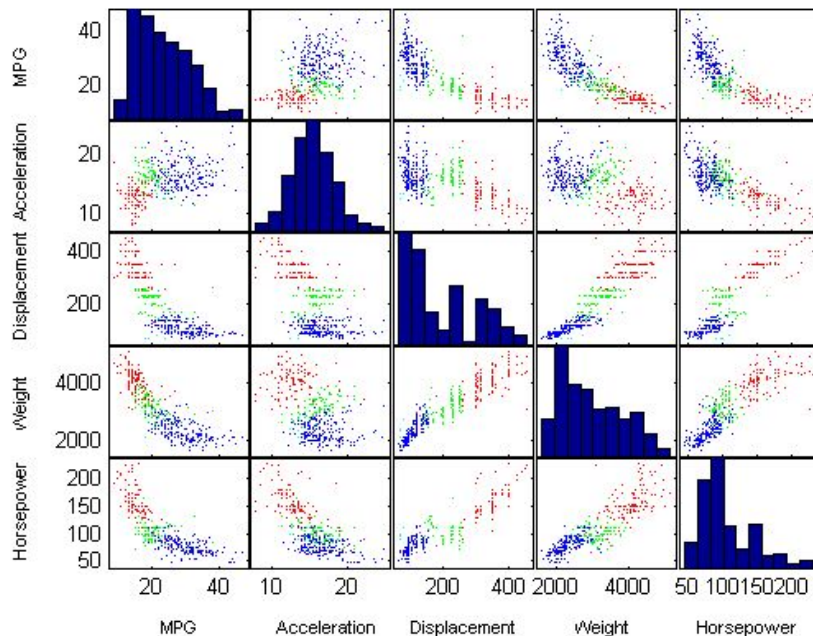


# Convex Hulls, Voronoi Diagrams, & k-Means Clustering

## Today

- Reading: “LineUp: Visual Analysis of Multi-Attribute Rankings”
- Let’s do Computational Geometry
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  - Voronoi Diagram
  - k-Means Clustering
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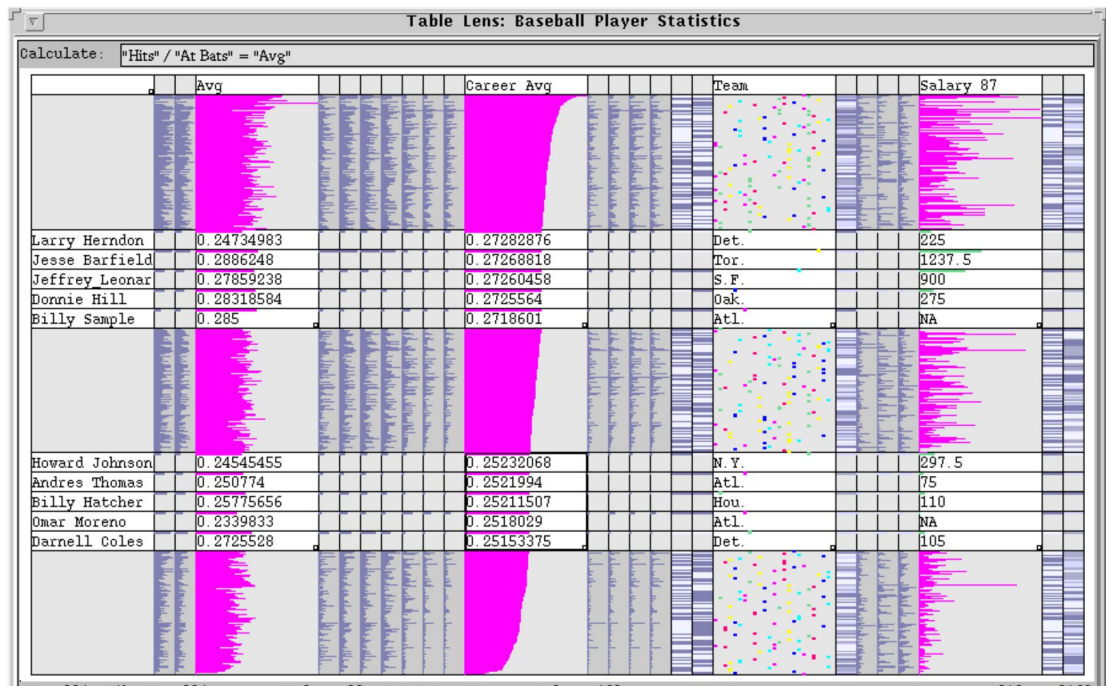
# Scatterplot Matrix

