Instructions: To solve these problems, you are allowed to consult your class textbook (The Design of Approximation Algorithms by Williamson and Shmoys) and notes, but no other sources. This exam is individual work; you should not talk about it with anyone except the course instructor. Please write clearly and concisely.

(1) [10 points] You are a crafty spy in charge of a spy agency, and are choosing a team for a long-term mission. You are given a universe set $U$ of the spies that you can choose for your mission, with $|U| = n$. You are also given a set of $m$ tasks that your spies will have to perform on your mission. Each task $i$ can only be performed by either set $S_i \subseteq U$, or a set $T_i \subseteq U$. Therefore, for every $i = 1 \ldots m$ you must choose either $S_i$ or $T_i$ to include in your team. Your goal is to minimize the total size of your team, i.e., the union of all the sets chosen.


(2) [10 points] Consider the following problem.

**Dominating Set:** We are given an undirected graph $G = (V, E)$, as well as weights $w(v)$ for each $v \in V$. For each vertex $v \in V$, define $N(v) = \{w \in V | (v, w) \in E\} \cup \{v\}$. In other words, $N(v)$ is the set of all neighbors of $v$, plus the node $v$ itself. A dominating set $C \subseteq V$ is a set of vertices such that for every vertex $v \in V$, either a neighbor of $v$ is in $C$, or $v$ itself is in $C$. The goal is to find a minimum-weight dominating set.

(a) Give an LP-relaxation and its dual for the Dominating Set problem. Your primal linear program should have a variable $x_v$ for every $v \in V$; those are the only variables you should need for that LP.

(b) Let $\Delta$ be the maximum degree of $G$ (assume $\Delta \geq 1$). Give a primal-dual approximation algorithm for Dominating Set with an approximation ratio of $\Delta$.

Hint: This algorithm should add only a single new node to $C$ in every iteration. Begin by forming a $(\Delta + 1)$-approximation algorithm, and then improve it to a $\Delta$-approximation.