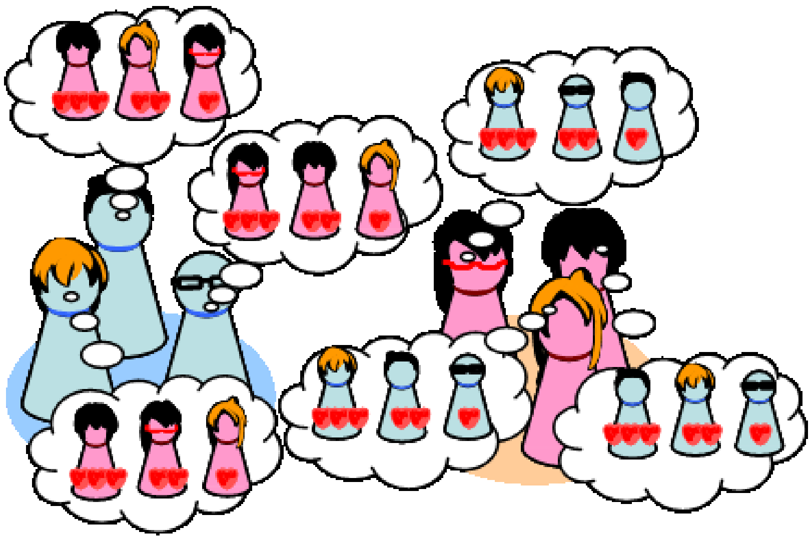
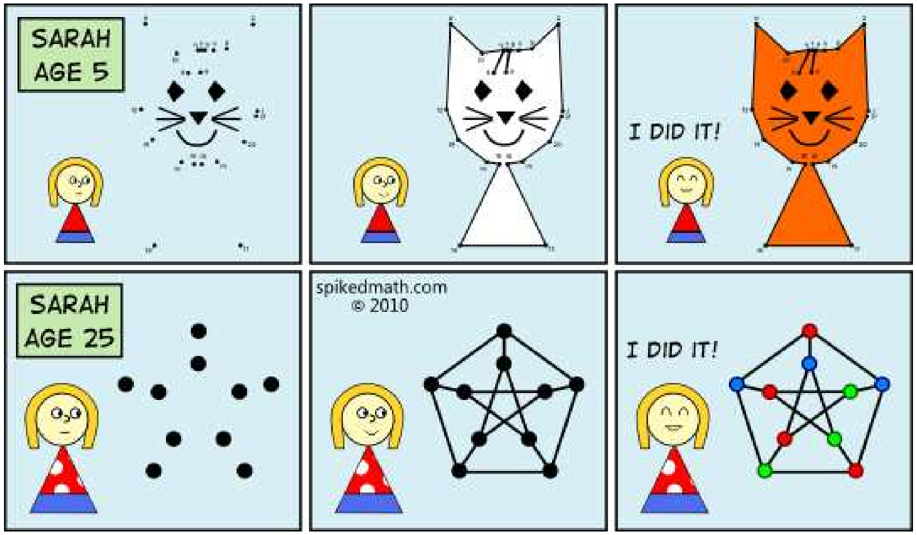
# Foundations of Computer Science
# Lecture 12

## Matching and Coloring

Bipartite Graphs and Matching
Stable Marriage
Conflict Graphs and Coloring
Other Graph Problems.





# Last Time

1. What is a graph?
2. Equivalent graphs: graph isomorphism.
3. Notation: path, degree, cycle,
4. Some common graphs: $K_n, K_{n,m}, C_n, L_n$.
5. The Handshaking Theorem: $\sum_{i=1}^{n} \delta_i = 2|E|$.
6. Different kinds of graphs: trees; planar; multigraphs; weighted; directed.
7. Problem solving with graphs: first pose the problem as a graph.

# Today: Matching and Coloring

1. Matching.
   - Sex in America.
   - Bipartite graphs.
   - Stable marriage: the mathematics of dating.

2. Coloring.
   - Conflict graphs.

3. Other graph problems.
   - Connected components, spaning tree, Euler cycle, newtork flow. (EASY)
   - Hamiltonian cycle, facility location, vertex cover, dominating set. (HARD)

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**Which survey is right?**

*Mathematicians should stick to numbers. Real people run the world.* No, No, No!

