Given a heptagon (i.e., seven-sided polygon) with side length $S$ and area $A$.

If we double $S$, what is the area $A'$ of the larger heptagon?
EXAMPLES — MODELING COVID-19

A square gets infected if two or more adjacent neighbors (N/S/E/W) are infected.

Given six initial gray infections, who gets infected?
Minimum number of infections to infect everyone?
Given a finite number of vaccinations available, who should we immunize?
How do we model this?

Answering these (and other important) questions requires discrete math!
EXAMPLES — PÓLYA’S MOUSE

“A mouse tries to escape from an old-fashioned cage. After many futile attempts bouncing back-and-forth, thumping his body against the cage bars, he finally finds one place where the bars are slightly wider apart. The mouse, bruised and battered escapes through this small opening, and to his elation, finds freedom.” – Pólya

Connect tiles of the same letter with wires

Wires cannot cross, enter tiles, or leave the box

Can this be done?

If not, can you prove it is impossible?

Answering these (and other important) questions requires discrete math!

EXAMPLES — PÓLYA’S MOUSE

Understand and model the problem you are trying to solve

Tinker to better understand the problem (look for easy cases or counter-cases)

Be bold and formulate a conjecture about the problem

Prove the conjecture (if you can...)
EXAMPLES — SHUTTLE BUS TIMINGS

Given two bus stations A and B — assume buses run non-stop

Bus station A dispatches buses every six minutes

At bus station B, the manager rolls a standard six-sided die every minute and dispatches a bus if the roll is a three

How many buses are dispatched per day?
What is the average time between bus dispatches?
If you arrive at a bus stop, how many minutes do you have to wait on average?

Answering these (and other important) questions requires discrete math!
EXAMPLES — FRIENDSHIP (SOCIAL) NETWORKS

Model friendships (or business relationships or etc.) using nodes to represent people and using edges to represent the friendship relationship

Who is popular? Who is not…?

Who would you advertise a new product to?

**Definition:** in a friendship clique, everyone is friends with everyone else

How many friendship cliques are there?

What might a friendship clique represent?

**Definition:** a friendship clique with \( n \) nodes is called an \( n \)-person-clique

Answering these (and other important) questions requires discrete math!

EXAMPLES — FRIENDSHIP (SOCIAL) NETWORKS

These two graphs are equivalent (i.e., isomorphic):

Is that helpful…?
EXAMPLES — FRIENDSHIP (SOCIAL) NETWORKS

Is it possible to construct a graph without any 3-person-cliques?
If so, how many distinct non-isomorphic (i.e., different) such graphs are there? (Prove this.)
If not, can you prove such a construction cannot be done?

Definition: a connected graph is a graph in which each node
is reachable from every other node

Definition: node x is reachable from node y if there is at least one
sequence of edges leading from y to x

Construct a connected graph with 13 nodes containing exactly two 4-person-cliques

What is the minimum number of edges needed to construct this graph? (Prove this.)

This is Problem #2 of Problem Set 0
A (TRICKY) PROBLEM TO WORK ON...

Take any circle with diameter $D$ and inscribe within it an equilateral triangle with side length $L$

Select a chord at random…

Recall that a chord is a line segment with its two endpoints on the circumference of the circle.

What is the probability that the length of the chord is greater than $L$…?
WHAT NEXT...?

Tinker with the problems shown on the last few slides
Do Problem Set 0 for Wednesday (TOMORROW!)
Get the textbook…
Read the Preface and Chapters 0 and 1 of the textbook
Turn on Submitty email notifications
Re-read the syllabus slides and post any questions on the Discussion Forum
Email me directly (goldsd3@rpi.edu) about any registration/SIS issues
See you again in lecture on Friday…