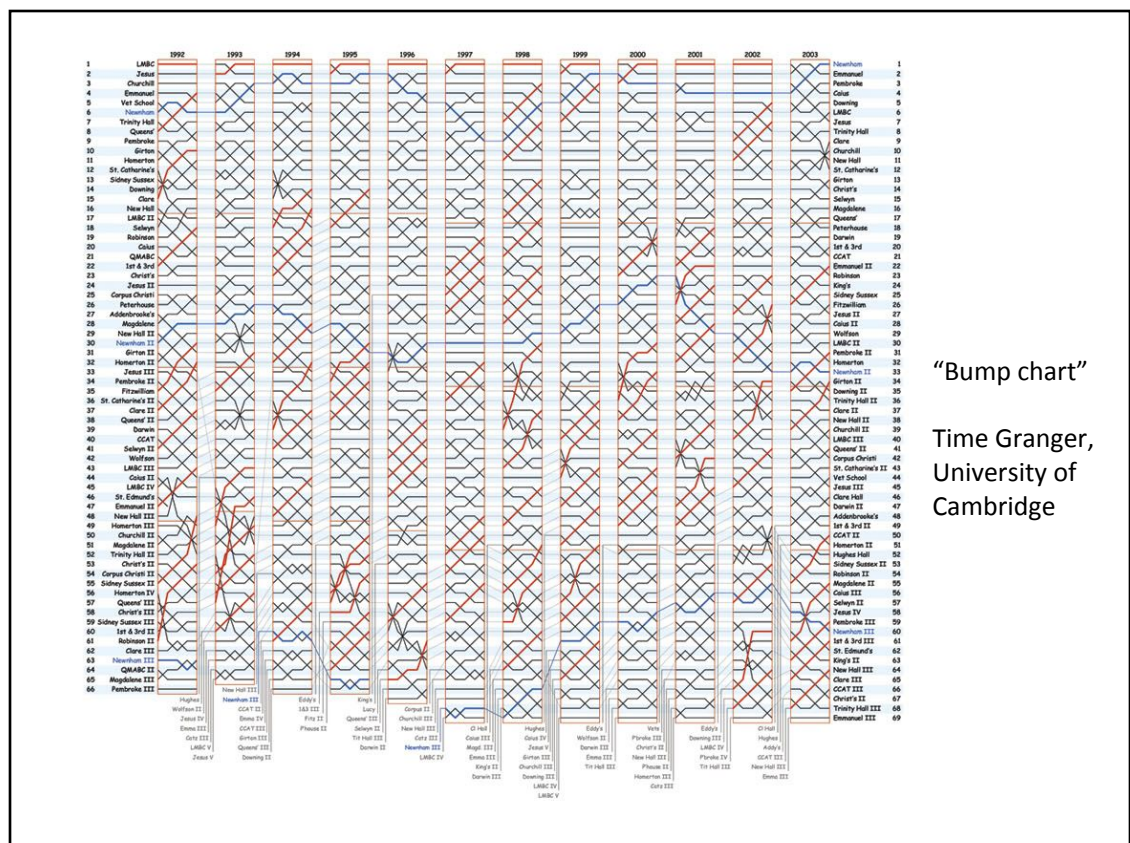
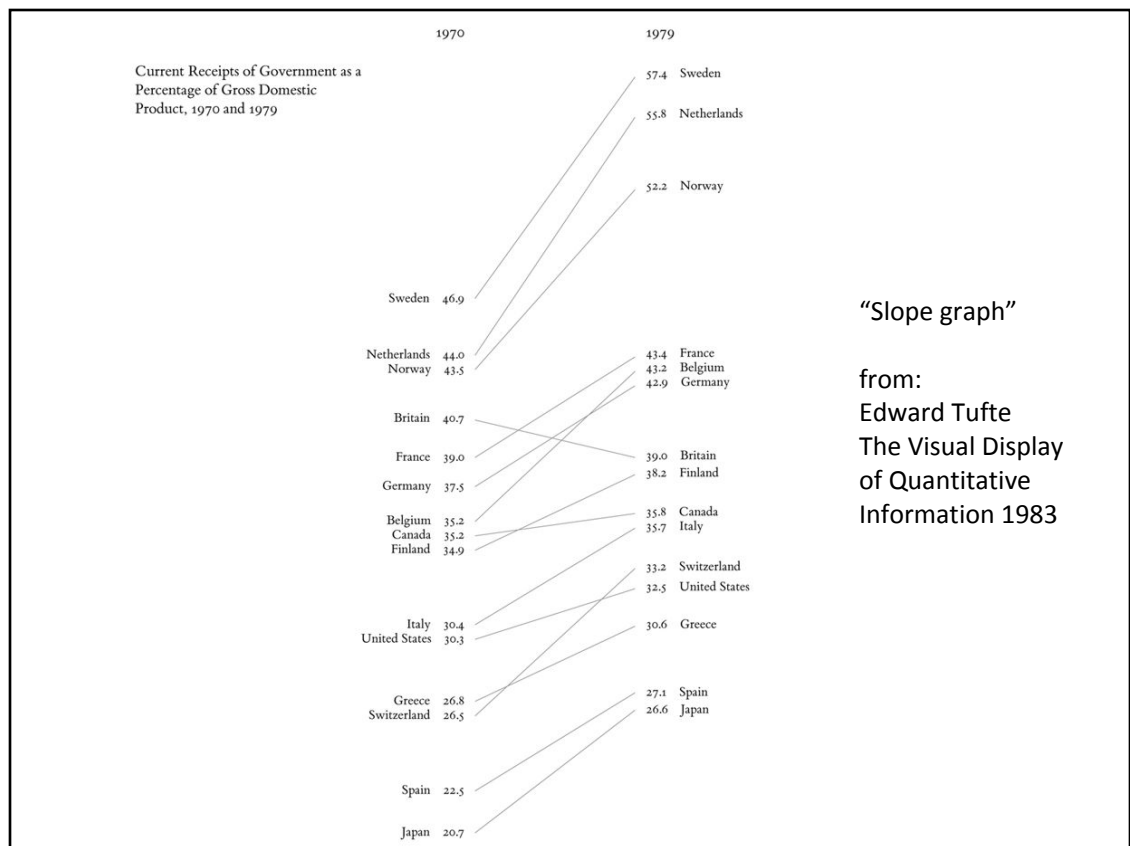


<http://www.mathworks.com/help/stats/examples/visualizing-multivariate-data.html>