Contentious sensational issues are exactly where mathematics is needed. We *must* face them head on with cold reason instead of flaring emotions.

# Sex In America: Resolving the Issue

Modeling asumptions:

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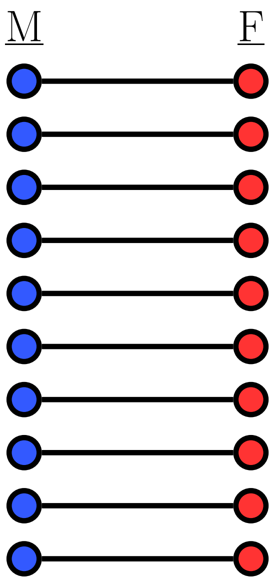

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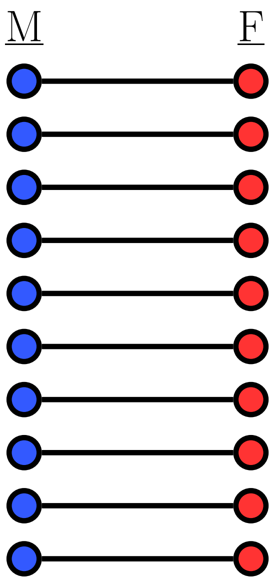


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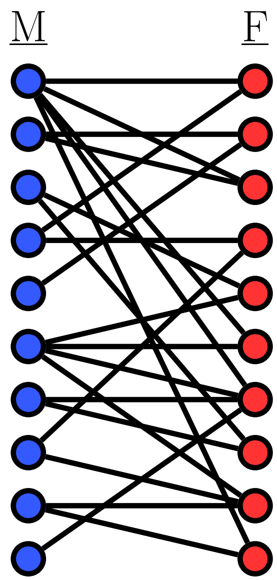

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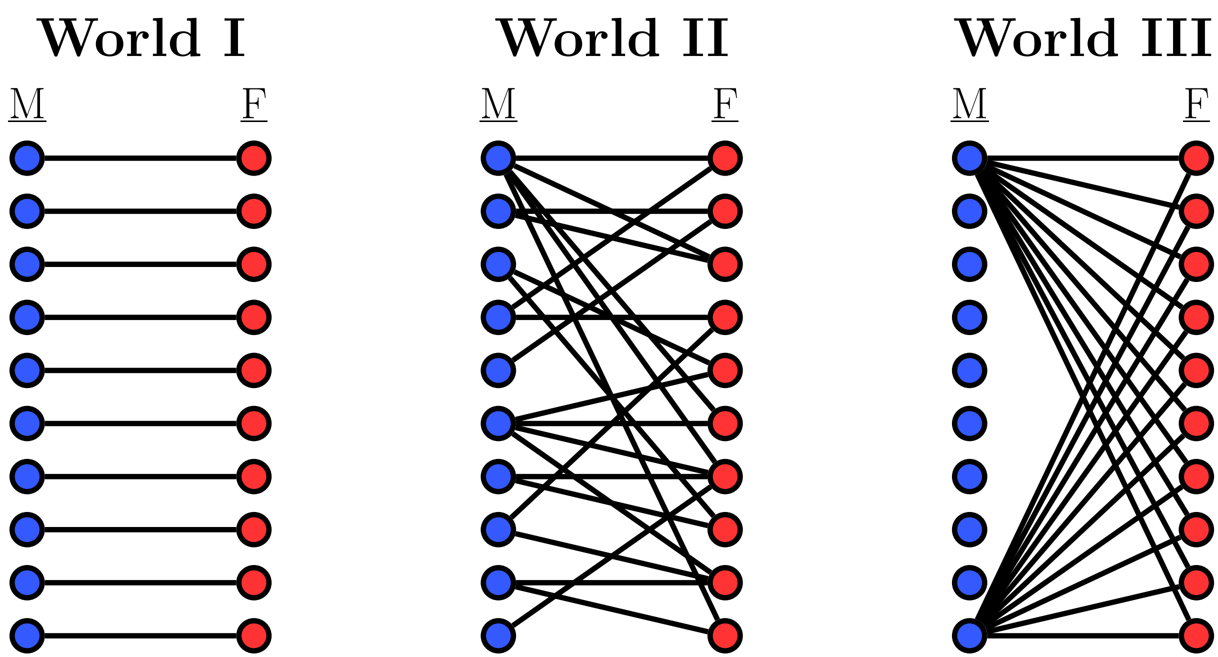

