1,0	0 1,1	1 1,2	1 1,3	1.4 1,4
1 0,0	1 0,1	1.4 0,2	∞ 0,3	∞ 0,4

propagate fixed value to neighbors

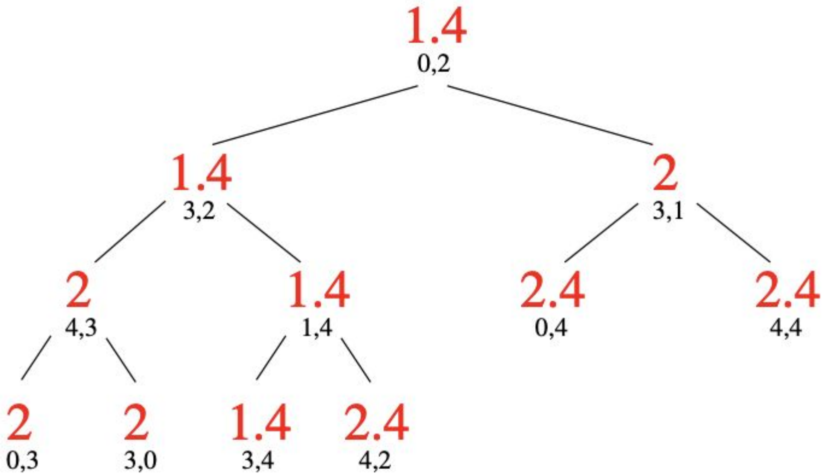
Lock its value, and update its immediate neighbors

Fast Marching Method Implementation (DS HW!)

Grab the next pixel in the priority queue and repeat….



after fixing all pixels <= 1



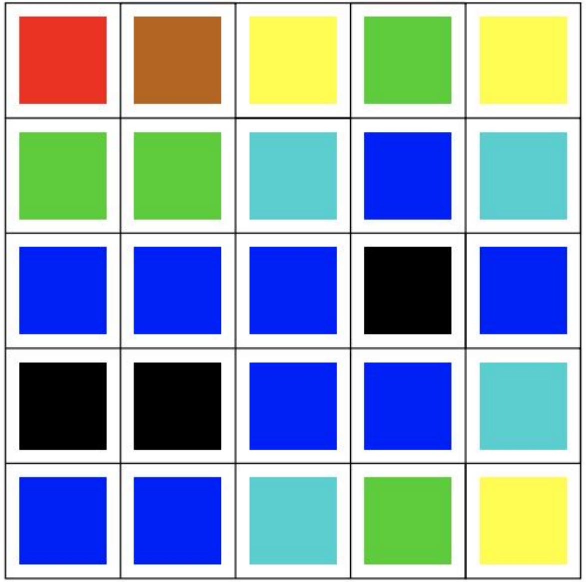
priority queue after fixing all pixels <= 1

Fast Marching Method Implementation (DS HW!)

Final result: Every pixel stores the (approximate) shortest distance to the original surface (black pixels)

3 4,0	2.8 4,1	2.4 4,2	2 4,3	2.4 4,4
2 3,0	2 3,1	1.4 3,2	1 3,3	1.4 3,4
1 2,0	1 2,1	1 2,2	0 2,3	1 2,4
0 1,0	0 1,1	1 1,2	1 1,3	1.4 1,4
1 0,0	1 0,1	1.4 0,2	2 0,3	2.4 0,4

final distance field



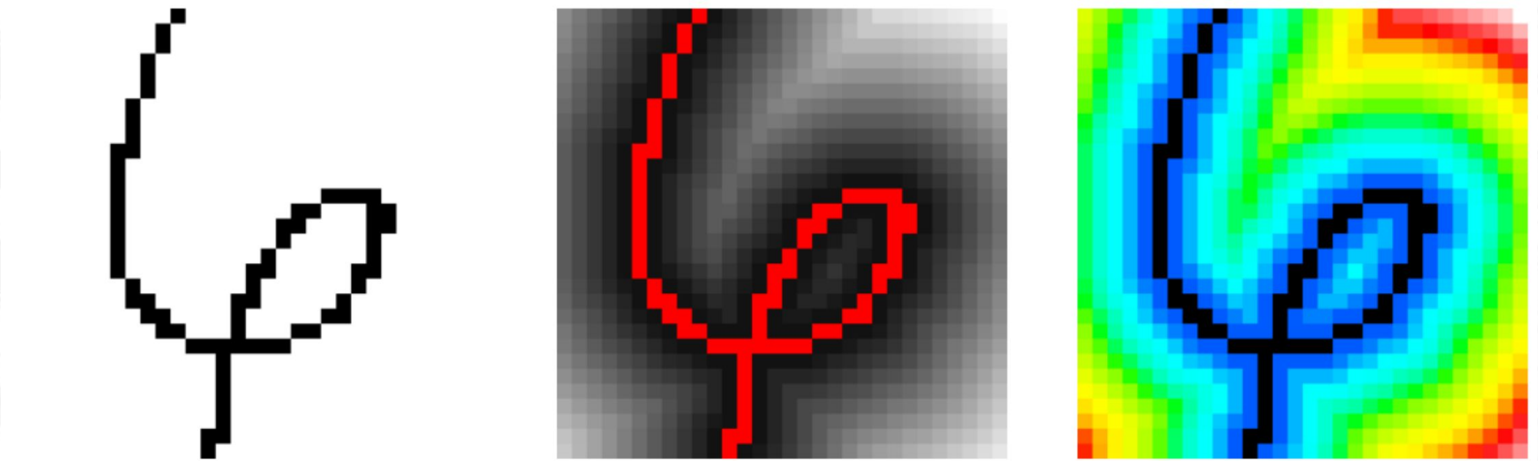
output image

Analysis of Fast Marching Method

- For an image/grid of size *w* x *h*, with *t* pixels/triangles:
- Naive:

  → *O (# of volume grid samples * # of surface elements) =* $O(w^2h^2)$
- Fast Marching:

  →

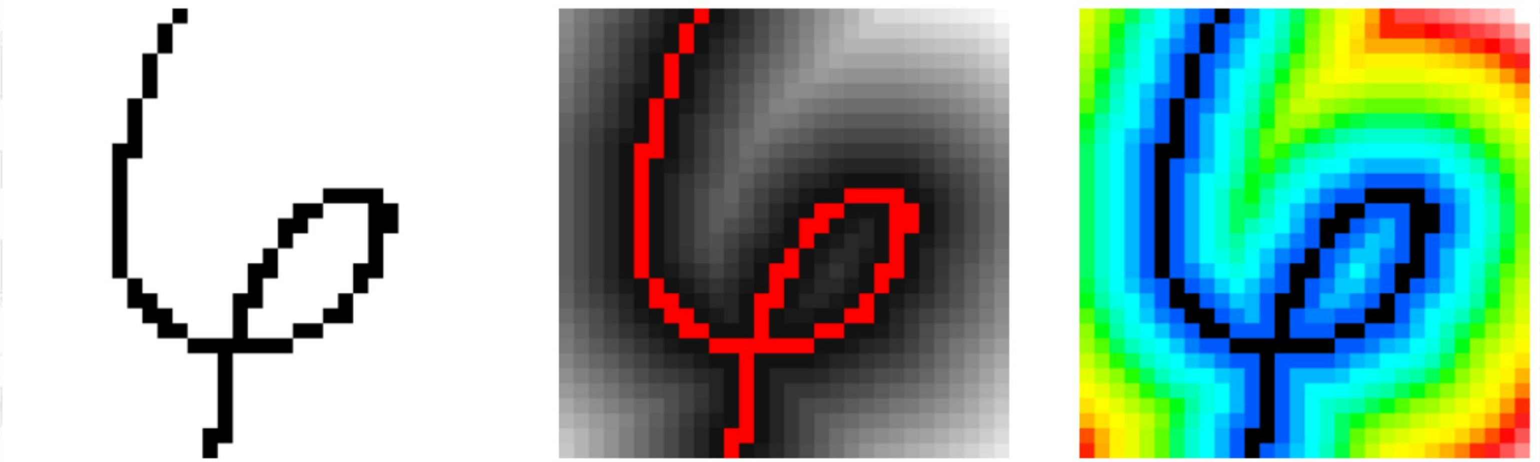


Analysis of Fast Marching Method

- For an image/grid of size *w* x *h*, with *t* pixels/triangles:
- Naive:

  → *O (# of volume grid samples * # of surface elements) = O(*w^2h^2*)*
- Fast Marching:

  → *O (# of volume grid samples * log active front) = O (w*h * log(t))*



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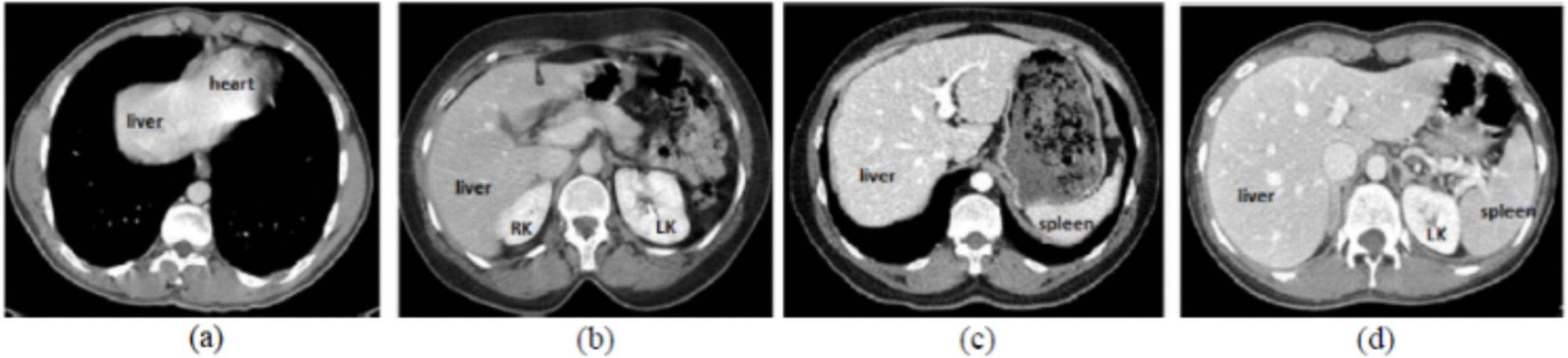
Motivating Application: Medical Imaging

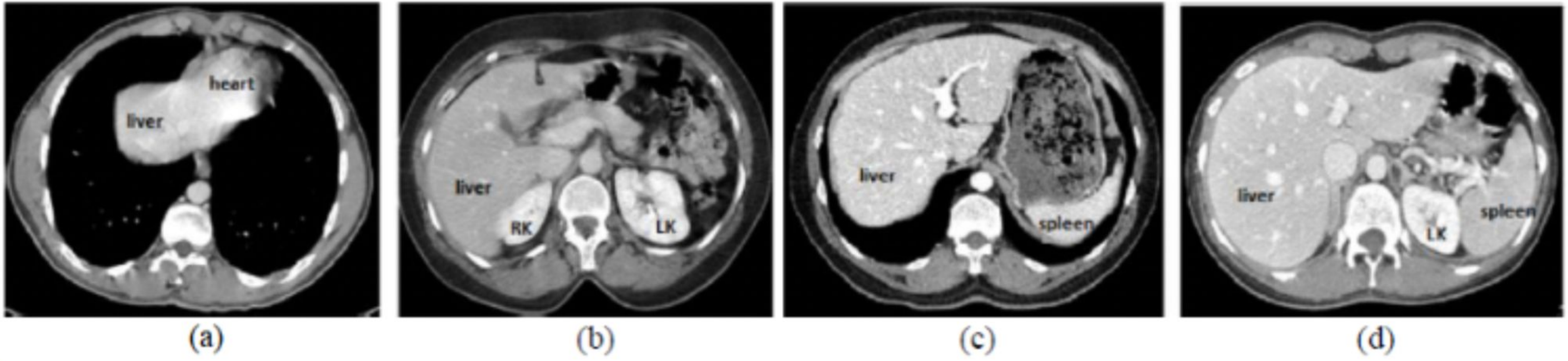
- Problem Statement: Convert 2D slices of MRI or CT image data into a 3D triangle mesh of the different organs and structures
- This will facilitate more intuitive visualization



