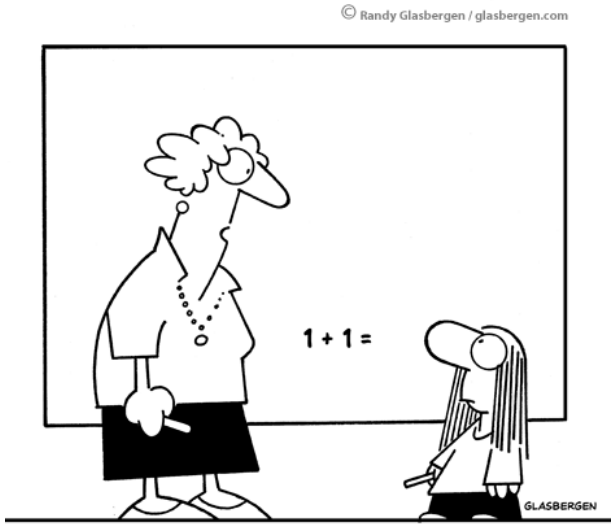
Foundations of Computer Science
Lecture 1

Warmup: A Taste for Discrete Math and Computing

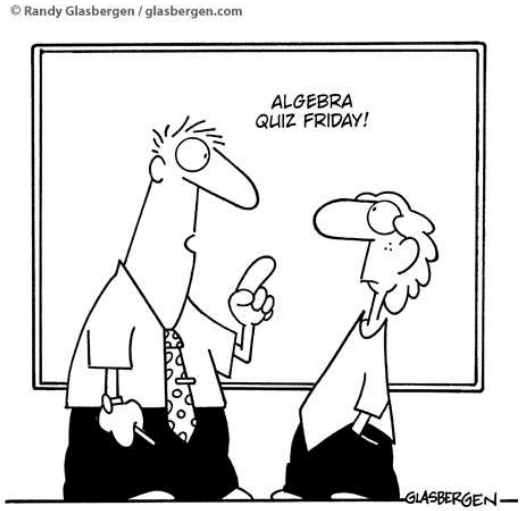
Background

Disease spread, speed-dating, friendship networks

3 Challenge Problems



"Yes, this will be useful to you later in life."



"It's important to learn math because someday you might accidentally buy a phone without a calculator."

"In 1953 you were my math teacher. You promised that algebra would come in handy someday. How much longer do I have to wait?"

(Today) Warmup: A Taste for Discrete Math and Computing

Resources and Rules

1. Web Page: www.cs.rpi.edu/∼magdon/courses/focs.php
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   CS II (data structures)
   Calc I (Calc II STRONGLY recommended)

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8. **Rules:** No food, no electronics, no cheating.

The Storyline

1. Discrete objects.

- concepts/concrete
- proof/theory/abstract
- theory of computation

our language will be mathematics . . .
. . . it will be everywhere

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2. Reasoning about discrete objects

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Background

Programming, numbers, geometry, algebra, calculus, ...

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- Does this set of *positive* numbers have a minimum element:

$$\{25, 97, 107, 100, 18, 33, 99, 27, 2014, 2200, 23, \ldots\}$$

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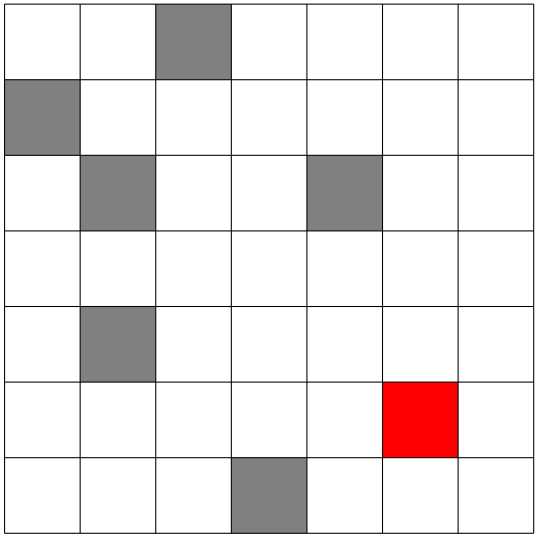
- Does this set of *positive* numbers have a minimum element:

$$\{25, 97, 107, 100, 18, 33, 99, 27, 2014, 2200, 23, \ldots\}$$

Any (non-empty) set containing only **positive integers** has a minimum element.

Two-Contact Ebola on a Grid

A square gets infected if two or more neighbors (N,S,E,W) are infected.



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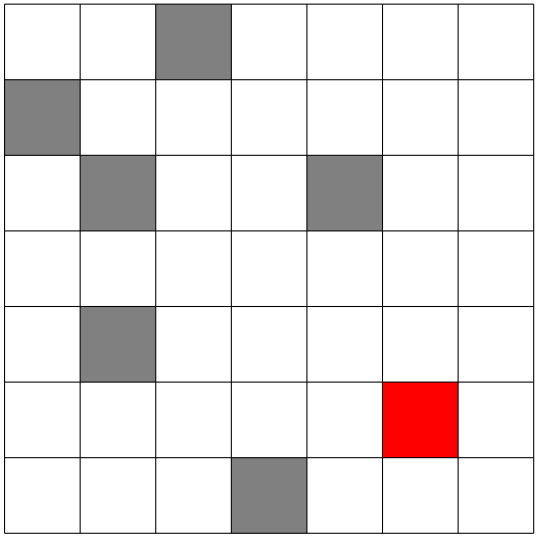

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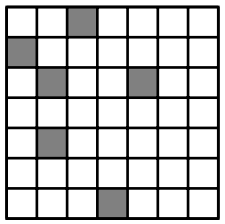

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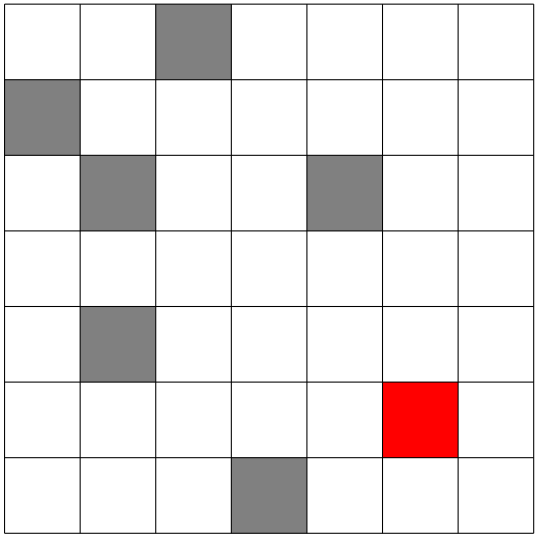