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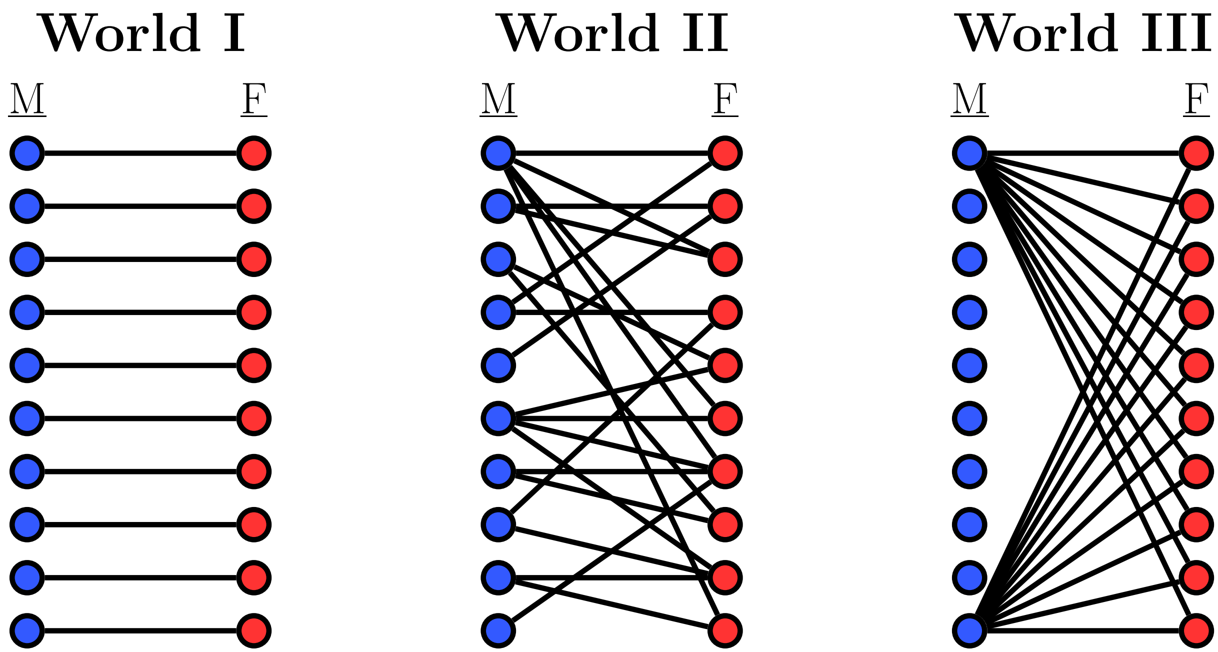


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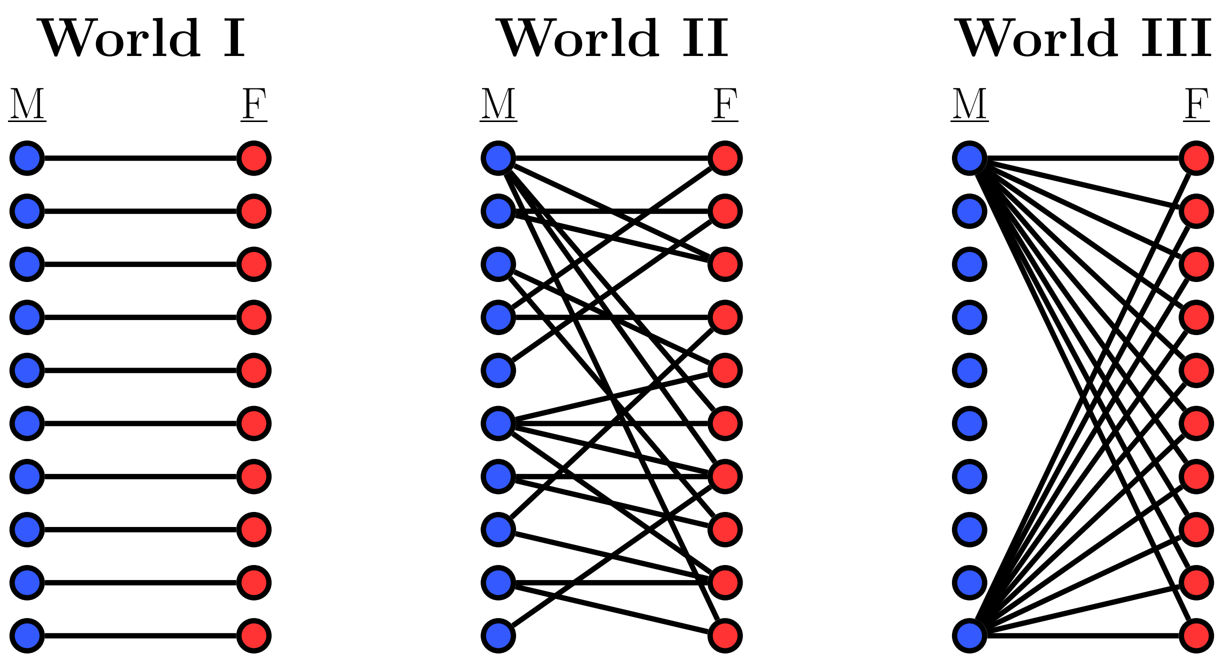


