ln(2) ≈ 0.693; ln(3) ≈ 1.098. What is ln(12)?

What is \( \int_0^T dx \frac{1}{1 + x^2} \)?

Define the function \( f(t) = \int_0^t dx \sin(1 + x^2e^x) \). What is \( \frac{d}{dt} f(t) \)?

Compute the limits:
\[
\lim_{x \to 0} \frac{e^x - 1}{\sin(2x)}; \quad \lim_{x \to 0} \frac{e^x - 1}{1 + x}; \quad \lim_{x \to 0} \frac{e^x - 1}{\sin(x^2)}; \quad \lim_{x \to 0} \frac{e^x - 1}{x + x^2}; \quad \lim_{x \to 0} \frac{e^x - 1}{e^{2x} - 1}; \quad \lim_{x \to \infty} \frac{e^x - 1}{e^{2x} - 1}; \quad \lim_{x \to \infty} \frac{e^x - 1}{x^3 + 2e^x}; \quad \lim_{x \to \infty} \frac{e^x}{x^x}.
\]

What are the solutions to \( x^2 - 5x + 6 = 0 \)?

\( 2^{10} \approx 10^3 \). What is \( 2^{20} \)?

What is the Taylor expansion of \( f(x) = \frac{1}{2 + \sin(x)} \) around \( x = \frac{\pi}{2} \)?

What is \( 1 + 2 + 3 + \cdots + 1000 \)? What is \( \sum_{i=1}^{1000} i \)? What is \( \sum_{k=1}^{1000} k \)? What is \( \sum_{i=1}^{1000} i \)?

What is \( 1 + 2 + 3 + \cdots + n \)? What is \( \sum_{k=1}^{n} k \)?

What is \( 1 + 2 + 3 + \cdots + k \)? What is \( \sum_{i=1}^{k} i \)?

What is \( \sum_{i=1}^{k} \ln(i) \)? What is \( \prod_{i=1}^{k} i \)? What is \( 0! \)? What is \( 2^0 \)? An empty sum is 0. An empty product is 1.

What is \( \frac{1}{7} + \frac{1}{7^2} + \frac{1}{7^3} + \frac{1}{7^4} + \cdots \)?

What is the prime factorization of 252?

What is the minimum element in the set \( \{8, 9, 3, 10, 19\} \)?

Does this set of positive numbers have a minimum element:
\[
\{25, 97, 107, 100, 18, 33, 99, 27, 2014, 2200, 23, \ldots \}
\]

\( \ldots \)
Two-Contact Ebola on a Grid

A square gets infected if two or more neighbors (N,S,E,W) are infected.
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Given initial gray infections, who ultimately gets infected?
Two-Contact Ebola on a Grid

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day 1  day 2
Two-Contact Ebola on a Grid

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Given initial gray infections, who ultimately gets infected?

day 1  day 2  day 3
Two-Contact Ebola on a Grid

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Given initial gray infections, who ultimately gets infected?

day 1  day 2  day 3  day 4
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Two-Contact Ebola on a Grid

A square gets infected if two or more neighbors (N,S,E,W) are infected.

- Given initial gray infections, who ultimately gets infected?
- Minimum infections to infect everyone?
Two-Contact Ebola on a Grid

A square gets infected if two or more neighbors (N,S,E,W) are infected.

- Given initial gray infections, who ultimately gets infected?
- Minimum infections to infect everyone?
- Given few vaccines, who to immunize?

---

day 1  day 2  day 3  day 4  day 5  day 6  day 7  day 8

Creator: Malik Magdon-Ismail
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- Given initial gray infections, who ultimately gets infected?
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- What were the “entry points”?

---

day 1  day 2  day 3  day 4  day 5  day 6  day 7  day 8
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Minimum infections to infect everyone?

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What were the “entry points”?

Answering such questions involves discrete math.
Scheduling Speed Dates

In each round 4 people “group”-speed-date around a table. (4 rounds in all)
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**Answering such questions involves discrete math.**
People are circles and links are friendships.
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Who would you advertise to? You wish to maximize adoption of your new technology.
Friendship Networks and Ads

People are circles and links are friendships.

Who would you advertise to? You wish to maximize adoption of your new technology.

Answering such questions involves discrete math.
General Model. Desktop in your dorm-room, smartphone in your hand, fitbit, . . .
Simple Model. So we can analyze it.
**General Model.** Desktop in your dorm-room, smartphone in your hand, fitbit, ...  
**Simple Model.** So we can analyze it.

We wish to ask deep questions:

1. What can we compute?
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We wish to ask deep questions:

1. What can we compute?
2. What *can’t* we compute?
Modeling Computers

**General Model.** Desktop in your dorm-room, smartphone in your hand, fitbit, . . .

**Simple Model.** So we can analyze it.

We wish to ask deep questions:

1. What can we compute?
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3. Are there things we can compute in principle, but it takes too long?
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We wish to ask deep questions:

1. What can we compute?
2. What *can’t* we compute?
3. Are there things we can compute in principle, but it takes too long?

To answer these questions, we must become good at discrete math.
“A mouse tries to escape from an old fashioned cage. After many futile attempts bouncing back-and-forth, thumping his body against the cage bars, he finally finds one place where the bars are *slightly* wider apart. The mouse, bruised and battered escapes through this small opening, and to his elation, finds freedom.” – Polya
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Connect tiles of the same letter with wires. Wires cannot cross, enter tiles, or leave the box. How can it be done? If it can’t be done, why not?
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To solve such problems, “You need brains and good luck. But, you must also sit tight and wait till you get a bright idea.” – Polya.