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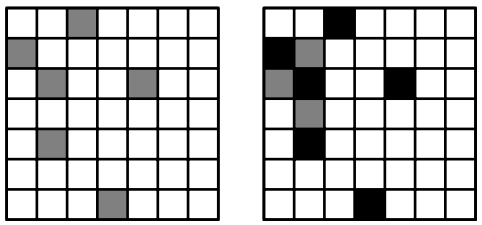

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day 2

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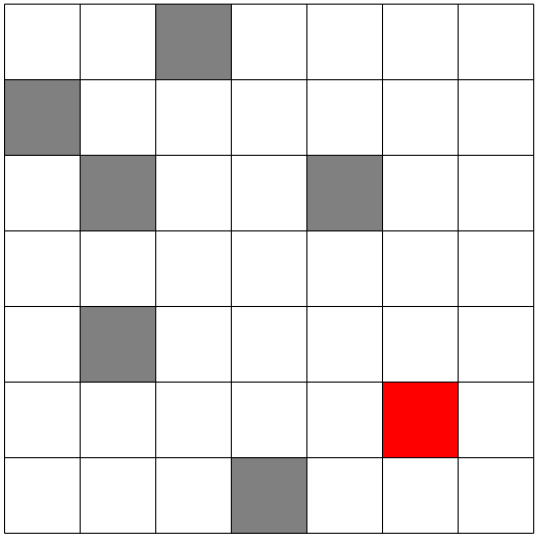

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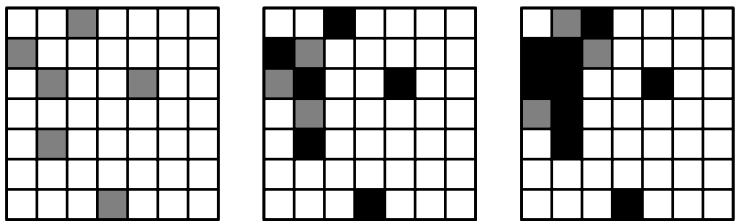

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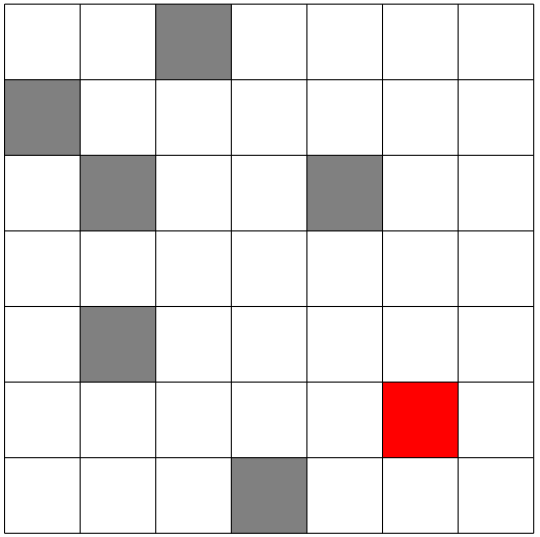

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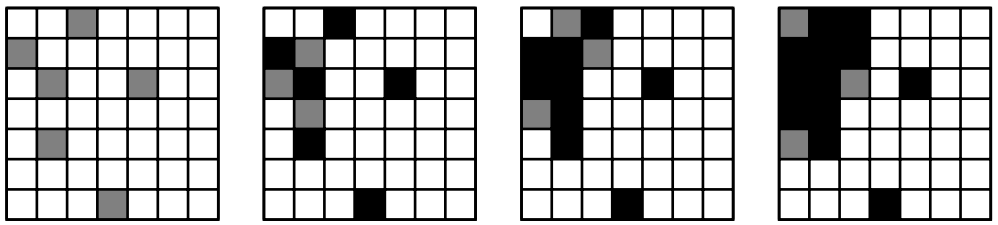

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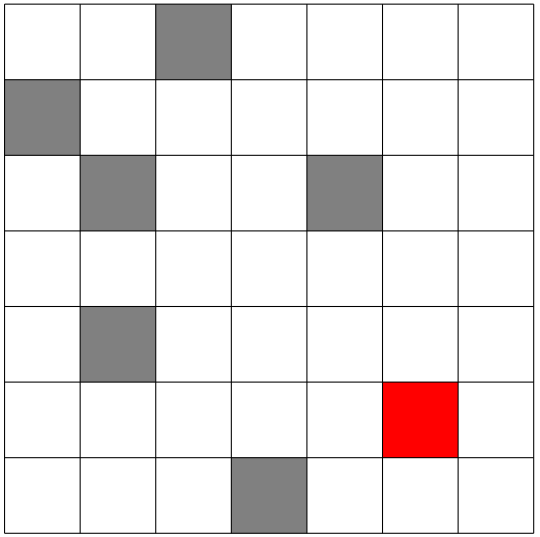

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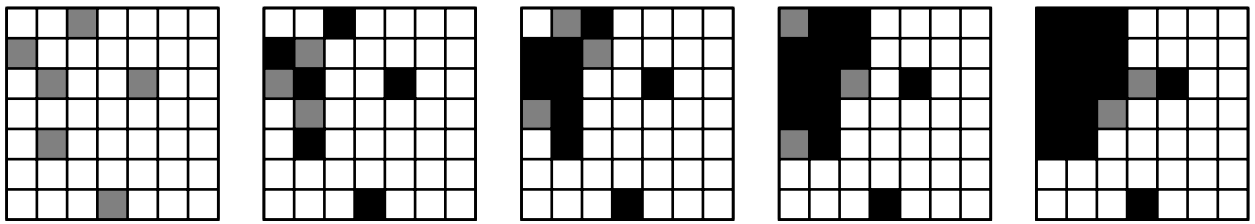

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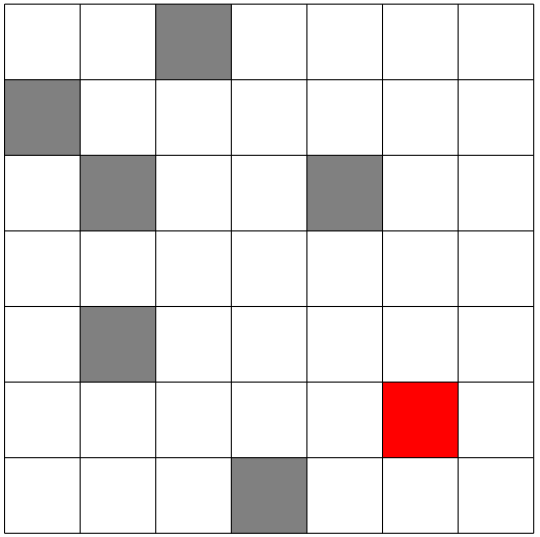