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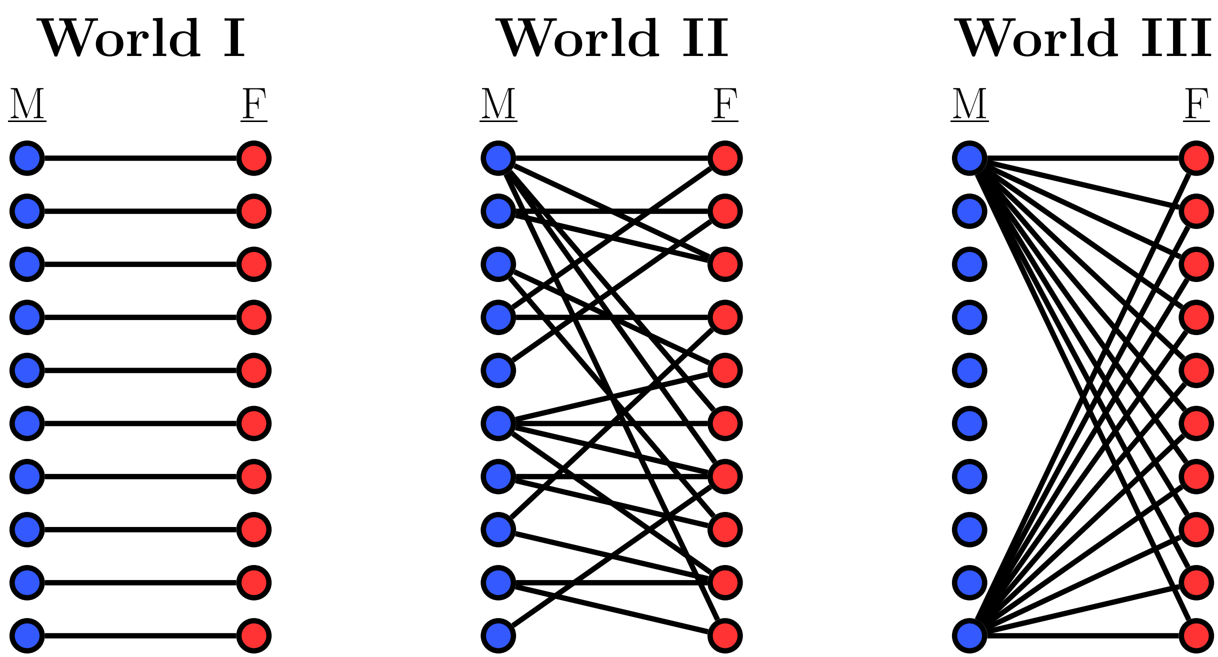


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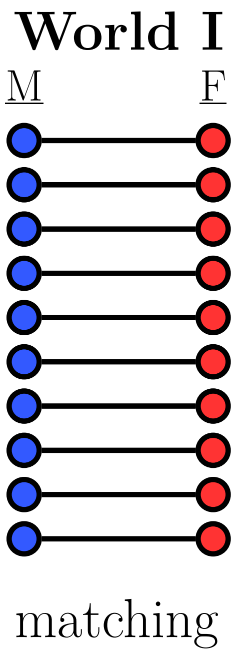
**SOS:** "Now, there is a basic adding up constraint that these gender differences seem to violate. Logically, men should have the same number of female sex partners as women have male sex partners. We note that this inconsistency has been found, as well, in several other surveys in recent years in the United States, the United Kingdom, France, Finland and elsewhere. The inconsistency constitutes an important puzzle for which we, like others, have no good answer."

