MIDTERM: 120 Minutes

Last Name: _______________________
First Name: ______________________
RIN: ____________________________
Section: _________________________

Answer ALL questions. You may use one double sided $8\frac{1}{2} \times 11$ crib sheet.

NO COLLABORATION or electronic devices. Any violations result in an F.

NO questions allowed during the test. Interpret and do the best you can.
You MUST show CORRECT work, even on multiple choice questions, to get credit.

GOOD LUCK!

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Circle one answer per question. 10 points for each correct answer.

(a) Compute the sum \( \sum_{n=1}^{4} 5^{n+1} \).

A. 3900.
B. 3901.
C. 3905.
D. 3906.
E. None of the above.

(b) Compute \( 20^{20} \mod 7 \)

A. 1.
B. 3.
C. 4.
D. 6.
E. None of the above.

(c) A graph has degree sequence \([6, 6, 4, 3, 3, 2, 2]\). How many edges does this graph have?

A. 13.
B. 25.
C. 30.
D. Not enough information to say.
E. Such a graph does not exist.

(d) Suppose a connected planar graph has 4 vertices and splits the plane into 3 regions. Which of the following are possible degree sequences for the graph?

A. \([2, 2, 2, 2]\).
B. \([3, 3, 3, 3]\).
C. \([3, 3, 2, 2]\).
D. None of the above.
E. No such graph exists.

(e) What is the last digit of \(103^{192}\).

A. 0
B. 1
C. 2
D. 3
E. 4
(f) Which of the following numbers evenly divides $569 - 1$?

A 4  
B 5  
C 11  
D 23  
E None of the above

(g) The converse of “If induction is appropriate then the base case is true and the inductive step holds” is:

A If the base case is false and the inductive step is false, then induction is not appropriate.
B If the base case is false or the inductive step does not hold, then induction is not appropriate.
C If induction is not appropriate, then the base case is false or the inductive step does not hold.
D If the base case is true and the inductive step holds then induction is appropriate.
E None of the above.

(h) Which claim below is not true?

A $2n^2 + n \in \Theta(n^2)$.
B $4^n \in \Theta(2^n)$.
C $f \in \Theta(n)$ and $g \in \Theta(n) \Rightarrow f + g \in \Theta(n)$.
D None of these claims are true.
E All of these claims are true.

(i) Suppose $f(x) > 0$ for all $x$, and $f(i + 1)/f(i) \leq r$, where $0 < r < 1$. For which of the following $g$ is $\sum_{i=1}^{n} f(i) \in \Theta(g(n))$?

A $g(n) = 1$.
B $g(n) = 2^r$.
C $g(n) = \ln(r)$.
D $g(n) = r^n$.
E None of the above.

(j) Which of these sums are $O(n^2)$: (a) $\sum_{i=1}^{n} (1 + i)^2$ (b) $\sum_{i=1}^{n} 2^i$ (c) $\sum_{i=1}^{n} \frac{i}{1+r^i}$ (d) $\sum_{i=1}^{n} (-1)^i i$ ?

A $a, b$  
B $c$  
C $a, b, c$  
D $a, c$  
E $c, d$
Let $d = \gcd(m, n)$, where $m, n > 0$. Bezout’s Theorem gives $d = mx + ny$ where $x, y \in \mathbb{Z}$. Prove or disprove that it is always possible to find $a, b \in \mathbb{Z}$ for which $ax + by = 1$. 
A leaf is a vertex with degree 1. Let $\Delta$ denote the maximum degree in a tree $T$. Use the hand-shaking theorem to prove that $T$ has at least $\Delta$ leaves.
Prove, or disprove: $n! \in \Theta(2^n)$. 
For $k \in \mathbb{N}$, show that $2^k + 1$ and $2^k - 1$ are relatively prime.
Let \( n \geq 1 \) be a natural number. Prove that \( 2^{(1/2)^n} \) is not rational.
SCRATCH