

Combinatorial Identities

One of my fascinations is with combinatorial Identities. Especially the ones I happen upon while doing other things. Here are some of the ones that I have discovered. Many such identities and general methods for deriving identities amongst combinatorial coefficients can be found in the beautiful little book by Graham et al, [1, Chapter 5].

1.

$$\sum_{r=0}^n r \binom{2n}{n+r} = \frac{n}{2} \binom{2n}{n}$$

2.

$$\sum_{r=0}^n \binom{2n}{r} = \frac{1}{2} \binom{2n}{n} + 2^{2n-1}$$

3.

$$\sum_{r=0}^n \frac{2n+1}{2n+1-r} \binom{2n}{r} = 2^{2n}$$

4.

$$\sum_{r=0}^{n-\alpha} \frac{2n+1-2r}{2n+1-r} \binom{2n}{r} = \binom{2n}{n+\alpha} = \binom{2n}{n-\alpha}$$

5.

$$\sum_{k=0}^n \binom{2k}{k} \binom{2(n-k)}{n-k} = 2^{2n}$$

References

- [1] R. L. Graham, D. E. Knuth, and O. Patashnik. *Concrete Mathematics: A Foundation for Computer Science, 2nd Edition*. Addison Wesley, New York, 1998.