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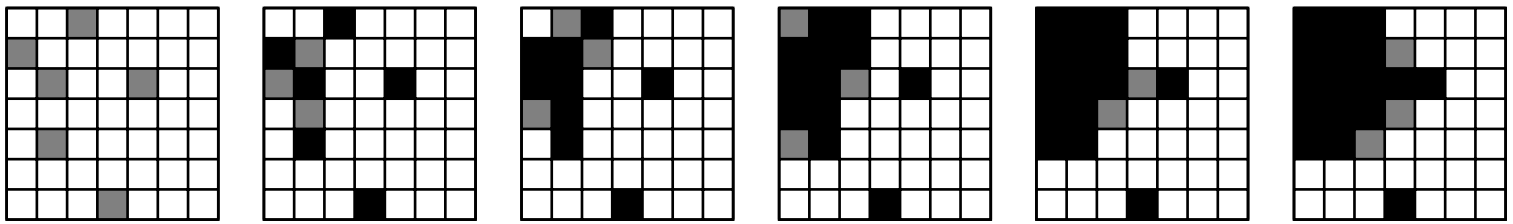

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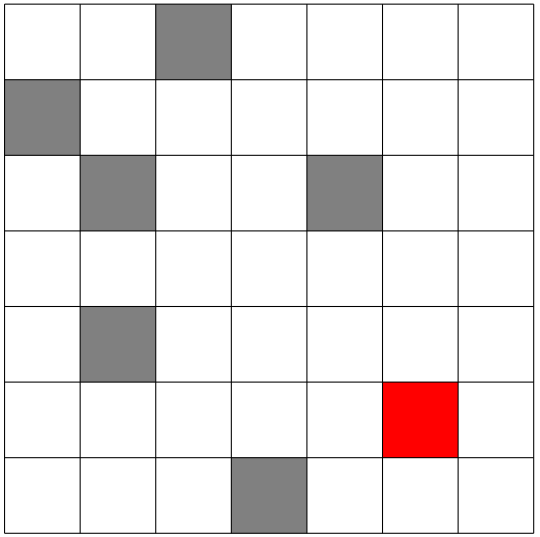

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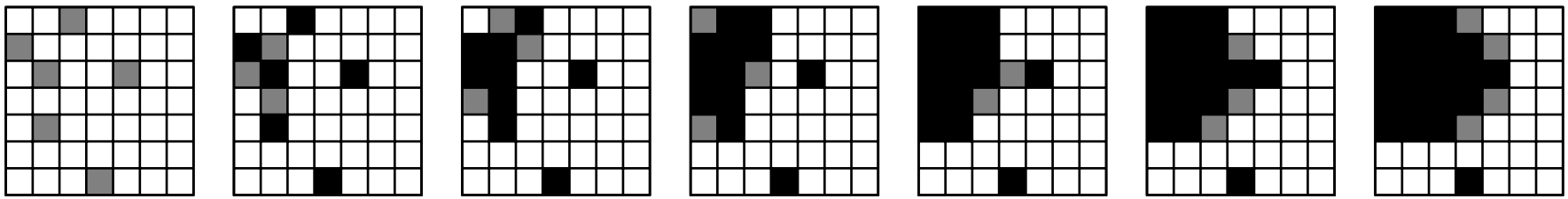

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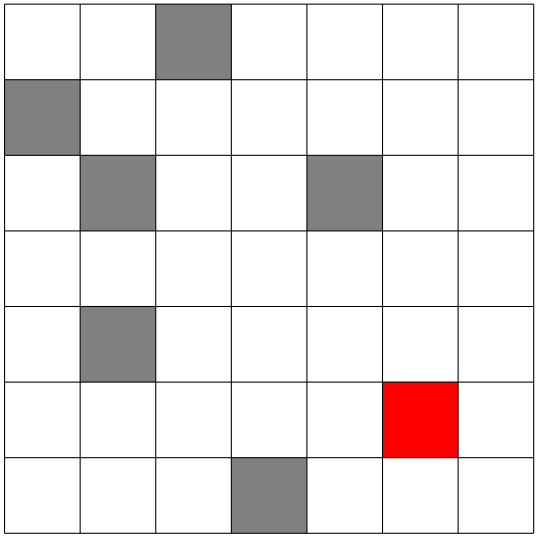

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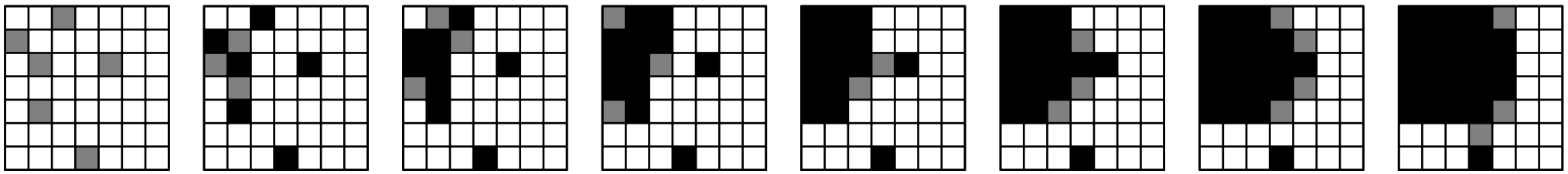

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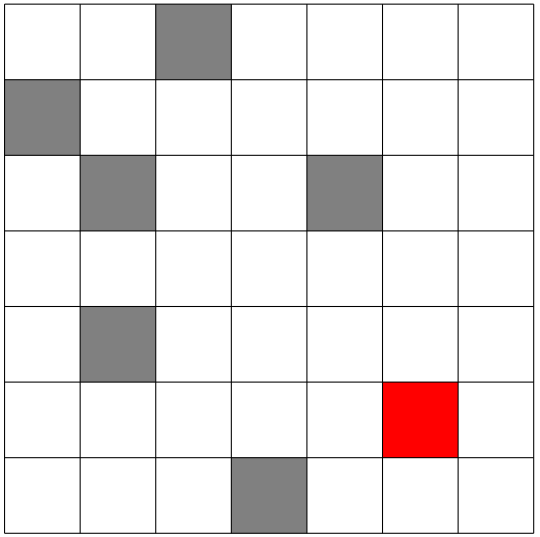
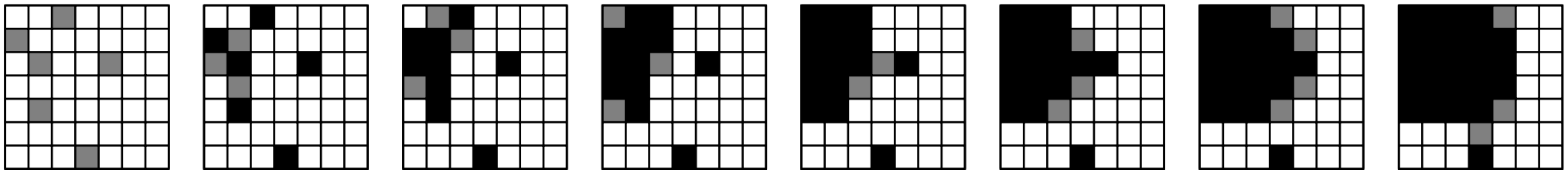


- Given initial gray infections, who ultimately gets infected?
- Minimum infections to infect everyone?