https://chaos.grand-challenge.org/Data/

Motivating Application: Medical Imaging

- Input: a stack of 2D images, closely spaced parallel "slices" of the 3D object
- Step 1: Segment the different regions (by density / color / texture)



https://chaos.grand-challenge.org/Data/

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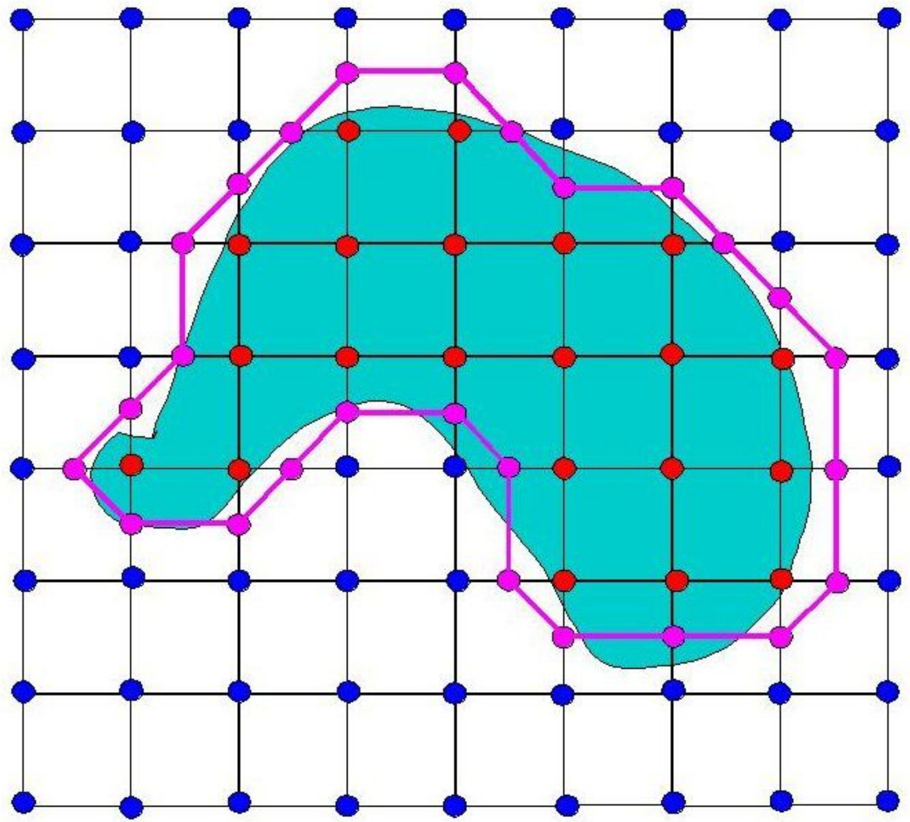
Marching Cubes

- Each point in the 3D grid is labeled "inside" (red dots) or "outside" (blue dots) the unknown surface.
- Any cell in the grid that has at least one red vertex and at least one blue vertex, must be crossed by the unknown surface.

- We can piecewise construct an approximation of the surface.