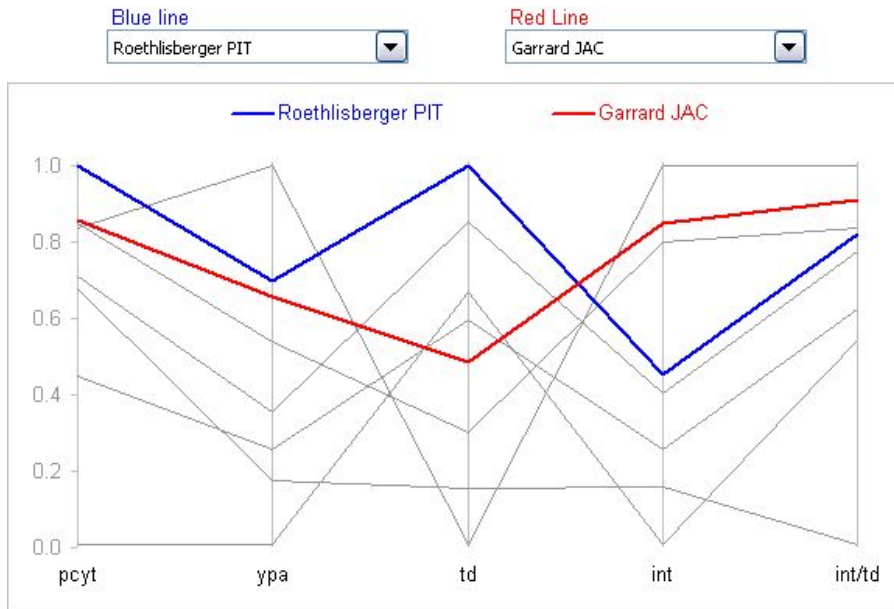
“The table lens: merging graphical and symbolic representations in an interactive focus + context visualization for tabular information” Ramana Rao, SIGCHI Conference on Human Factors in Computing Systems 1994

Focus+context





# Parallel Coordinates



<https://peltiertech.com/Excel/Charts/ParallelCoord.html>

## RankExplorer: Visualization of Ranking Changes in Large Time Series Data

Conglei Shi, Weiwei Cui, Shixia Liu, *Member, IEEE*, Panpan Xu, Wei Chen, and Huamin Qu, *Member, IEEE*

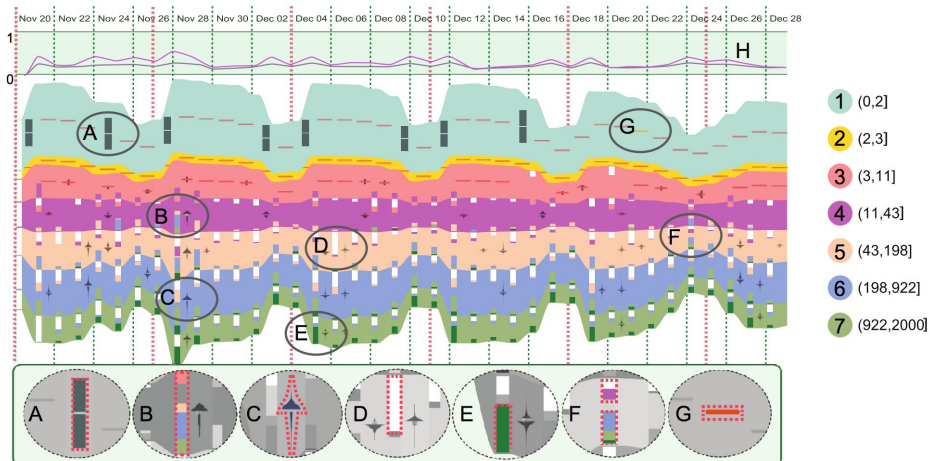


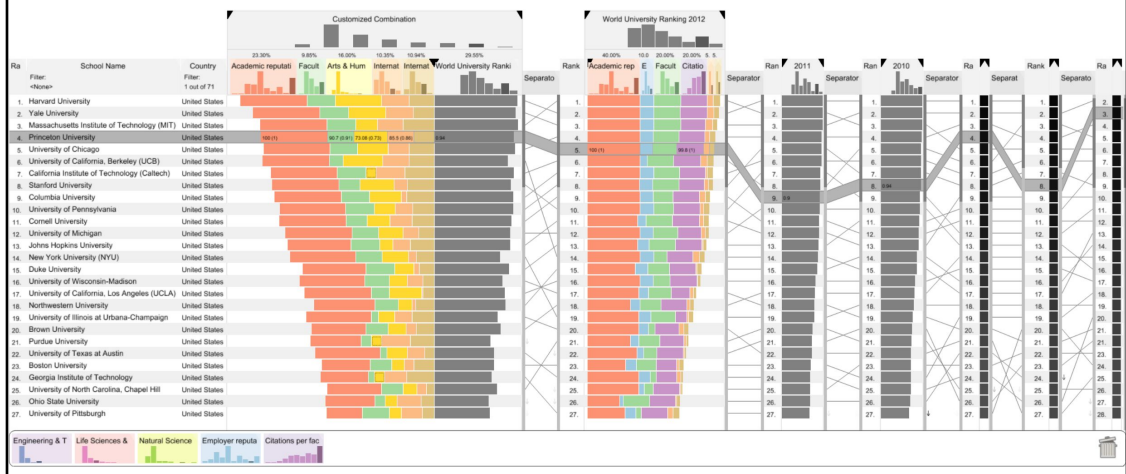
Fig. 1. RankExplorer visualization of the top 2000 Bing search queries from Nov. 20 to Dec. 29 in 2011. All queries are divided into seven categories. The width of each layer at a time point encodes the total query count at that time. The color bar and glyphs encode the content changes in each ranking category. From the color bar, we can observe: 1) the change between layers (the bar segments with the colors of other layers in B and F); 2) new queries coming in (the white segment in D); 3) recurring queries (the dark green segment in E). From the changing glyphs, we can see: 1) a non-change pattern (only red line in G); 2) a swap pattern (the two equal-height segments in A represent that the two queries swap their rankings); 3) a shift pattern (the increasing part is significantly larger than the decreasing part in C). From the trend curve (H), we can see the degree of ranking change over time.



