General Announcements

• Quiz 1 was handed back in 2/28 recitations; also in lecture 3/1; pick up your Quiz 1 in my office hours

• Quiz 1 solutions are posted on the course website

• Homework 5 due by 11:59PM Tuesday 3/6

• Our Midterm Exam will be Wednesday 3/7 (6:00-7:50PM in DCC 308)

• Homework solutions will be posted by Saturday (bring grading questions/grievances to your recitation TA during his/her office hours)

• Please check (refresh/reload) the weekly schedule before attending office hours; rooms may change

Getting Help in this Course

• Look to the textbook, also to Rosen’s recommended textbook

• Look to the specific examples in the textbook

• Look to Piazza for answers (and keep posting specific questions)

• Attend any/all recitations

• Attend any/all office hours

• Work on warm-up, recitation, and old problems with your classmates

• Practice
Handshaking Theorem (Proof)

Proof. We prove that any graph with \( m \) edges has \( \sum_{i=1}^{n} \delta_i = 2m \) by induction on \( m \).

**Base case.** For \( m = 0 \), every \( \delta_i = 0 \), so \( \sum_i \delta_i = 0 = 2m \).

**Induction step.** Assume the claim is true for any graph with \( m \) edges and consider an arbitrary graph \( G \) with \( m + 1 \) edges.

Let \( e = (v_1, v_2) \) be an edge. Remove edge \( e \), keeping endpoint vertices \( v_1 \) and \( v_2 \), thereby forming another graph \( G' \) with \( m \) edges.

Let \( \delta_i \) and \( \delta'_i \) be the vertex-degrees in \( G \) and \( G' \), respectively. Since \( G' \) has \( m \) edges, by the induction hypothesis, the sum of its vertex degrees equals \( 2m \):

\[
2m = \sum_{i=1}^{n} \delta'_i = \delta'_1 + \delta'_2 + \sum_{i=3}^{n} \delta'_i = \delta'_1 + \delta'_2 + \sum_{i=3}^{n} \delta_i
\]

Since we removed edge \( e \), \( \delta'_1 = \delta_1 - 1 \) and \( \delta'_2 = \delta_2 - 1 \).

Therefore,

\[
2m = (\delta_1 - 1) + (\delta_2 - 1) + \sum_{i=3}^{n} \delta_i = \sum_{i=1}^{n} \delta_i - 2
\]

Rearrange this to obtain: \( 2(m + 1) = \sum_{i=1}^{n} \delta_i \).

Since \( G \) was an arbitrary graph with \( m + 1 \) vertices, the theorem follows by induction.

Graph Theory — Proofs by Induction

When we apply induction to a graph problem, we typically remove an edge or vertex to get a smaller graph (to which you apply the induction hypothesis).

We must start from a general larger graph and perform this removal. Do not start from a general smaller graph and add a vertex or edge, because such an addition does not guarantee that we have a general larger graph.

Bipartite Graph Matching

The neighborhood of left-vertex \( x \) is denoted \( N(x) \) and contains all right-vertices linked to \( x \).

Cycles in a Graph

Define a cycle as a path consisting of vertices and edges that do not repeat except for one vertex.