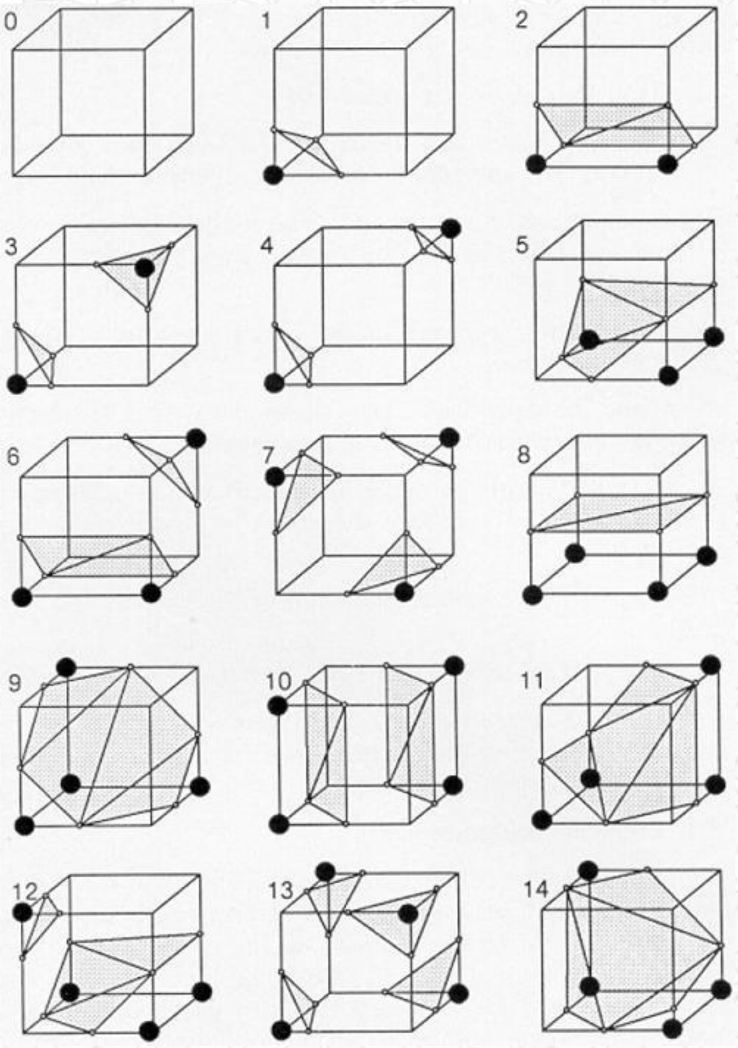


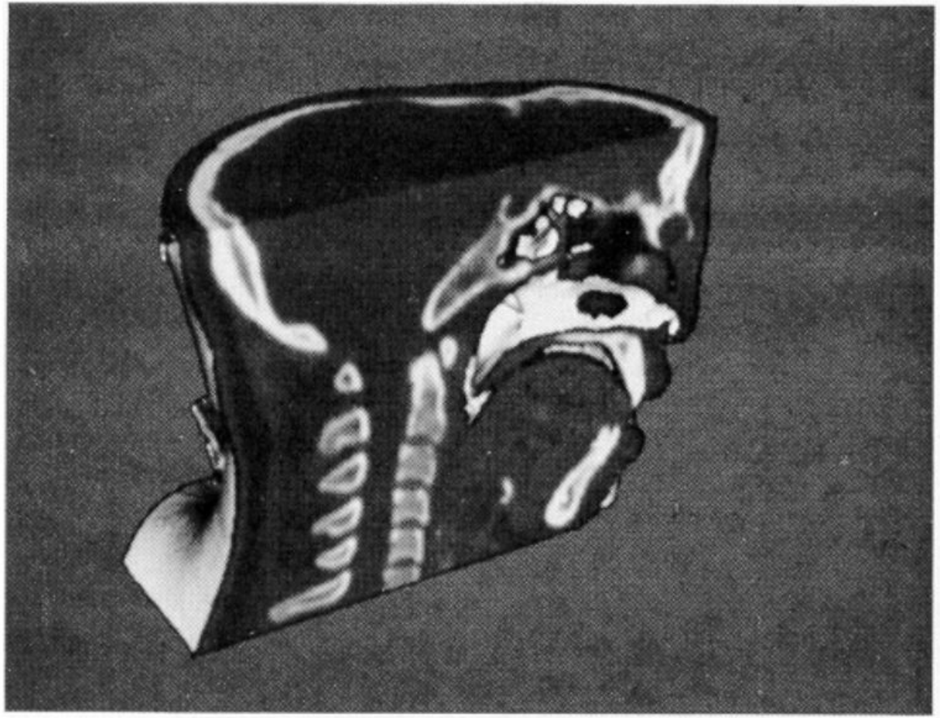
http://www.cs.carleton.edu/cs_comps/0405/shape/marching_cubes.html

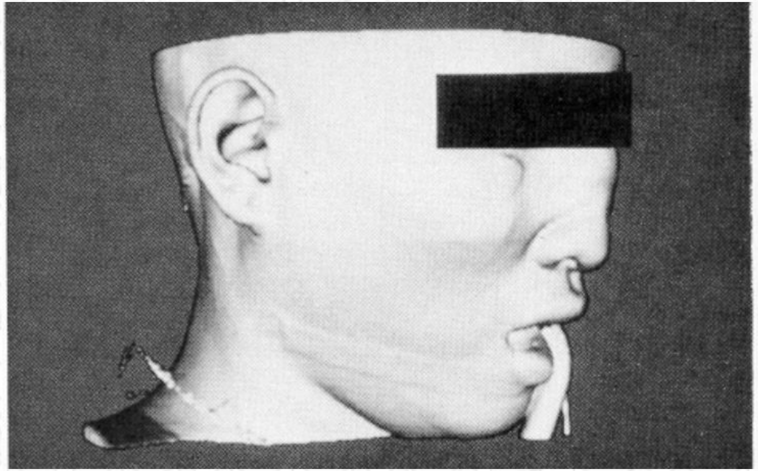
Marching Cubes

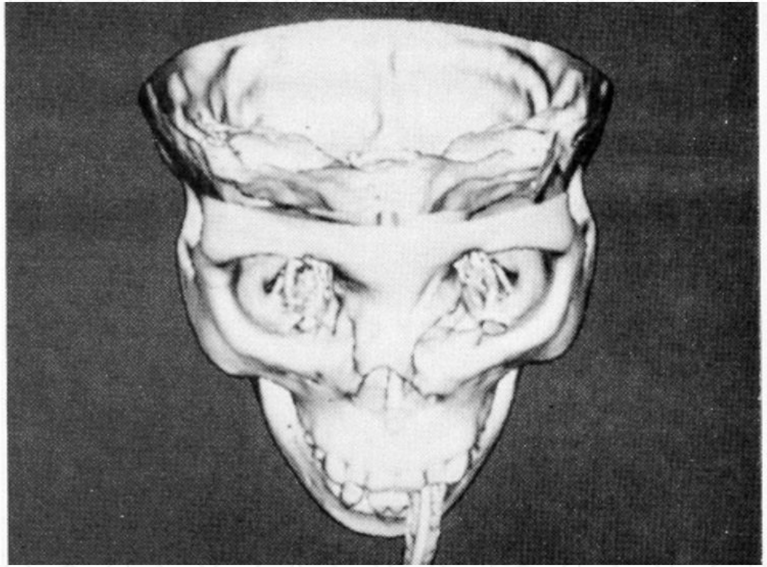
- 256 possible inside/outside labelings of each grid cube.
- Merging rotations…
  15 unique cases to implement

"Marching Cubes: A High Resolution 3D Surface Construction Algorithm", Lorensen and Cline, SIGGRAPH '87.