<http://www.tableausoftware.com/>



- “LineUp: Visual Analysis of Multi-Attribute Rankings”, Gratzl, Lex, Gehlenborg, Pfister and Streit, IEEE INFOVIS 2013



- Visualizing overall ranking is simple, interpretation of ranking is not simple
- Alternative rankings exists
- Customize weights of multiple heterogeneous attributes
- Customize function for a particular attribute: linear, log  
is this easy for the user to do? Give a larger set of typical options rather than type javascript?
- Compare different publications rankings, compare rankings over time
- Cost-benefit analysis for improving rankings
- Rankings should be personalized, we each have our own priorities
- Take “rankings” to a new level
- Previously thought bar charts were boring, not creative

## Requirements analysis

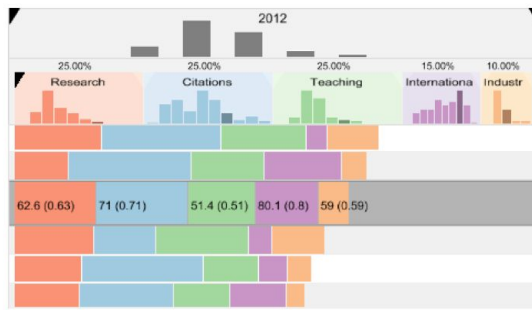
- List of 10 detailed requirements, including:
  - Nested model for visualization design and validation
  - Domain problem characterization
  - Data must be normalized, allow filters of data, handle missing data (unfortunately discussion/solution rather vague in paper)
  - But keeping track of 10 criteria was a little hard to follow
  - Some of the 10 requirements seemed redundant
- Did good job developing & describing the problem & requirements
- Formalizes general (intuitive, agreed upon) criteria
- Interaction is key to effective user interpretation of data

# Features

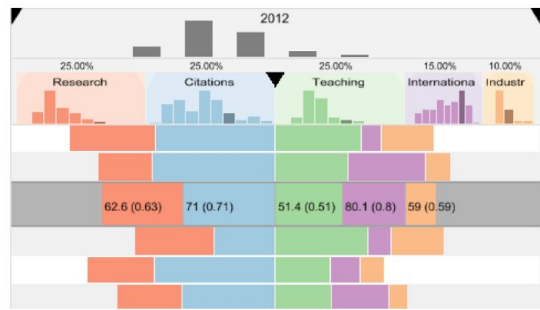
- Column types
  - Rank columns
  - Textual attribute column – labels, nominal attributes
  - Categorical attribute column
  - Numerical attribute column – encoded as bars!
  - Combined attribute column
- Histogram above the attributes – confusing, not sufficiently explained
- Interactivity
  - Interactive bar graph plotter with customizable filters & parameters
  - Immediate feedback when weights are changed
  - Animated transitions, color highlight – strength/length of time applied, based on amount of change
  - “Snapshot” for comparing rankings before/after weight edits
  - Collapsing/compressing columns
    - Use fewer pixels to display same data (can be done with bar graphs, not words/raw numbers)
    - “Fisheye” view allowing a gradual falloff in level-of-detail/screen space
    - Clean up clutter
    - Replace connection lines to invisible targets with arrows

## Features II

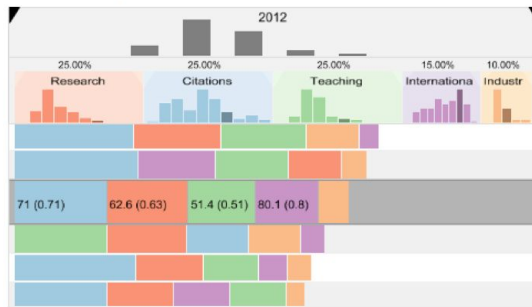
- Memo pad: place to put attributes not currently in use.
- Consistent, customize-able mapping between attribute and color
  - too many colors might become too busy/confusing
- Parallel mapping editor
- Orthogonal mapping editor
- Allows user to infer the importance of attributes to ranking
- Is this too many features in one industrial strength application?  
Would it be better to have a collection of individual tools/plugins?
- Too many controls, not enough guidance for novices on how to use them
  - Tool (and/or presentation of tool) is needlessly complex?



(a) Classical stacked bars



(b) Diverging stacked bars



(c) Ordered stacked bars



(d) All-aligned bars

## Evaluation

- Heavy focus on User Study in paper
- Compare to existing tools: Confirmed that experts in Excel could complete tasks that novices would be asked to perform with LineUp – However, did not do formal user study comparison between Excel & Lineup because expertise is such a huge component
- Check for colorblindness – why?
- 12 separate tasks
- NASA TLX workload assessment questionnaire, task completion times
- Justification for their experiments was early in paper & detailed, synced well with study & results, study measured the appropriate things
- Well designed study, but only a tiny # of participants (8)
- Users in study were unrealistic? Target audience is “expert amateurs”?