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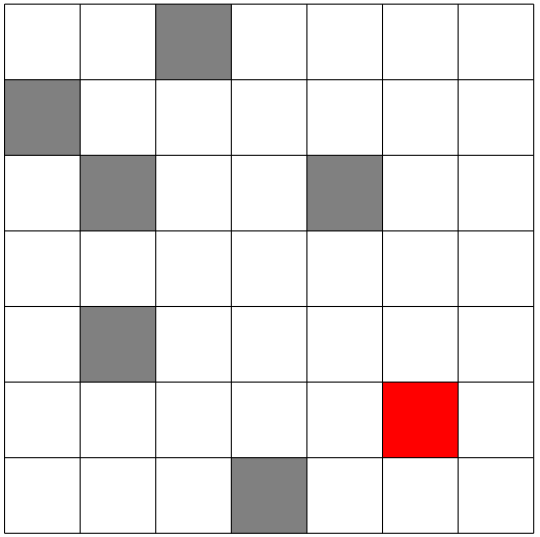

- Given initial gray infections, who ultimately gets infected?
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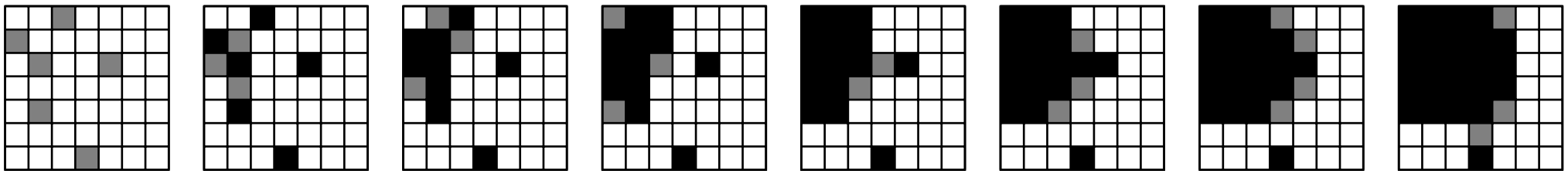
- Given few vaccines, who to immunize?



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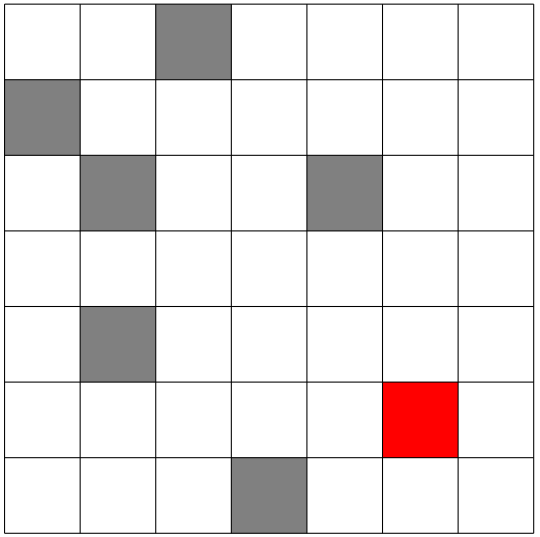

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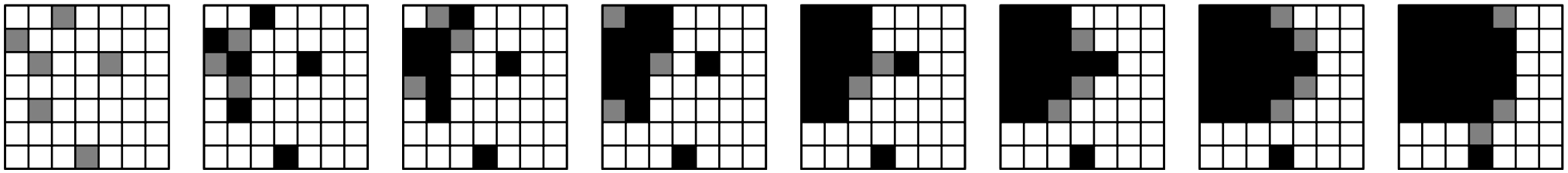

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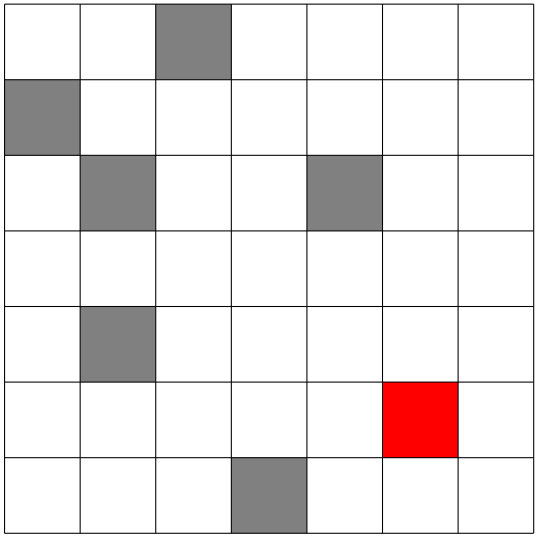

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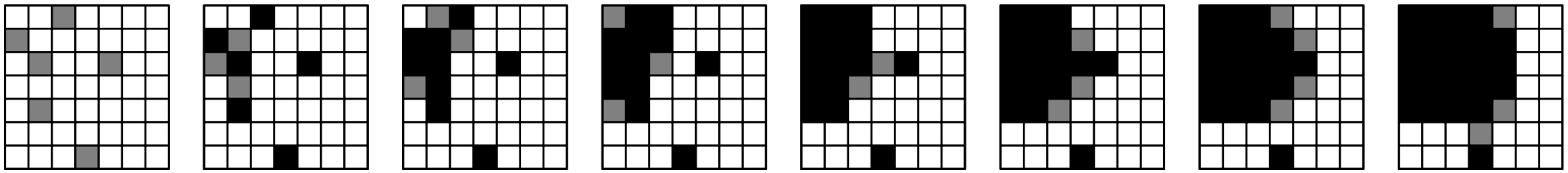
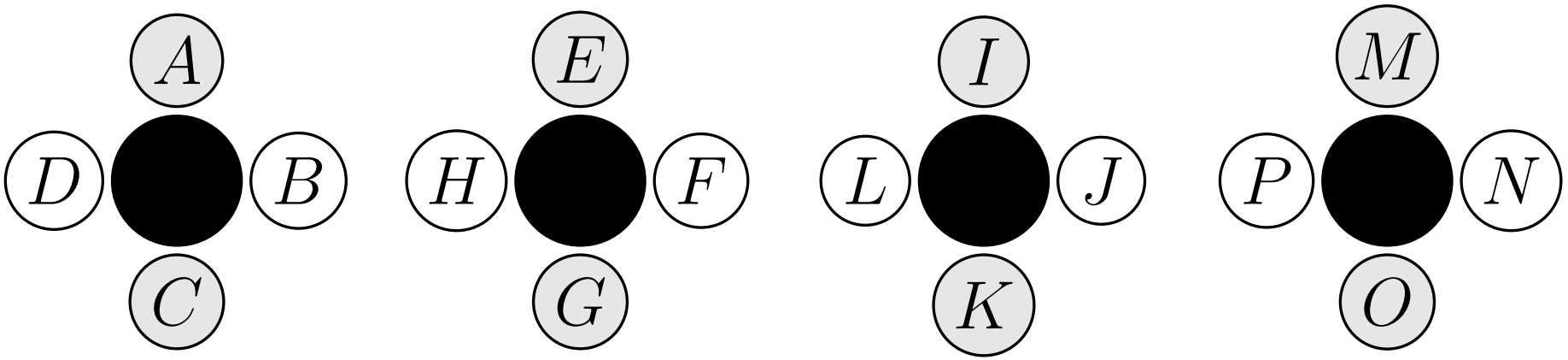