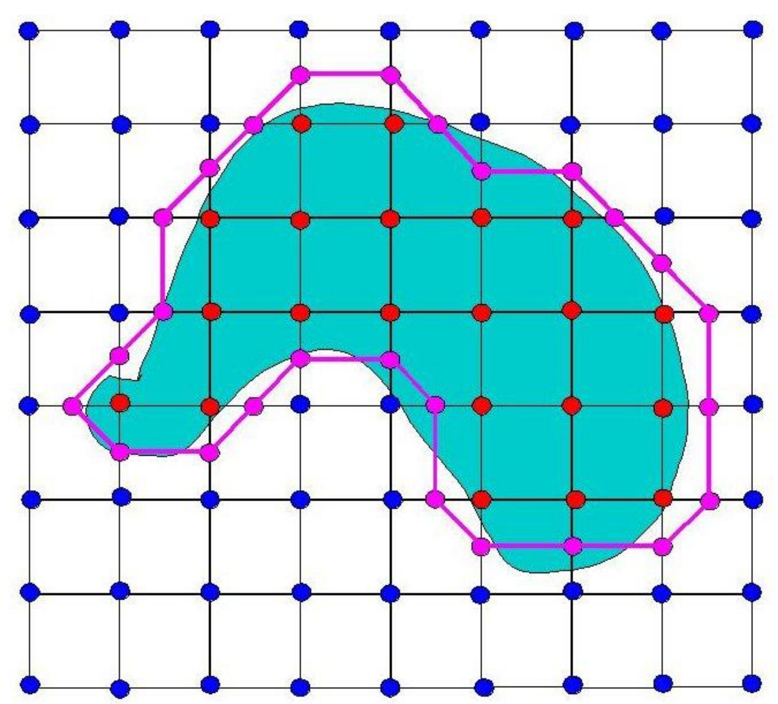






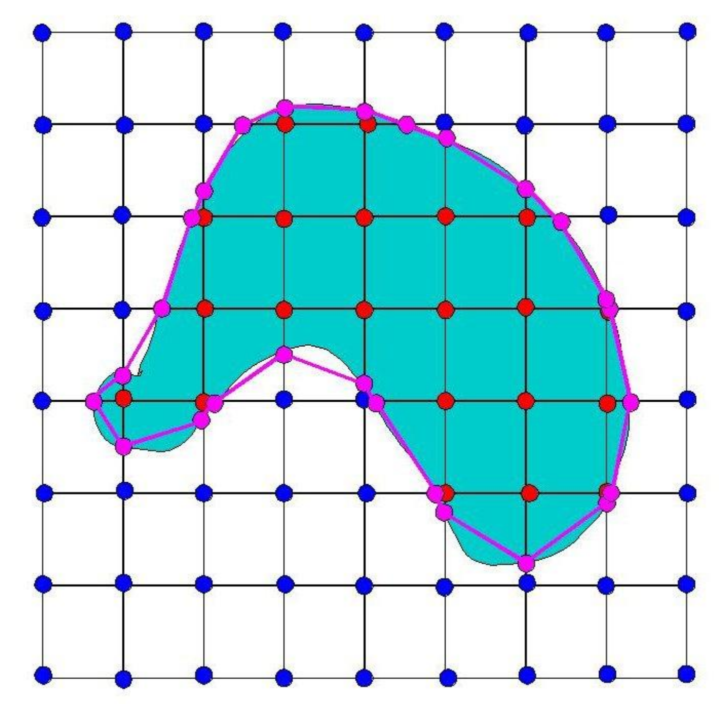
"Marching Cubes: A High Resolution 3D Surface Construction Algorithm", Lorensen and Cline, SIGGRAPH '87.

More than Binary – Signed Distance Data!