# Results

- Authors surprised that filter task took longer than most other tasks
- Users thought it was visually pleasing, potentially helpful, easy to understand, and (compared to Excel) would save time, help gather more insights
- Drag & drop is easier to create complex rankings than typing formulas
- Some negative comments about rankings change from user who did not [ know to ] use the snapshot feature
- Does this scale to bigger datasets? Is it visually clear? Is it reasonable to navigate?

# Future work

- Features to help optimize rankings, what needs to be changed to achieve a certain ranking?
  - Why is this an important feature? (ask any university administrator...)
- Statistical techniques to deal with large number of attributes
- Large differences in rankings become steep slopes which are hard to interpret
- Need to test actual users in real-world analyses and observe adoption rates
  - Web-based implementation
  - Apply tool for original purpose: ranking of genes, clusters, pathways for analysis of genomic data

# Writing Quality

- Paper was well written
- Justification for their experiments was early in paper & detailed, synced well with study & results, study measured the appropriate things
- Who is the audience?
  - Clients designing rankings
  - Or people who need to look at rankings
  - Domain experts?
  - Novices?
- Lots of figures! Great. But some details in some figures were confusing/hard to follow
- Would have been nice to include algorithm in paper
- Multiple real-world examples good -- Nutrition example helped explain the data mapping editor
- Scrolling up to the top of the paper every time they mention Figure 1 or a specific # requirement from the list of 10 was annoying
- Good use of bold text
- Some parts were repetitive

## Definitions / Misc.

- Likert scale: on a survey, response from a set of choices on a scale, commonly: strongly disagree, disagree, neither agree nor disagree, agree, strong agree
- Algorithm design, encoding/interaction design
  - “A nested model for visualization design and validation”, Tamara Munzer, TVCG 2009
  - <http://www.cs.ubc.ca/labs/imager/tr/2009/NestedModel/NestedModel.pdf>

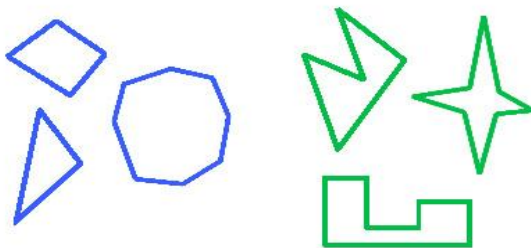
# Today

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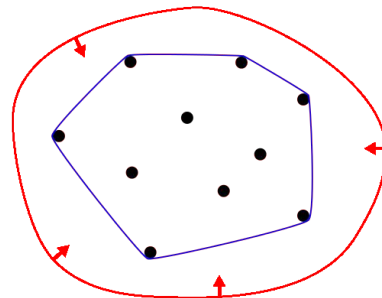
## Convex vs. Concave, Convex Hull

**Convex:** Shape has no inward corners or curving faces.

**Concave:** Has inward corner(s) or inward curving face(s).



<http://img.sparknotes.com/figures/B/b333d91dce2882b2db48b8ad670cd15a/convexconcave.gif>

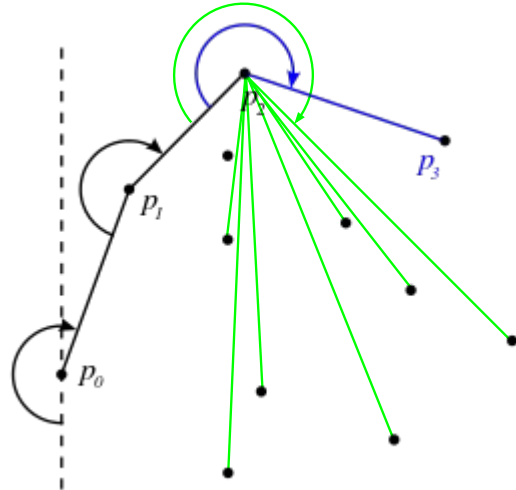


<http://en.wikipedia.org/wiki/File:ConvexHull.svg>

**Convex Hull:** The smallest convex shape that contains all of the input points / elements. 2D, 3D, higher dimension.

## “Gift Wrapping”

- Find left most point
- “Walk around” the point set in the clockwise direction



- Find the next point  $p_3$  on the hull.
  - smallest outer angle between lines  $p_1p_2$  &  $p_2p_3$

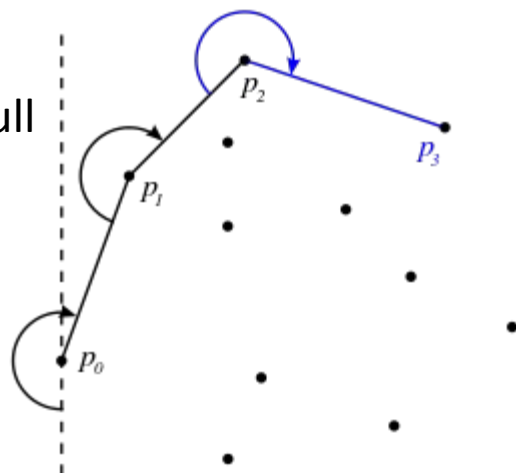
## Gift Wrapping Analysis?