day 1 day 2 day 3 day 4 day 5 day 6 day 7 day 8

Scheduling Speed Dates

In each round 4 people "group"-speed-date around a table. (4 rounds in all)



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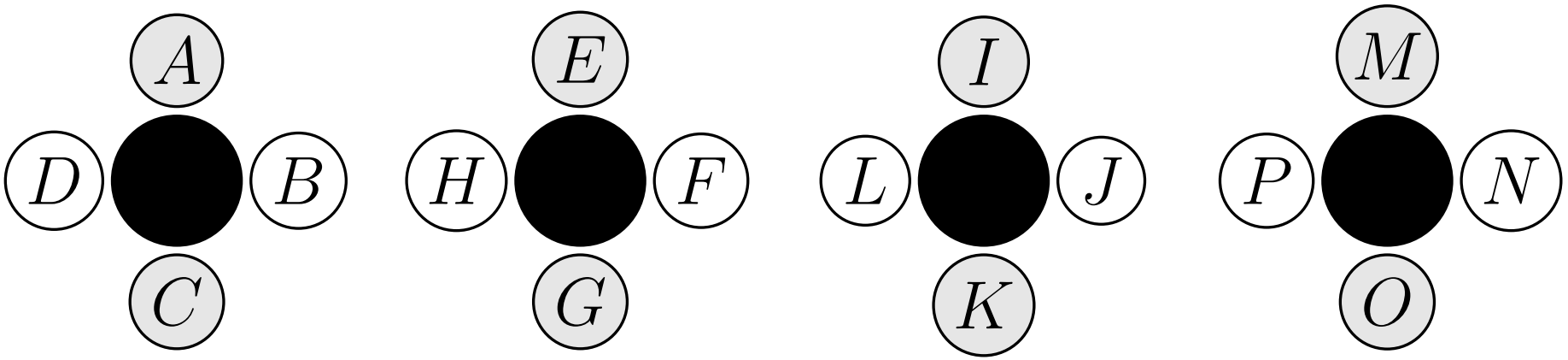

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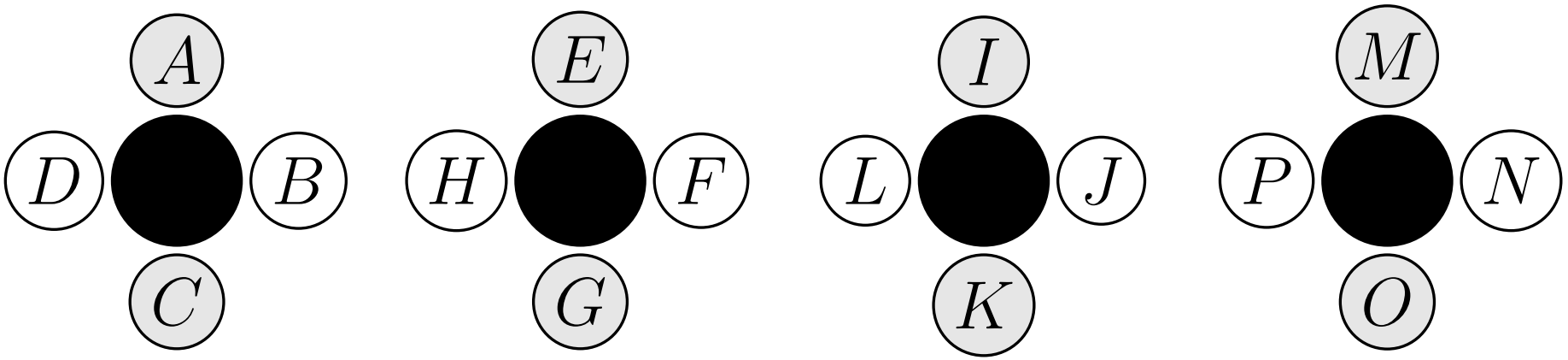

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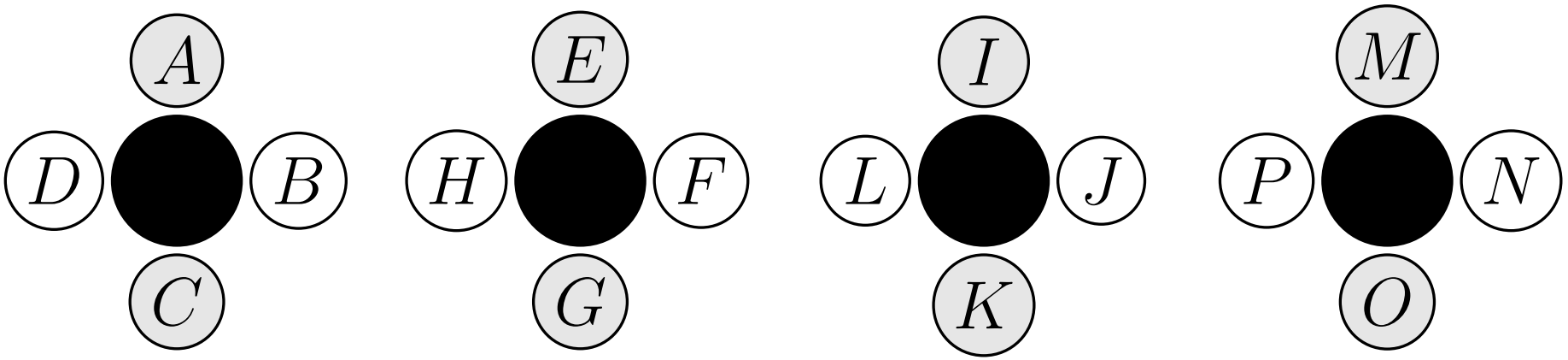

