General Announcements

• Quiz 2 is on Wednesday 4/11 from 6:00PM to 7:50PM in DCC 308 (see the course website for more details)
• If you did not pick up your midterm exam in class 3/29, feel free to pick up your exam during my office hours
• For multiple choice regrades, please see any TA or me during office hours
• For questions about grading on Q11, please see Shruthi during office hours
• For questions about grading on Q12/Q13, please see Dan during office hours
• For questions about grading on Q14/Q15, please see Mengwen during office hours
• Note that all midterm exam grades will be increased by 2 points for Question 11(a) — this will happen automatically in “Rainbow Grade,” which will hopefully be available by next Monday 4/9 (nope, but later this week)
• No office hours for me on Thursday 4/12
• NO CLASS ON THURSDAY 4/12

Homework 5 — Solutions to Chapter 11 Problems

11. [7 POINTS] Problem 11.3: A graph is regular if every vertex has the same degree. Which of these graphs are regular: $K_6; K_{4,5}; K_{5,5}; L_6; S_6; W_4; W_5$?

**SOLUTION:** In a complete graph (i.e., $K_6$), each vertex is linked to every other vertex. Therefore, each vertex has the same degree and the graph is regular.
In a complete bipartite graph on $n$ and $\ell$ vertices (i.e., $K_{4,5}$ and $K_{5,5}$), each of the $n$ vertices is linked to every one of the $\ell$ vertices. Therefore, if $n = \ell$ (i.e., $K_{5,5}$), the graph is regular.
In a path or line (i.e., $L_6$), consecutive vertices are linked, with two endpoint vertices. Therefore, the degree sequence is always $[2, 2, \ldots, 2, 1, 1]$. These graphs are not regular.
In a star (i.e., $S_6$), a central vertex is linked to every other vertex. Therefore, in $S_{n+1}$, the central vertex has degree $n$ and all other vertices have degree 1. These graphs are not regular.
In a wheel (i.e., $W_4$ and $W_5$), a central vertex is linked to every other vertex; further, the non-central vertices connect to form a cycle. Therefore, in $W_{n+1}$, the central vertex has degree $n$ and all other vertices have degree 3. Given this pattern, only $W_4$ is regular.

Though not part of this problem, note that a cycle (i.e., $C_n$) is a path with an additional link connecting the first and last vertices. All vertices in a cycle have degree 2; therefore, all such graphs are regular.
12. **[6 POINTS] Problem 11.8(a)-(c) [or F17 Problem 11.7(a)-(c)]:** Compute the number of edges in the following graphs: (a) $K_n$; (b) $K_{n,\ell}$; (c) $W_n$.

**SOLUTION:**

(a) $K_n$ is a complete graph, i.e., each vertex is linked to every other vertex. Each vertex has degree $n - 1$. Therefore, the number of edges is $\frac{n(n-1)}{2}$. Note that we divide by 2 here because we do not want to double-count each edge.

(b) $K_{n,\ell}$ is a complete bipartite graph on $n$ and $\ell$ vertices, i.e., each of the $n$ vertices is linked to every one of the $\ell$ vertices. Therefore, the number of edges is $n \times \ell$.

(c) $W_n$ is a wheel, i.e., a central vertex is linked to every other vertex; further, the non-central vertices connect to form a cycle. Each of the $n - 1$ vertices on the cycle contributes 2 edges, thus the number of edges is $2(n - 1)$. 