- For  $n$  input points, and output convex hull contains  $h$  points?

$$O(nh)$$

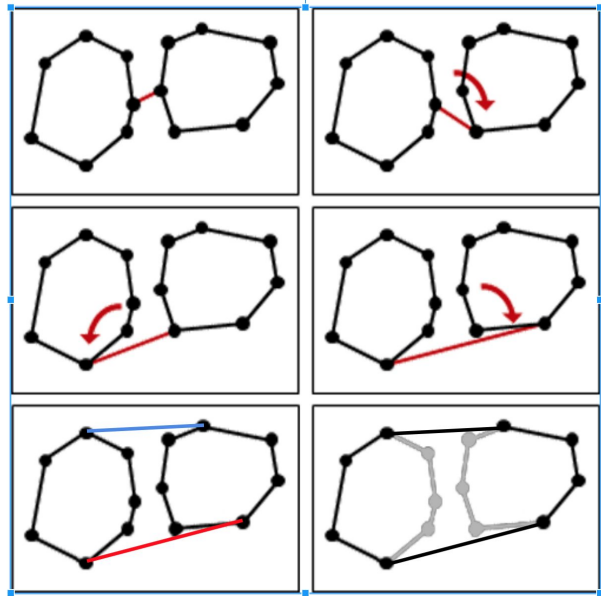
- Worst case?

$$O(n^2) \text{ most/all input points are on the convex hull}$$



## Divide & Conquer (*like merge sort*)

- Sort the x axis
- Split into 2 ~equal groups
- Recurse...
- Merge the 2 convex hulls
  - Find rightmost point in left hull, and leftmost point in right hull.
  - Walk down to find **lower tangent**
  - & up for **upper tangent**
  - Discard points in between upper & lower tangents
- Analysis?



$O(n \log n)$

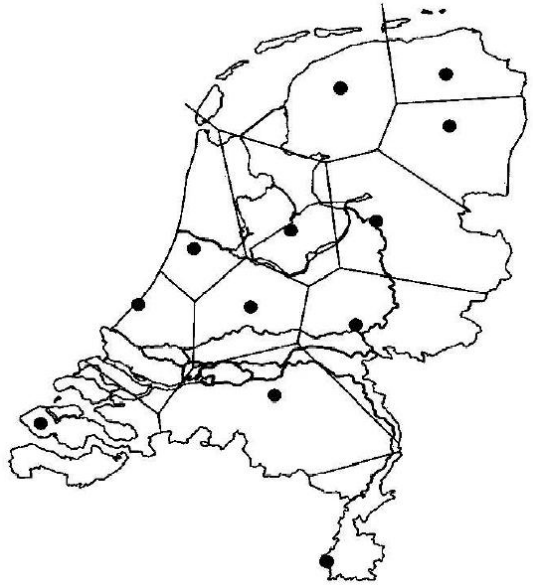
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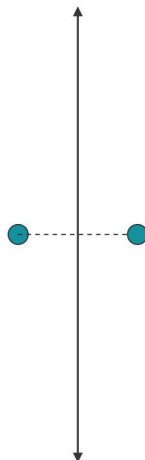
# Voronoi Diagram

- Intuition: Partition space into regions that all agree on which is the closest point from a set of input points.
- Examples
  - Re-district the Netherlands into provinces so that everyone reports to the closest capital
  - Multiple ambulance helicopters distributed across the county, which should we dispatch to help?

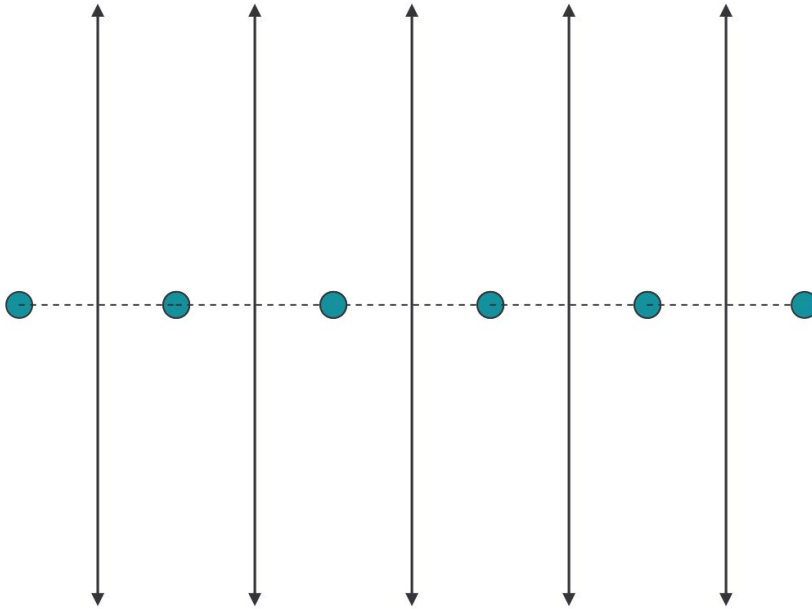


<http://ccc.inaoep.mx/~rodrigo/robotica/Trigui.pdf>

- If we have just 2 input points, we divide space into 2 regions.
- Region/Cell edges are the perpendicular bisectors of nearby points.



## Multiple input points... but all on a line



## Assume “General Position”

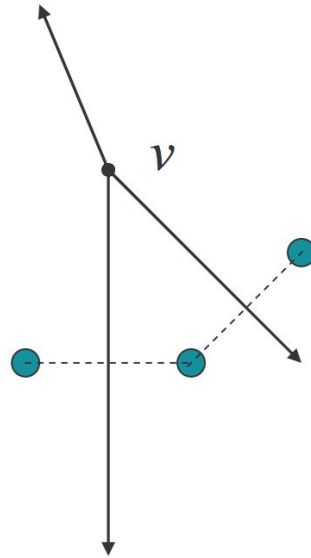
- What does this mean?
- We'll focus on the general case, where we don't have *coincidental alignments...*
  - No 2 points are exactly at the same position
  - No 3 points are exactly co-linear
  - No 4 points lie exactly on a circle
  - Etc.
- We don't have to worry about anything ever being a tie, there will only be one right answer.
- How to ensure general position in practice?