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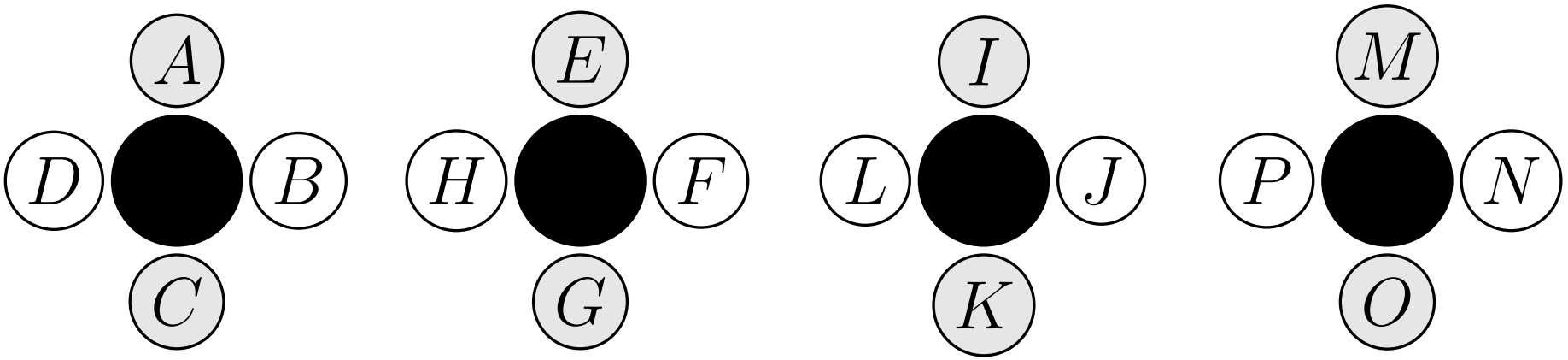

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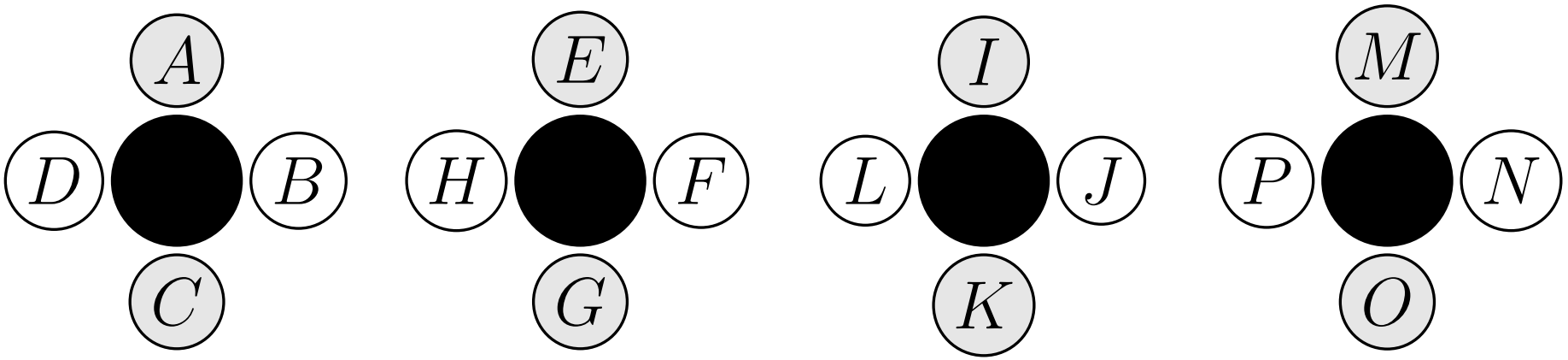

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Friendship Networks and Ads

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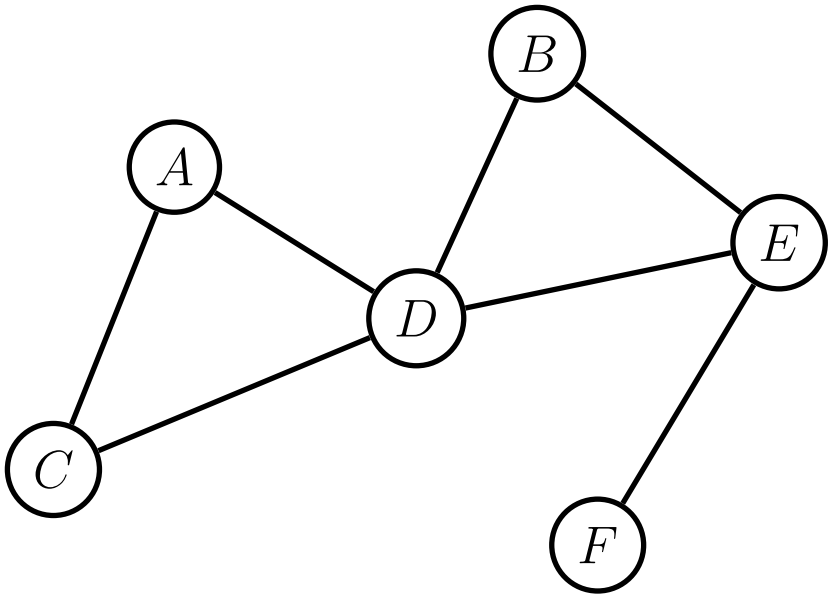

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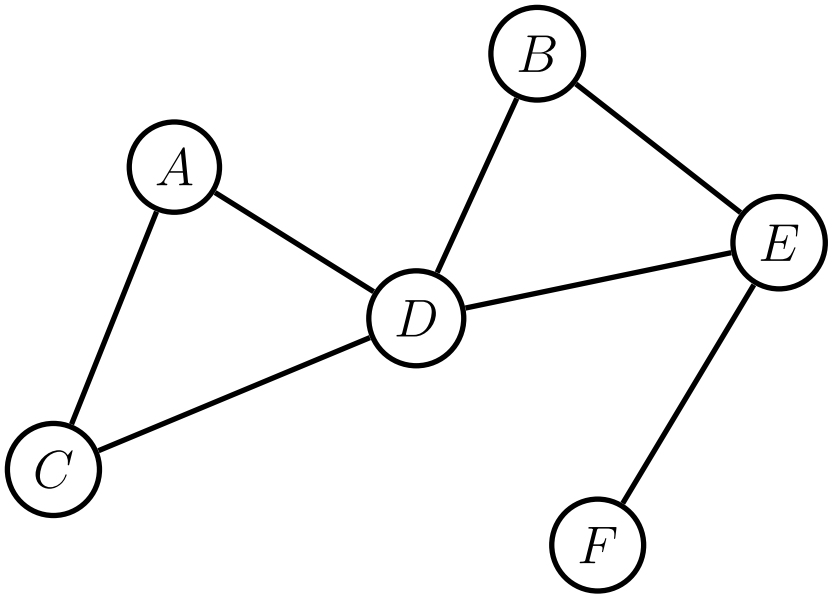