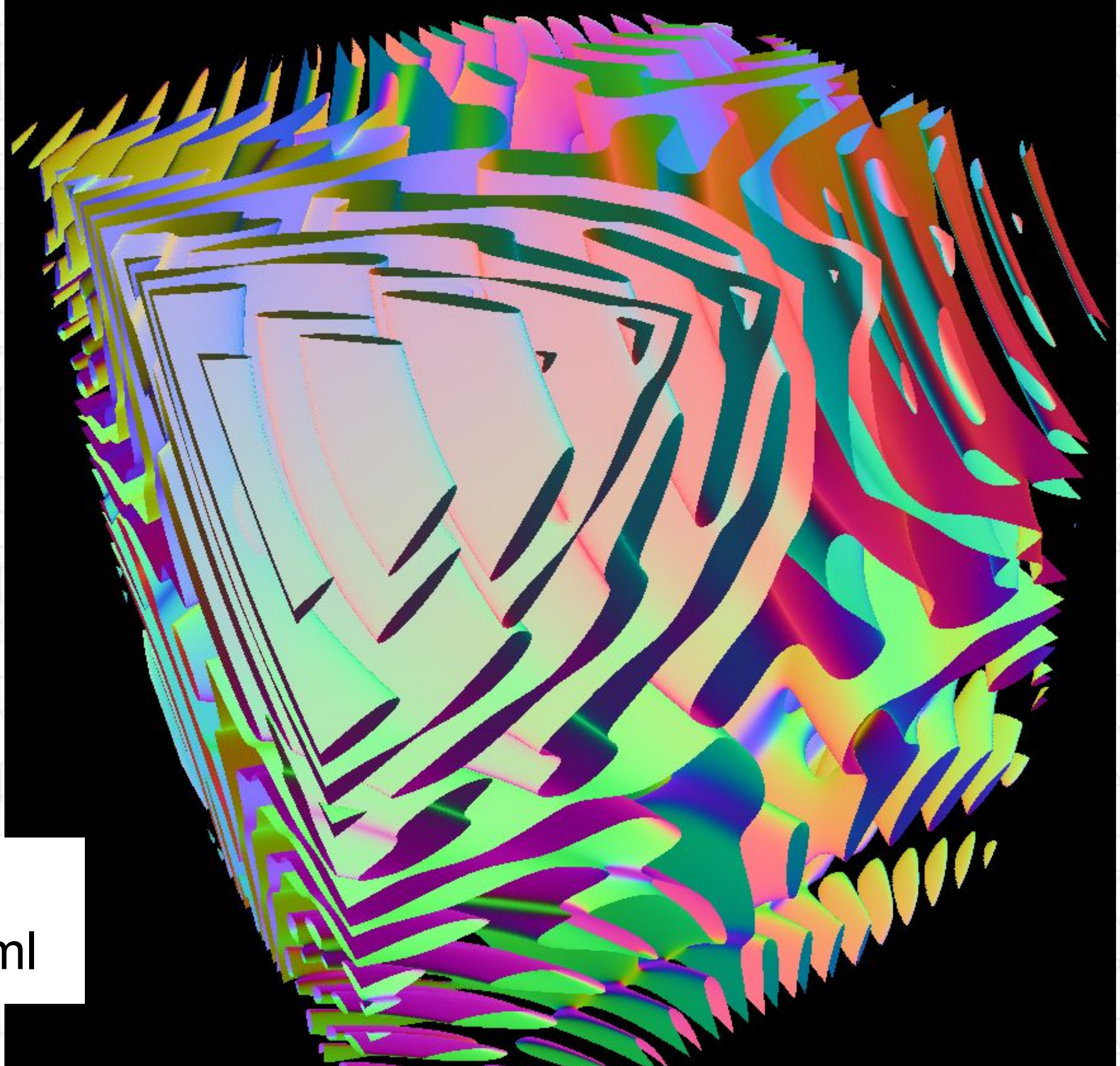


Crossing point should be placed not at the midpoint of each edge,

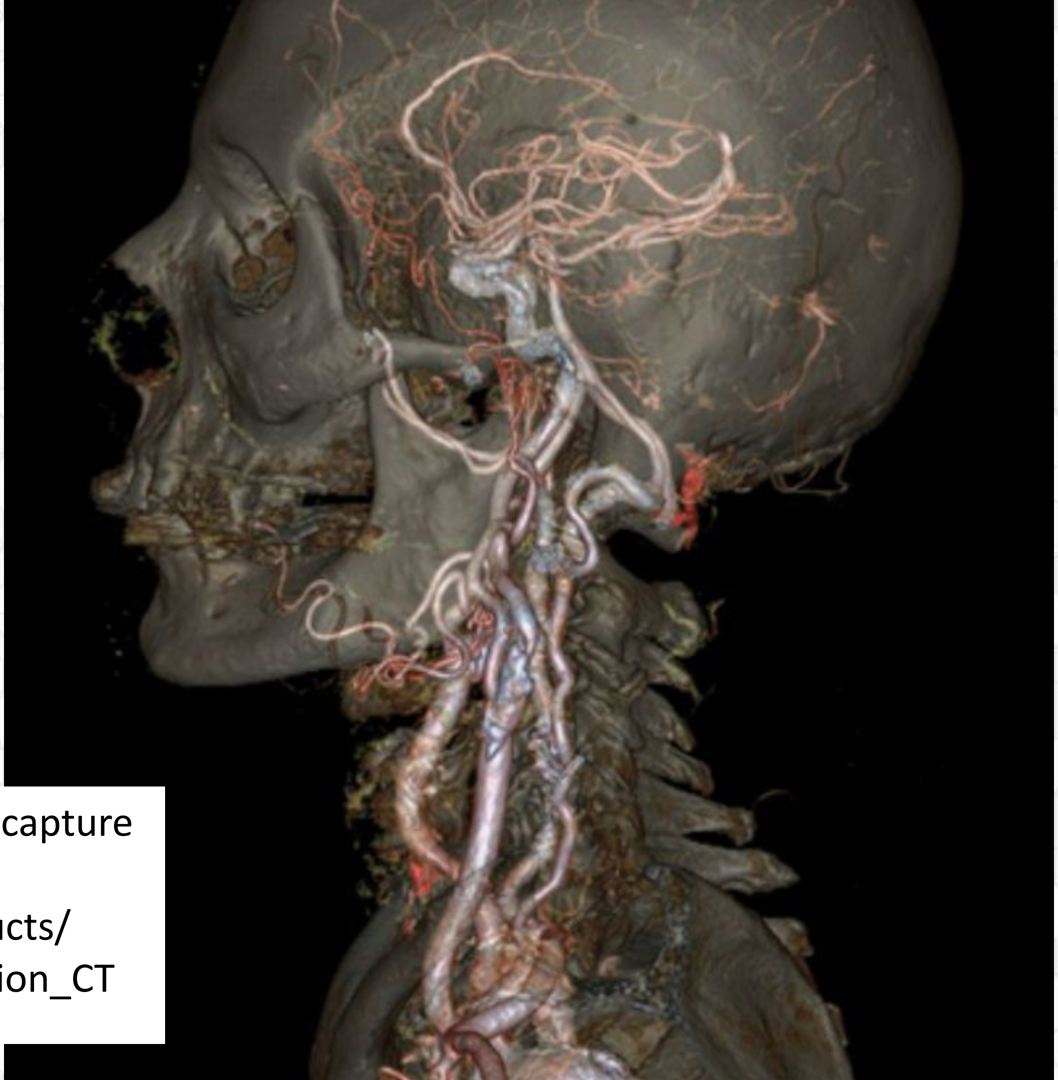
but at the estimated position of the level set!



http://www.cs.carleton.edu/cs_comps/0405/shape/marching_cubes.html



https://graphics.stanford.edu/~mdfisher/MarchingCubes.html



http://gizmodo.com/ges-new-fast-ct-scanner-captures-insane-images-in-a-he-1482904872
http://www3.gehealthcare.com/en/Products/Categories/Computed_Tomography/Revolution_CT

