

Problem Set 1 TA comments

1. Some students tried to use mathematical induction for problem 3.7.

Base case is trivial, but nobody succeeded to prove the induction step.

Two things are commonly overlooked in the induction step:

- (1). The number of nodes should always be even, so in the induction step 2 more nodes should be added;
- (2). After adding two nodes to the graph of k nodes, the previous graph may change. (e.g. one previous node with degree $k/2$ can be connected to the two new nodes and disconnected to one of its previous neighbor node, and its degree is increased from $k/2$ to $k/2+1$.)

In this case the induction hypothesis is not applicable since the subgraph (consisting of previous k nodes) of the new graph may have nodes with degree less than $k/2$ (e.g. $k/2-1$).