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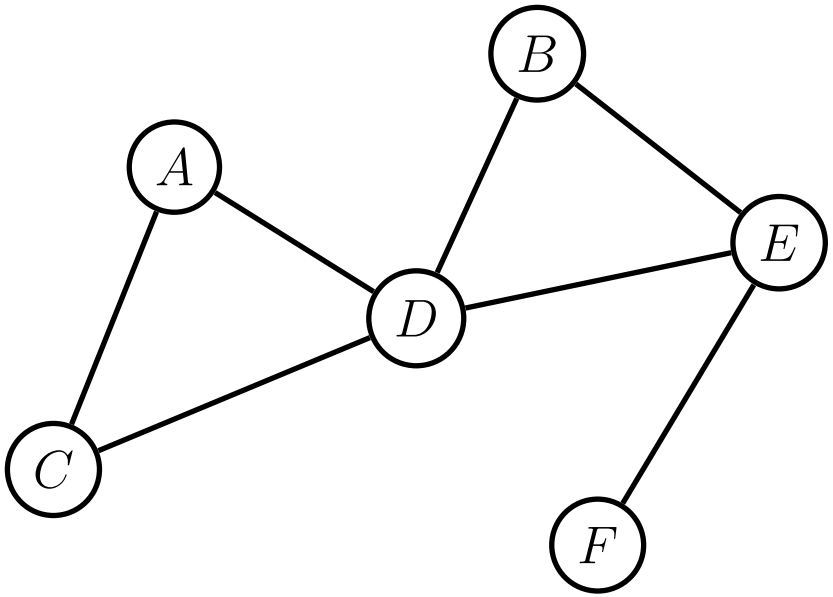

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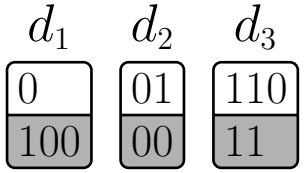
Modeling Computers

Desktop, smartphone, fitbit, ...

What is computing?

Dominos:



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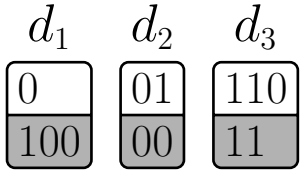
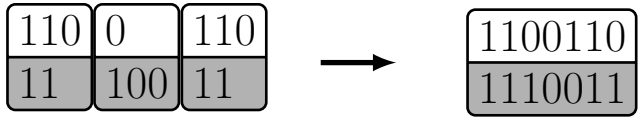


$d_3 d_1 d_3 =$



Domino puzzle: Want same top and bottom.

Modeling Computers

Desktop, smartphone, fitbit, . . .

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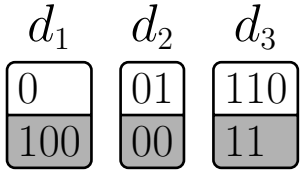

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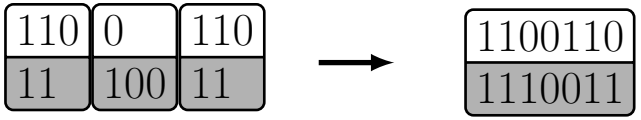

Domino puzzle: Want same top and bottom.

Domino program:

Input: dominos

Output: sequence that works

or

say it can't be done

Modeling Computers

Desktop, smartphone, fitbit, . . .

We have deep questions:

1. What can we compute?
2. What *can't* we compute?
3. How fast?

Answers involve discrete math.

Computing is Mathematics

"Too few people recognize that the high technology so celebrated today is essentially a mathematical technology."

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"The required techniques of effective reasoning are pretty formal, but as long as programming is done by people who don't master them, the software crisis will remain with us and will be considered an incurable disease. And you know what incurable diseases do: they invite the quacks and charlatans in, who in this case take the form of Software Engineering Gurus."

– Edsger Dijkstra

Polya's Mouse

"A mouse tries to escape from an old fashioned cage. After many futile attempts bouncing back-and-forth, thumping his body against the cage bars, he finally finds one place where the bars are *slightly* wider apart. The mouse, bruised and battered escapes through this small opening, and to his elation, finds freedom." – Polya

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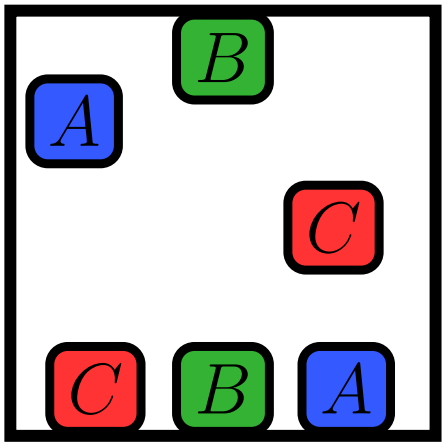

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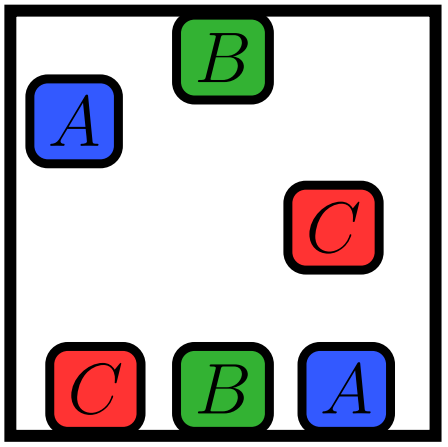

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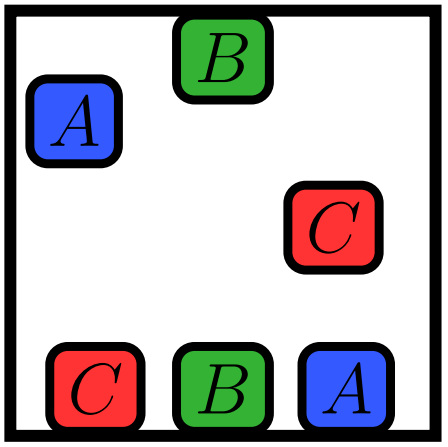
To solve such problems, "*You need brains and good luck. But, you must also sit tight and wait till you get a bright idea.*" – Polya.



Getting Good at Discrete Math

The *professional*'s workflow in addressing a discrete math problem:

1: Model the problem your are trying to solve using a discrete mathematical object.