*Add a small amount of random noise to every point.*

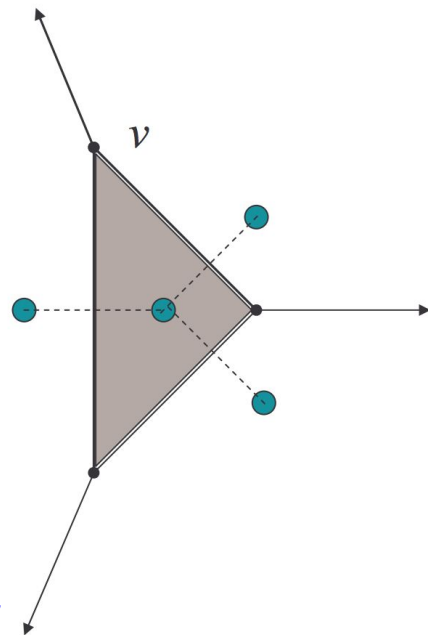
*Small enough to not change the result.*

*But large enough so things don't align.*

- Where the perpendicular bisectors intersect, we create corners for our Voronoi cells



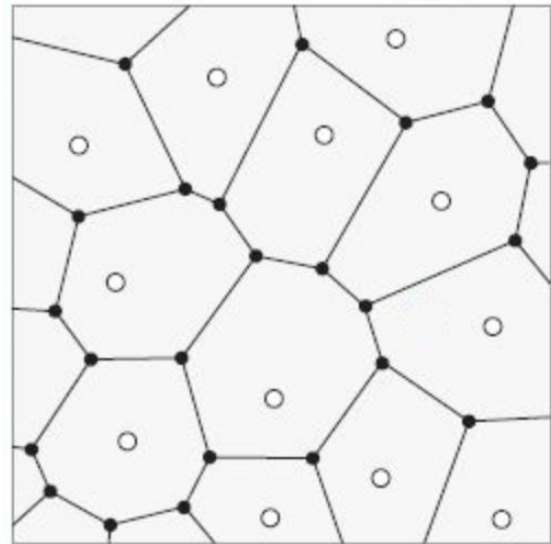
- A Voronoi cell for an input point is bounded by the perpendicular bisectors with its closest neighboring input points
- Each region will be convex!



- *A Voronoi cell can have any number of edges.*
- *If the input points are evenly spaced, most cells will have approx 6 sides, with approx equal length sides and approx equal  $120^\circ$  angles.*

# Nearest Neighbor Query

- Once we have built a Voronoi Diagram
- We can find the **nearest neighbor** in the input point set
- a.k.a. locate which Voronoi Cell contains the query point
- ***In  $O(\log n)$  time***  
... algorithm details omitted



## Incremental Algorithm for Voronoi construction

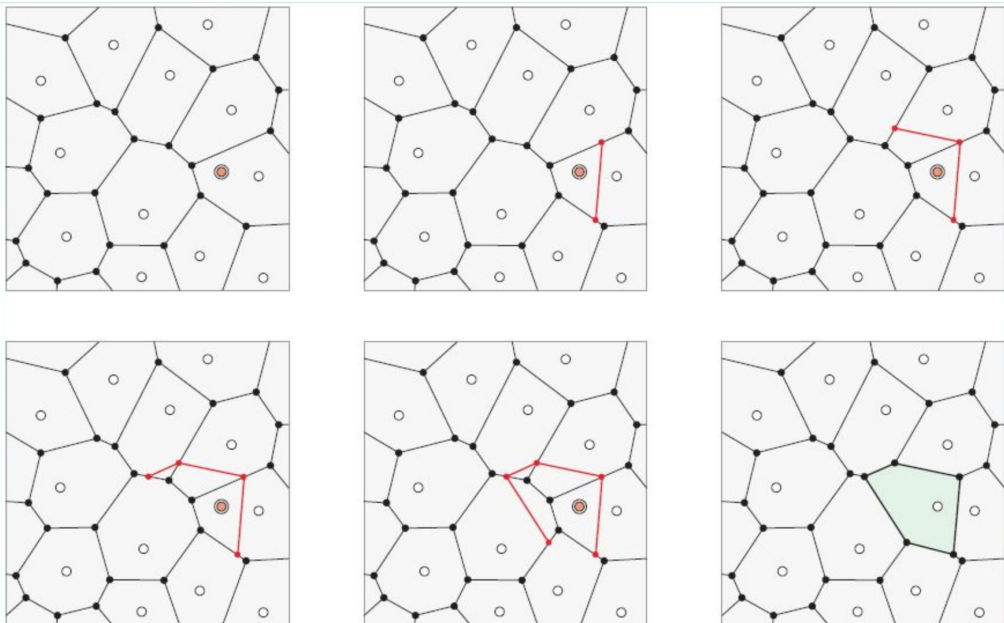


Figure 4.5: The incremental algorithm for creating Voronoi diagrams.

# Analysis of Incremental Algorithm?

- Find the cell containing new point
- Calculate perpendicular bisector with the nearest neighbor and points in the neighboring cells

- Average case to insert 1 point?

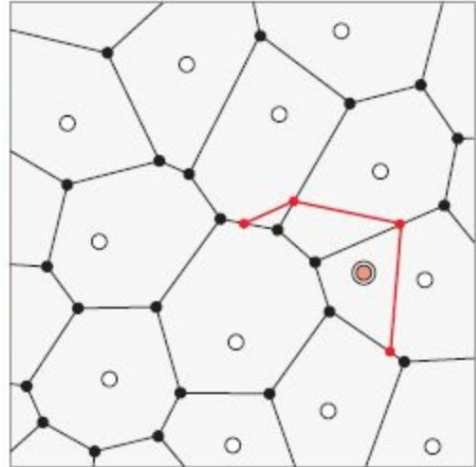
$O(\log n)$

- Worst Case to insert 1 point?