# Bipartite Matching and Hall's Matching Condition



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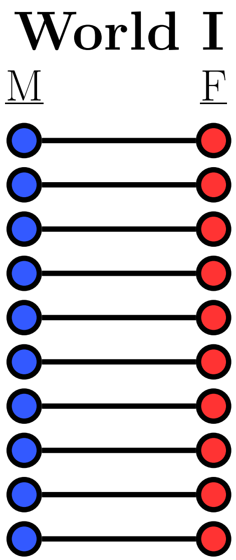


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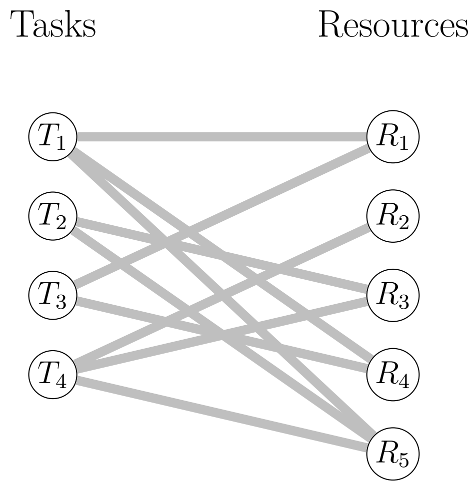

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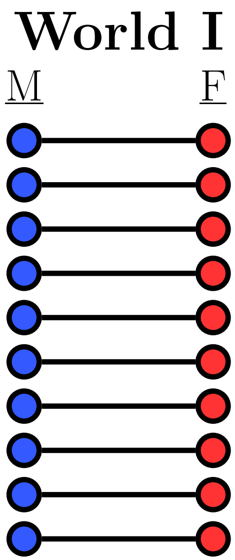


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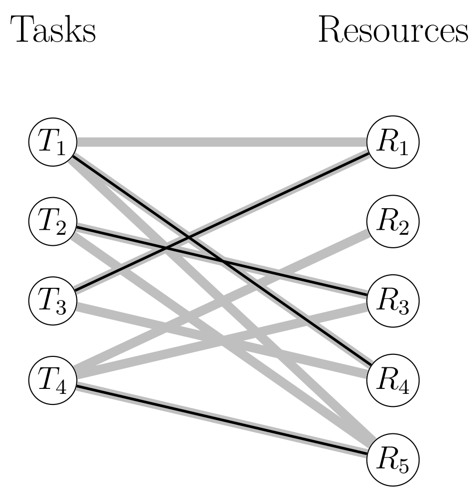


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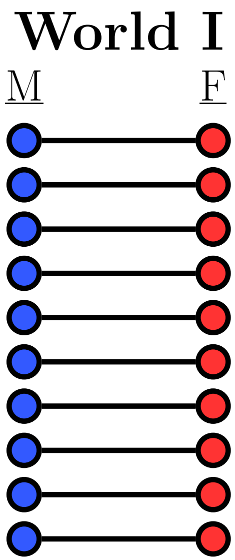


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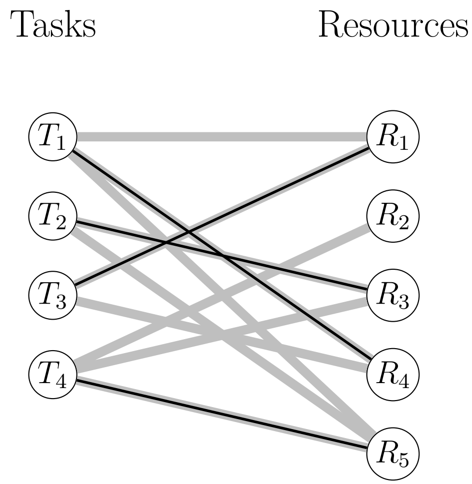