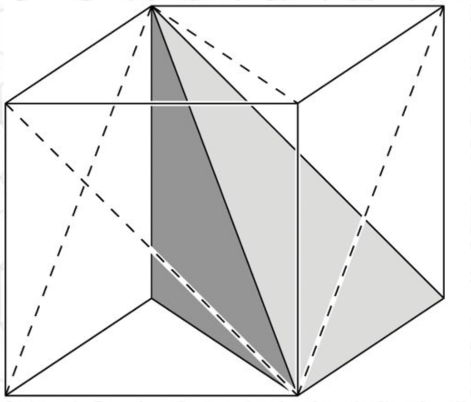
Outline for Today

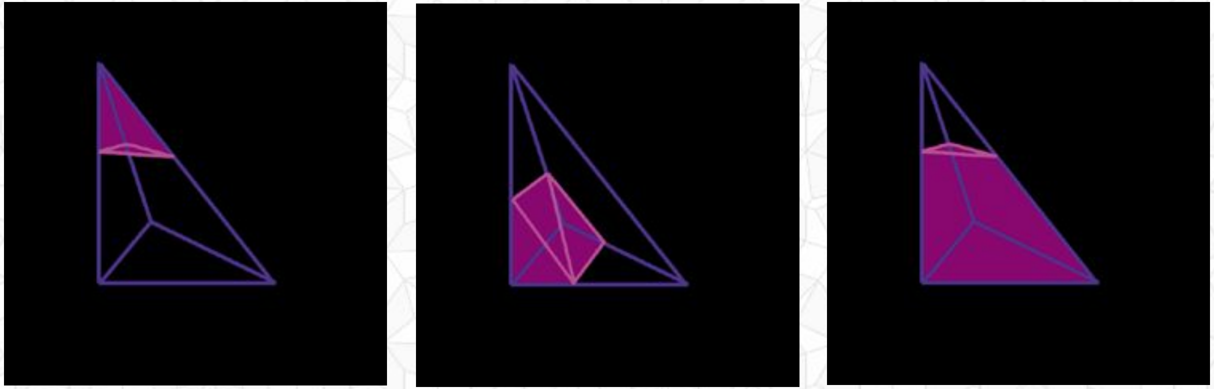
- Homework 5 Questions?
- Last Time: Quad Trees
- Explicit vs. Implicit Surface Representations
- Signed Distance Field
- Level Sets (Surface → Signed Distance)
- Fast Marching Method
- Medical Imaging
- Marching Cubes (Signed Distance → Surface)
- Marching Tetrahedra
- Next Time: ?

Marching Tetrahedra

Jules Bloomenthal
"An implicit surface polygonizer"
Graphics Gems IV

- Implementation Shortcut: Chop every grid cube into 6 tetrahedra….

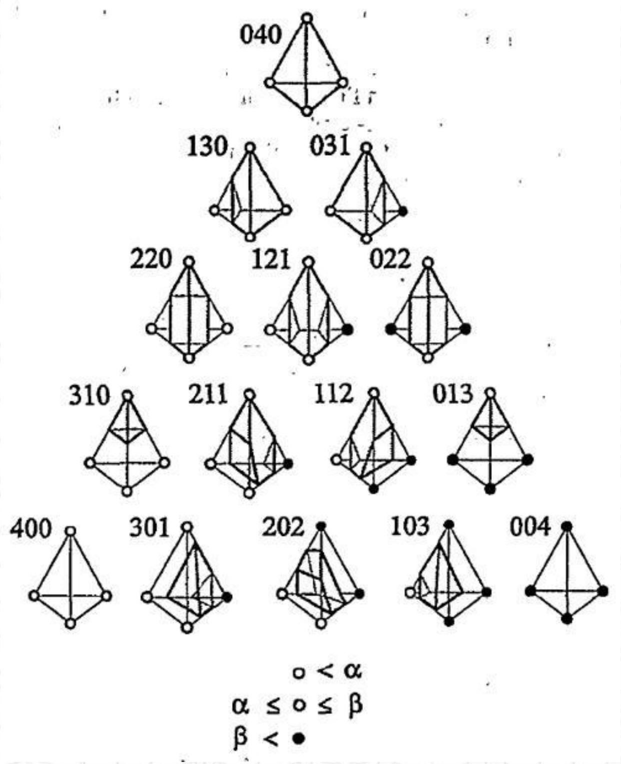




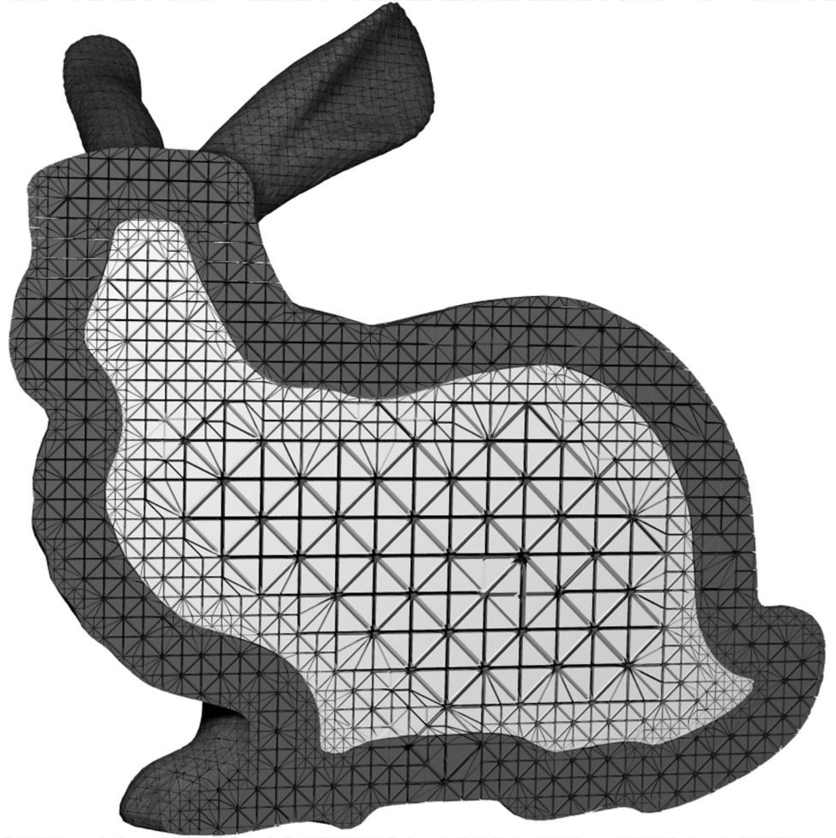
- Now only 3 unique cases for tetrahedra!

