General Announcements

- Midterm Exam average: 74.6/100 (236 exams)
- Exams are organized by section and sorted by RCS ID
- If you did not pick up your exam in class 3/29, feel free to pick up your exam during my office hours
- For multiple choice regrades, please see any TA or me during office hours
- For questions about grading on Q11, please see Shruthi during office hours
- For questions about grading on Q12/Q13, please see Dan during office hours
- For questions about grading on Q14/Q15, please see Mengwen during office hours
- Note that all midterm exam grades will be increased by 2 points for Question 11(a) — this will happen automatically in “Rainbow Grade,” which will hopefully be available by Monday

Pop Quiz 4 — Question 3

A box has 10 coins, of which six are fair coins and four are biased with a probability of heads being $\frac{2}{3}$. What is $P[\text{exactly two heads}]$ in each case below:

(a) Pick a single random coin and flip it three times.

**SOLUTION:** Draw a tree of outcome possibilities (showing all possible outcomes). Then label each edge with the appropriate edge-probability.

Or use the Law of Total Probability:

$P[A] = P[A|B] \times P[B] + P[A|\overline{B}] \times P[\overline{B}]$

Case 1. $B$: You picked one of the fair coins
Case 2. $\overline{B}$: You picked one of the biased coins

Note that $A$ represents the given problem (i.e., pick a single random coin and flip it three times)

$P[B] = \frac{6}{10}$

$P[\overline{B}] = \frac{4}{10}$

$P[A|B] = \frac{3}{8}$

(i.e., three equally likely cases HHT, HTH, THH out of eight)

$P[A|\overline{B}] = \frac{3}{8}$

(i.e., three cases HHT, HTH, THH, each with probability $\frac{2}{3} \times \frac{2}{3} \times \frac{1}{3}$)
Plugging everything in yields:

$$P[A] = P[A|B] \times P[B] + P[A|\overline{B}] \times P[\overline{B}]$$

$$= \frac{3}{8} \times \frac{6}{10} + \frac{4}{9} \times \frac{4}{10}$$

$$= \frac{29}{72}$$

(b) Repeat three times: pick a coin at random, flip it, then put the coin back in the box.

**SOLUTION:** Draw a tree of outcome possibilities (showing all possible outcomes!).

Then label each edge with the appropriate edge-probability.

Hmmmm, this is a lot of work....

Consider one flip.

The probability of one flip landing heads is:

$$\frac{4}{10} \times \frac{2}{3} + \frac{6}{10} \times \frac{1}{2} = \frac{17}{30}.$$  

Therefore, the probability of one flip landing tails is:

$$1 - \frac{17}{30} = \frac{13}{30}.$$  

We know that the three cases of interest are: HHT, HTH, and THH.

The probability of HHT is $$\frac{17}{30} \times \frac{17}{30} \times \frac{13}{30} = \frac{3757}{27000}.$$  

The same probability applies to HTH and THH.

Therefore, the probability of the three flips landing heads is:

$$3 \times \frac{3757}{27000} = \frac{11271}{27000} \approx 0.4174.$$  

See the example simulation code on the course website....