$O(n)$

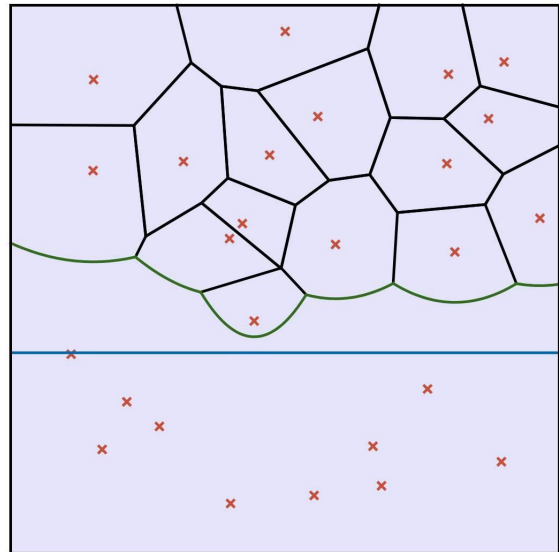
- Average case to build complete diagram?

$O(n \log n)$



## Sweep line algorithm for Voronoi construction

- Sort the points by y-axis
- Move a “sweep line” from top to bottom,
- Keep track of the “beach line”: all points equidistant from the sweep line & one of the points on the top. (A curve made from parabolas)
- Above the beach line the diagram has been finalized.
- *Running time:*  $O(n \log n)$





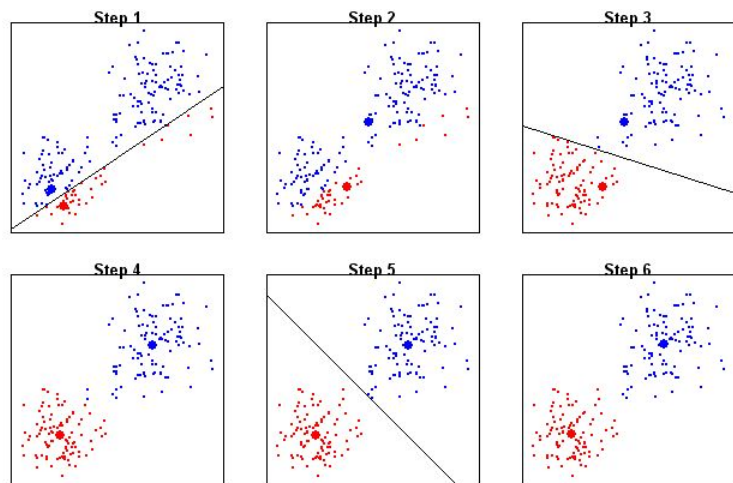
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## K-Means Clustering

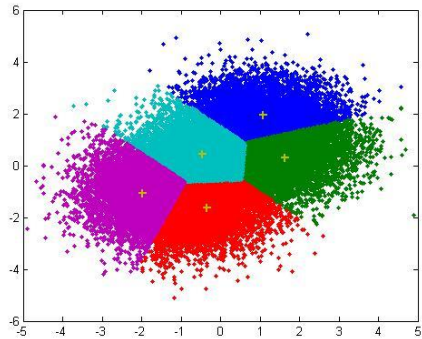
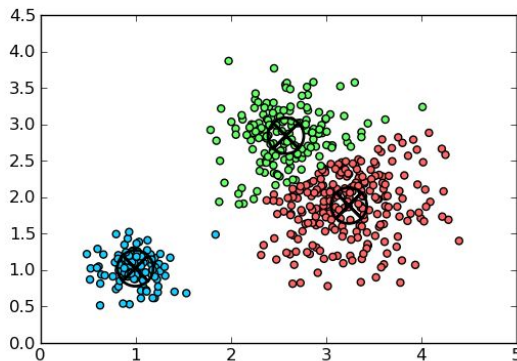
For a set of  
2D/3D/ $n$ D points:

- Choose  $k$ , how many clusters
- Select  $k$  points from your data at random as initial “team” representatives
- Every other point determines which team representative it is closest to and joins that team
- Average the positions of all team members, this is the team’s new representative
- Repeat 3-5 times until change  $<$  threshold



# K-means Clustering

- Works quite well, when the data can be meaningfully classified (and we know how many clusters to use).
- With dense data, output is visually similar to Voronoi diagram (k-Means chooses the data points that define the cells)



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# Homework Assignment 4:

## Data Collection & Data Preparation

- Identify an interesting (to you) and interestingly large, possibly high-dimensional data source that is currently “untapped”
  - Shouldn’t just be a “download a file” dataset
  - Strongly encouraged to be related to your research!
- Use your programming skills to:
  - Obtain the data (what should you collect?)
  - Parse/Organize/Simplify/Post-Process the data
- Visualize the data using a tool that’s new to you... Excel, LineUp, Tableau, Google Analytics, or Plotly
  - Try Parallel Coordinates? Convex Hull? Voronoi Diagram? k-Means Clustering?
- *Teams of 2 encouraged! [but optional]*  
*(... perhaps someone you didn’t know before this class?)*

## Reading for Tuesday

*Post a comment/question  
on LMS by Tuesday @10am*

"Angular Brushing of Extended  
Parallel Coordinates",  
Hauser, Ledermann, and  
Doleisch, InfoVis 2002

