Answer **ALL** questions.

**OPEN BOOK** (notes, assignments, and textbook) and electronic devices allowed.

**NO COLLABORATION** or Internet use. Any violations result in an F.

**NO questions** allowed during the test. Interpret and do the best you can.

**GOOD LUCK!**

Circle at most one answer per question.

**10 points** for each correct answer.

You **MUST** show **CORRECT** work to get full credit.

**When in doubt, TINKER.**

| Total | 200 |
1. What is the expected number of times a six appears when a fair die is rolled ten times?

   A $\frac{2}{3}$
   B $\frac{1}{6}$
   C $1\frac{2}{3}$
   D $1\frac{1}{3}$
   E None of the above

2. A test has twenty-five multiple-choice questions worth four points each and fifty True-False questions worth two points each. The probability that Katie answers a multiple choice question correctly is 0.8 and for a True-False question this probability is 0.9. What is her expected score on the test?

   A 200
   B 150
   C 100
   D 170
   E None of the above

3. We roll $n$ fair dice. The $i$-th dice has $x_i$ sides, so takes on one of the values $1, 2, \ldots, x_i$. What is the expected sum of the values of these $n$ dice?

   A $\frac{n}{2} + \frac{1}{2} \sum_{i=1}^{n} x_i$
   B $\frac{n}{2} + \sum_{i=1}^{n} x_i$
   C $\frac{1}{2} \sum_{i=1}^{n} x_i$
   D $\frac{n+1}{2}$
   E None of the above

4. $X$ is a random variable that represents a roll of a fair six-sided die. What is the variance of $X$?

   A $\frac{7}{6}$
   B $\frac{11}{6}$
   C $\frac{19}{4}$
   D $\frac{21}{6}$
   E $\frac{35}{12}$

5. Which of the following are countable?
   (I) $\mathbb{Z} \times \mathbb{Z} = \{(u,v) \mid u \in \mathbb{Z} \text{ and } v \in \mathbb{Z}\}$
   (II) The set of unrecognizable languages
   (III) The set of solvable problems

   A I & III
   B I only
6. Which of the following strings match the regular expression \(\{0,01\}^* \cdot \{1,10\}^*\)?

(I) 101110 (II) 00111 (III) 00100 (IV) 01100

[A] II and IV  
[B] III  
[C] all except IV  
[D] all except I  
[E] all

7. What is the correct relationship between the cardinalities of these sets:

(I) \(\mathcal{A}\), the set of all languages  
(II) \(\mathcal{I}\), the interval \([0,1]\)  
(III) \(\mathcal{C}\), the set of C programs that compile successfully and halt eventually when run

[A] \(|\mathcal{C}| = |\mathcal{A}| < |\mathcal{I}|\)  
[B] \(|\mathcal{C}| = |\mathcal{A}| = |\mathcal{I}|\)  
[C] \(|\mathcal{I}| < |\mathcal{C}| = |\mathcal{A}|\)  
[D] \(|\mathcal{A}| = |\mathcal{I}| \leq |\mathcal{C}|\)  
[E] \(|\mathcal{C}| < |\mathcal{A}| = |\mathcal{I}|\)

8. Consider the following DFA. Which of these strings will it accept:  
(I) 011011  
(II) 100110  
(III) 111101

[A] I & II  
[B] II & III  
[C] III only  
[D] I only  
[E] none
9. Which is the following claims is true about the language $L = \{ \omega \# \omega^R \# \omega \mid w \in \{0,1\}^* \}$?

A. Its complement is regular
B. It is not decidable but is recognizable
C. It is context-free
D. It is not context-free but is decidable
E. It can be recognized with a PDA

10. Which of the following languages will not be accepted by this DFA?

\[
\text{start} \rightarrow q0 \xrightarrow{0} q1 \xrightarrow{0} q2 \xrightarrow{1} \text{end}
\]

A. $\{00\} \cdot \{1\}^*$
B. $\{00\} \cdot \{1^*00\}^*$
C. $\{0\} \cdot \{1\}^* \cdot \{0\} \cdot \{0\}^* \cdot \{1\}$
D. $\{100\} \cdot \{100\}^*$
E. $\{0\} \cdot \{10\}^* \cdot \{01\}$

11. If $L_1$ and $L_2$ are both undecidable but recognizable languages, which of the following are also recognizable:
   (I) $L_1^c$ (II) $L_1 \cap L_2$ (III) $L_1 \cup L_2$

Hint: Given recognizers for $L_1$ and $L_2$, how could you build recognizers for these languages?

A. I
B. I and II
C. II
D. II and III
E. III

12. How many strings of length four are accepted by this DFA?

\[
\text{start} \rightarrow q0 \xrightarrow{1} q1 \xrightarrow{0} \text{end}
\]

A. 5
B. 6
C. 8
D. 10
E. 12
13. Generate a random two digit binary string by choosing each digit independently and identically, selecting zero with probability $1/3$ and one with probability $2/3$. What is the probability that the automaton from the previous problem will accept a string generated in this manner?

A $\left(\frac{2}{9}\right)$  
B $\left(\frac{4}{9}\right)$  
C $\left(\frac{5}{9}\right)$  
D $\left(\frac{1}{3}\right)$  
E $\left(\frac{7}{9}\right)$

14. If the complement of a language is countable, which of the following are necessarily true: (I) the language is regular (II) the language is decidable (III) the language is context-free

A all  
B none  
C II only  
D I only  
E III only

15. Describe the language generated by this CFG.

1: $S \rightarrow A1B$
2: $A \rightarrow \varepsilon | 0A$
3: $B \rightarrow \varepsilon | 0B | 1B$

A The set of strings that starts with zero and contains a one
B The set of strings with an odd number of zeros
C The set of strings containing a one
D The set of strings with more ones than zeros
E None of the above

16. Consider the CFG

1: $S \rightarrow 0 | SA$
2: $A \rightarrow AA | S1$

Which string is in the language described by this CFG?

A 10101  
B 001  
C 011  
D 101  
E None of the above

17. Which of the following CFGs generates all finite binary strings?

(I) $S \rightarrow \varepsilon | 0S | 1S$
(II) $S \rightarrow \varepsilon|1|OS|S1$
(III) $S \rightarrow \varepsilon|0|S1|SS$

A I and II
B II and III
C I and III
D I
E all three

18. If $L$ is undecidable, which of the following cannot be true?

A There is a recognizer for $L$
B $L \subseteq L_{\text{HALT}}$
C $L$ is decidable
D $L$ is countable
E Any of the above could be true

19. Which CFG generates the same language as

1: $S \rightarrow 00S1$
2: $T \rightarrow 0S1$
3: $S \rightarrow 0T$
4: $S \rightarrow \varepsilon|01$

A $S \rightarrow \varepsilon|01|00S11$
B $S \rightarrow \varepsilon|01|0S1|00S1$
C $S \rightarrow \varepsilon|01|00S1$
D $S \rightarrow \varepsilon|01|000S11$
E $S \rightarrow \varepsilon|01|000S1$

20. Under which of the following operations is the class of decidable problems closed: (I) complementation (II) union (III) intersection (IV) Kleene-Star?

Hint: how would you construct deciders for languages defined using these operations?

A all except IV
B II and III
C all except I
D all
E none