Answer **ALL** questions. You may use one double sided $8\frac{1}{2} \times 11$ crib sheet. NO COLLABORATION or electronic devices. Any violations result in an F. NO questions allowed during the test. Interpret and do the best you can. You **MUST** show **CORRECT** work, **even on multiple choice questions**, to get credit.

**GOOD LUCK!**

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Circle one answer per question. 10 points for each correct answer.

(a) Compute the sum \( \sum_{n=1}^{4} 3^n \).

A 120.

B 121.

C 242.

D 243.

E None of the above.

(b) What is the last digit of 3^{11}?

A 1.

B 3.

C 7.

D 9.

E None of the above.

(c) A graph has degree sequence \([6, 6, 3, 3, 3, 2, 2]\). How many edges does this graph have?

A 12.

B 25.

C 30.

D Not enough information to say.

E Such a graph does not exist.

(d) Suppose a connected planar graph has 18 vertices, each of degree 3. Into how many regions does any planar representation of this graph split the plane?

A 6.

B 11.

C 27.

D 40.

E None of the above.

(e) Compute 102^{1211} \mod 5.

A 0

B 1

C 2

D 3

E 4
(f) Which of the following numbers evenly divides $102^{211} - 3^{211}$?

A 5  
B 17  
C 2  
D 99  
E None of the above

(g) The negation of “If Lassie vomits then she ate grass or she is sick” is:

A If Lassie didn’t eat grass and is healthy, she will not vomit.  
B Lassie vomited and did not eat grass and is not sick.  
C When Lassie eats grass or is sick, she does not vomit.  
D Lassie did not vomit and she ate grass and is sick.  
E None of the above.

(h) Which claim below is true?

A If $x, y \in \mathbb{Q}$ then $y^x \in \mathbb{Q}$.  
B $x$ is odd if and only if $x^2 - 1$ is divisible by 8.  
C If $p$ is prime, then $k^p - k$ is not divisible by $p$, for any integer $k$.  
D None of these claims are true.  
E All of these claims are true.

(i) Which of the following asymptotic relationships is correct?

A $(n + 1)! \in O(n!)$.  
B $(n + 1)! \in \omega(n!)$.  
C $(n + 1)! \in o(n!)$.  
D $(n + 1)! \in \Theta(n!)$.  
E None of the above.

(j) Which of the following recursions defines a sequence $T_n$ satisfying $T_n \in \Theta(2^n)$?

A $T_1 = 2; T_n = T_{n-1}^2$ for $n > 1$.  
B $T_1 = 2; T_n = 2 + 2T_{n-1}$ for $n > 1$.  
C $T_1 = 2; T_n = 2nT_{n-1}$ for $n > 1$.  
D All of the above.  
E None of the above.
Let $p$ be prime. Consider an integer $b \in [1, p - 1]$. Use Bezout’s Theorem to show that there exists an integer $x \in [1, p - 1]$ that satisfies $bx \equiv 1 \mod p$. 

Prove or disprove: every graph with $n$ vertices and $n - 1$ edges is a tree.
For any positive integer \( k \), prove that \( 1^k + 2^k + \cdots + n^k \in \Theta(n^{k+1}) \).
Let $A_n = \underbrace{1 \cdots 1}_n$ for $n \geq 1$. Notice that $A_n = 10A_{n-1} + 1$ for $n \geq 2$. Use induction to show that $A_n \equiv 3 \text{ mod } 4$ when $n \geq 2$. 
Determine the type of proof, and prove: every odd natural number is the difference of two perfect squares.
SCRATCH