"When the Blobs Go Marching Two by Two", *Jeff Lander,* Gamasutra

Volumetric & Multiple Materials

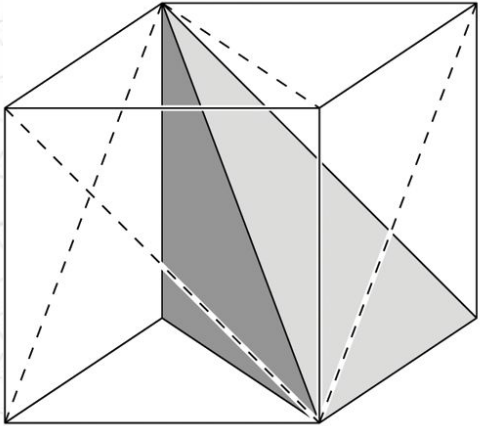


"Interval volume tetrahedrization"
Visualization '97, Nielson & Sung

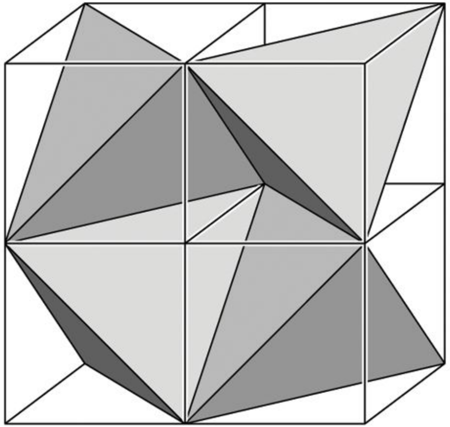


Implementation Details… Marching Tetrahedra

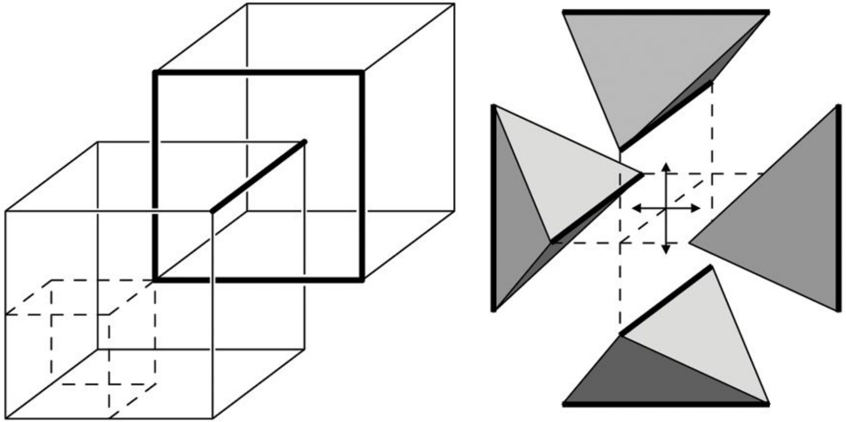
- Which cube → tetrahedra subdivision should we use?



6 tetrahedra
(all equal size & shape)
diagonal bias



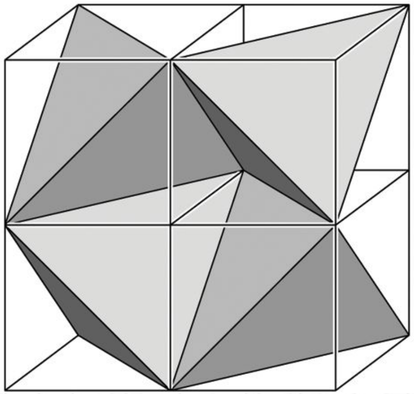
5 tetrahedra (1 equilateral that
is 2X the others in volume)
Orientation must be alternated

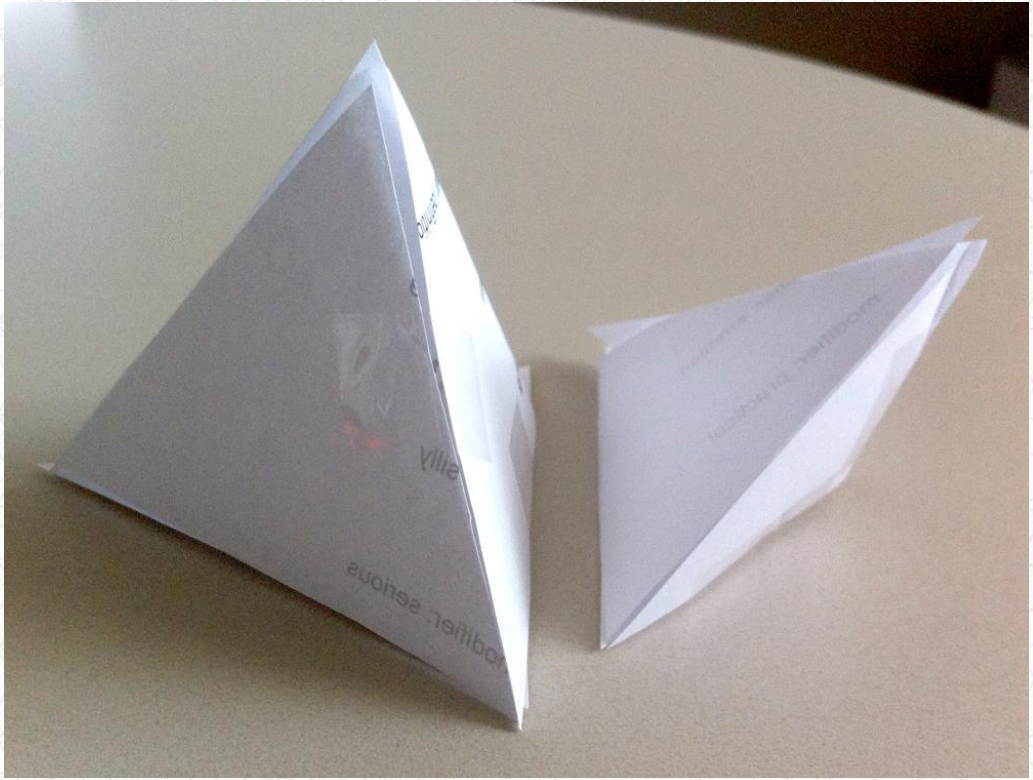


Crystal Lattice
All same size & shape,
but more complicated…

Debugging Marching Tetrahedra

- Drawing (in 2D) didn't work
- Creating an OpenGL visualization didn't work (even with transparency)
- Solution: build lots of paper & tape models





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