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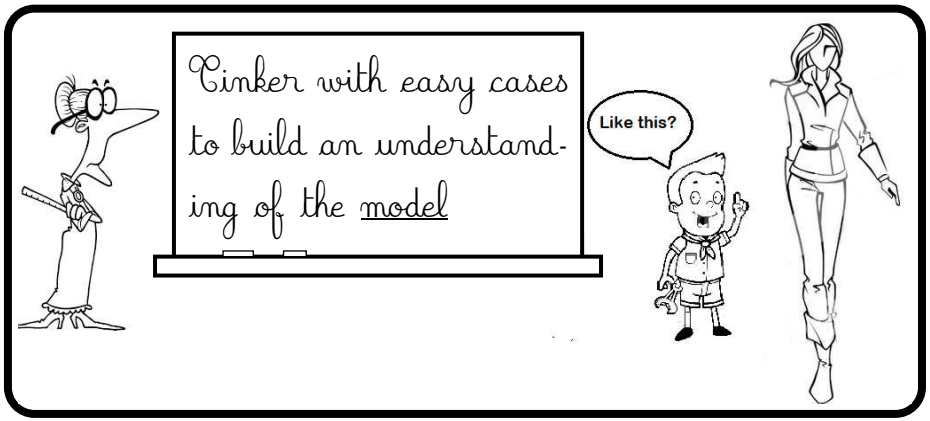
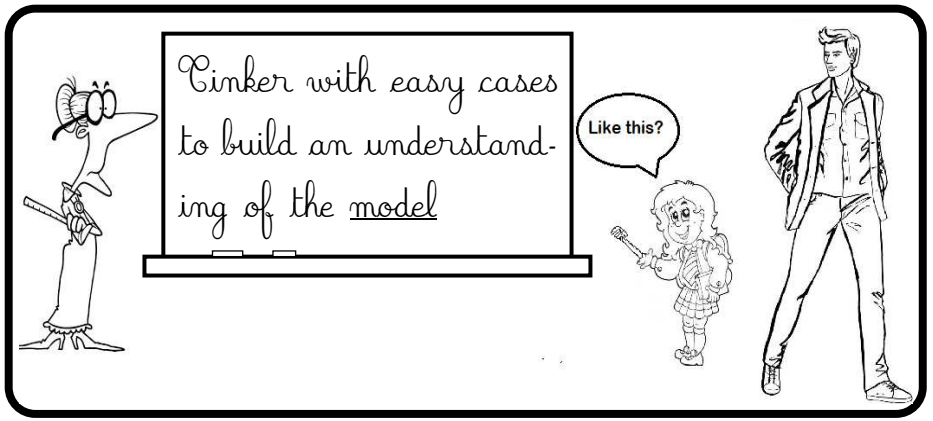
2: Tinker with easy cases to build an understanding of the model.
3: Based on the tinkering, formulate a conjecture about your problem/model.
4: Prove the conjecture and make it a theorem. You now *know* something new.

Tinker, Tinker, Tinker, Tinker!





Three Challenge Problems

$10

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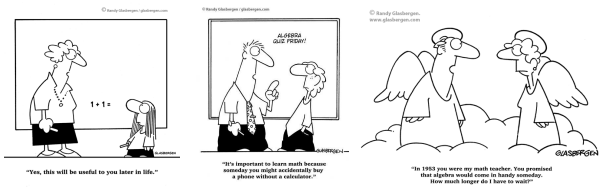

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$100

Distinct subsets with the same sum

571982539356796134655815 5629	179643969482421326695888 6393
548794588284315869667215 7984	636625253175995567694449 6585
476776653175425487422425 7763	854545854563689897436593 8274
185592435975773212586623 9784	336229118621152231856685 2576
428977642458919764751364 7977	846447386637547496734777 2855
796713196176885488959421 7186	289285756435526221996598 4217
257296727766613378922576 4888	429669393766126671538224 1936
129458714192195263969361 9381	863476461726572471638977 5433
476441363532391136169918 3586	841523424318278753412389 4858
147434364182347692266715 4474	226735325445487261618224 2154
257864976368491316342932 5833	468991184757874147318633 7883
516159698522656868197793 8754	442876678796483437179456 5542
224263269898168555152336 1879	714629518676416726843323 8125
747418961456741236751683 3398	227382381357296857746938 8278
621185567334594947174816 1445	668613272133686445763522 3349
494271623349877221925184 8674	316151829657648815899714 6221
551626435967275383653986 1178	191761142573992828514775 8625
585476271961854941776892 5747	351643153734338713535723 7754
531369117196395251812473 5471	754968465673294145694563 2221
673769175424123146975371 7635	239787667534997199495857 9984
429238861445414672824619 8812	467584425785737879299188 9317
446846371586674625897655 2344	283251524138293749861467 6246
263862173182236237316281 1879	875544277295326329936838 2378
125892226372929658978541 8839	983366282573462455573663 8328
448227972726479782765489 9397	529867125342542345461115 2788
874985532228537116298641 1895	985751287918118642182341 7538
111659945796197179668393 6952	147122614433131411447878 6593
387921327359632273599332 9751	354543937432166165138573 5599
921235913157415965716819 6759	673536761691562646227221 1264
335122318381871267369197 7472	214166575414547524965493 8214
885583532281251286889644 9976	848174725733251375828694 7416
433285948687125592255541 8653	996121723625357695279739 7966
242875158237196445338175 1663	994123799644582721866522 2824
673848186686895178788427 6161	624217749346348486191586 5966
879435317221317761293977 6215	434484351178291287584363 2652
298969424582747976915231 3629	756884256274813651861511 7797
611745442798775113146758 9412	277662155988214612511447 3423
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688421474699798597643369 5787	538758413152578761561756 3371
867182921838175741753686 2814	531769335337257228458824 2963
943115683724476832646893 8597	661214251555259366395596 6562
478844866467488588358518 4169	131492858771329249361662 5427
362475724773741477271137 2622	244682766728745168593917 3534
936181976428624318212196 3365	978669387873198453492455 8138
989331551615642258152935 4454	292671883874263477477871 3813
591362598985397528956215 8982	379142627449759664196914 2899
831389154856967281469285 8479	283172771517629996877495 1996
226586513851837911487461 3969	328128735346372529227191 6883
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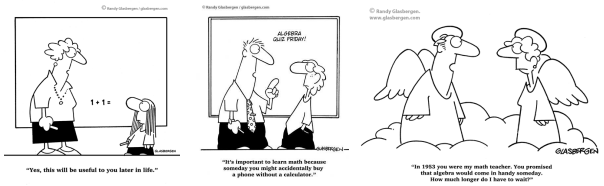

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$1,000

Domino Program

d_1	d_2	d_3
0 / 100	01 / 00	110 / 11

$d_3 d_1 d_3$ = 110 | 0 | 110 / 11 | 100 | 11

→ 1100110 / 1110011

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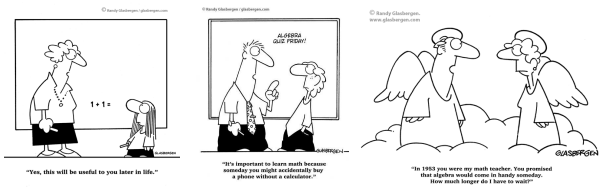

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