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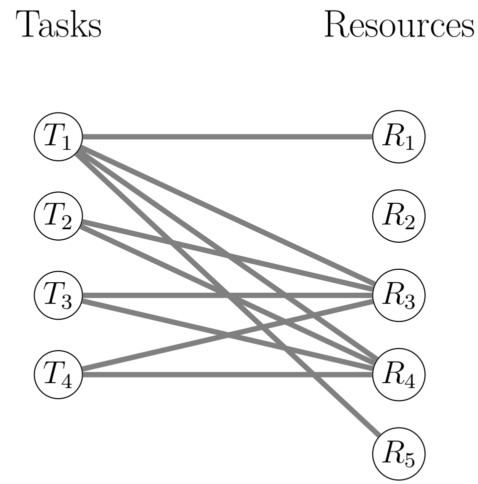


no matching

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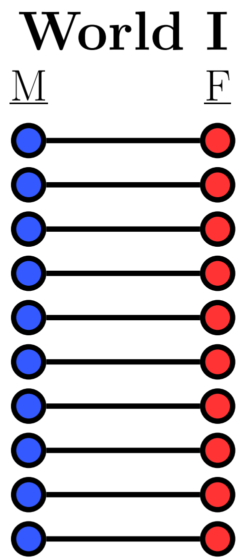


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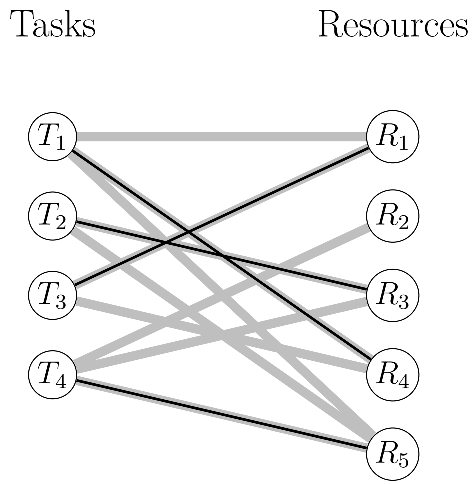


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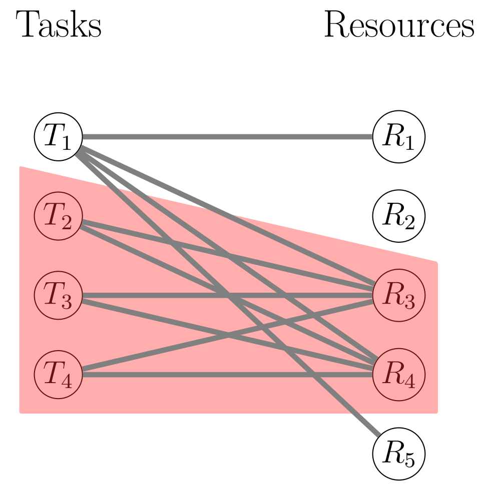


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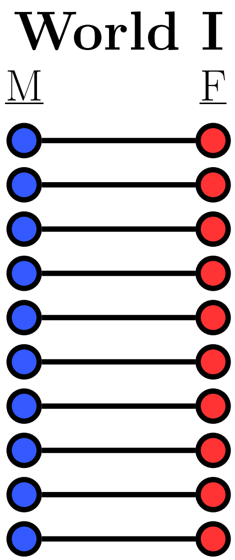


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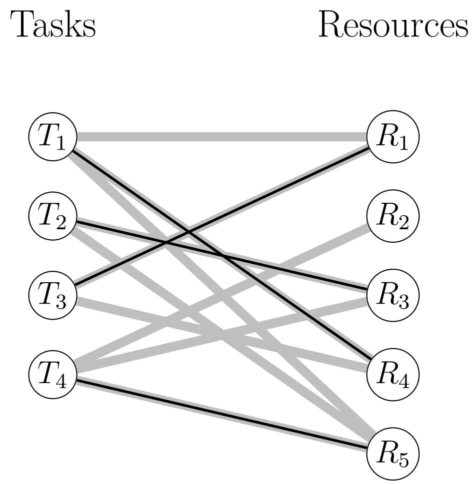


