General Announcements

- Quiz 1 will be graded and handed back in 2/28 recitations (and solutions will be posted)

Recursive Programs

Prove by induction that $T_n = 3n + 2$.

Base case. $T_0 = 3(0) + 2 = 2$, which is $T$.

Induction step. Assume $T_n = 3n + 2$ is $T$.

Then, $T_{n+1} = T_n + 3 = 3n + 2 + 3 = 3(n + 1) + 2$.

By induction, $T_n = 3n + 2$ for $n \geq 1$.

- Correctness: Does the program compute what you want it to?
- To prove correctness, use induction.

- Runtime efficiency: Does the program compute “quickly enough” for us to see results?
- To analyze runtime, solve a recurrence

Recurrences

A recurrence is a recursive function defined on natural numbers.
Exercise 7.12

Give recursive definitions for the set $S$ in each of the following cases.

(a) $S = \{3^0, 3^1, 3^2, 3^3, 3^4, \ldots \}$, i.e., the powers of 3.

(b) $S = \{\text{all binary strings that are palindromes}\}$, e.g., 010, 110011, 000, etc.

(c) $S = \{\text{all strings of matched parentheses}\}$, matched in the usual arithmetic sense,
  e.g., $[[]]$ and $[]$ are matched, but $[[]]$ and $[]]$ are not matched.

(a) Basis: $1 \in S$
  Constructor: $x \in S \rightarrow 3x \in S$

(b) Basis: $\varepsilon, 0, 1 \in S$
  Constructor: $x \in S \rightarrow 0x0 \in S$ and $x \in S \rightarrow 1x1 \in S$

(c) Basis: $\varepsilon \in S$
  Constructor: $x, y \in S \rightarrow [x]y \in S$

Rooted Binary Trees

A rooted full binary tree (RFBT) is a rooted binary tree (RBT) such that each node that is not a
leaf node has two child nodes.

Assume that we reach the given depth (e.g., 4 or $n$) completely.