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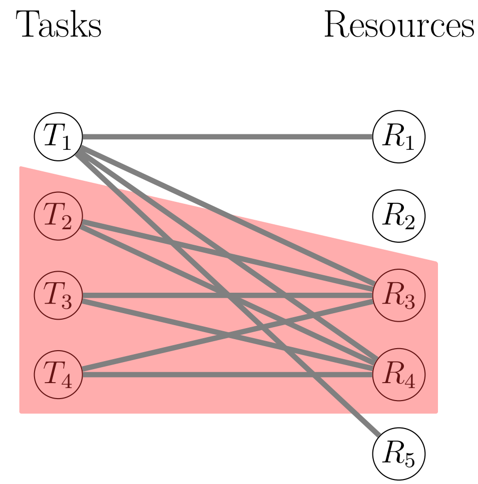


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**Hall's Theorem.**
Suppose that for *all* left-subsets $X$, $|X| \leq |N(X)|$ (Hall's "matching condition"). Then, there is a matching which covers every left-vertex.

Hall's Theorem says that the obvious necessary condition is also sufficient.

# Proof of Hall's Theorem: Induction Step

If for *all* left-subsets $X$, $|X| \leq |N(X)|$, then there is a left-matching.

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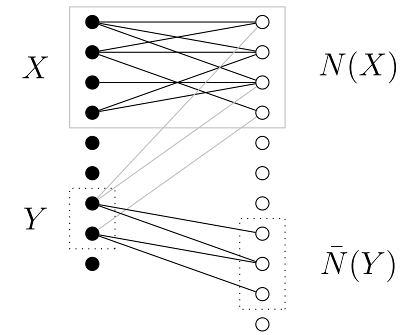
If for *all* left-subsets $X$, $|X| \leq |N(X)|$, then there is a left-matching.

**Case 1.** There is a proper left-subset $X$, with $1 \leq |X| \leq n$, for which $|X| = |N(X)|$.



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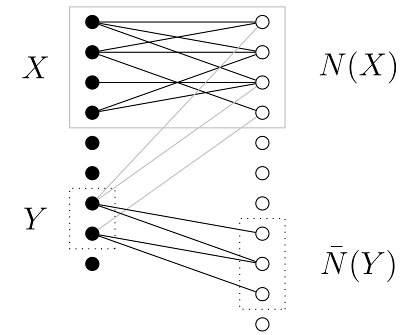

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- For any left-subset $Y \subset \overline{X}$, by the matching condition,

$$|N(X)| + |\bar{N}(Y)| = |N(X \cup Y)| \geq |X \cup Y| = |X| + |Y|$$
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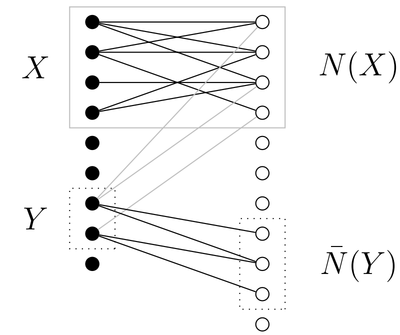

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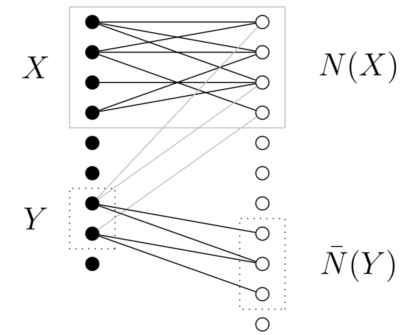

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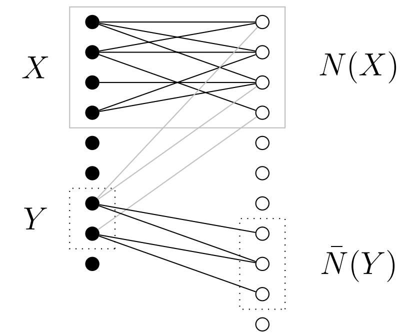
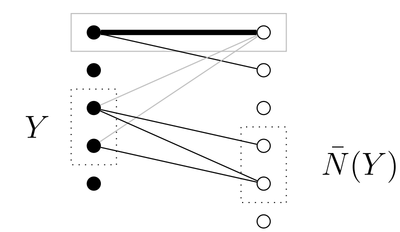



**<u>Case 2.</u>** For *every* proper left-subset $X$ (with $1 \leq |X| \leq n$), $|X| < |N(X)|$.



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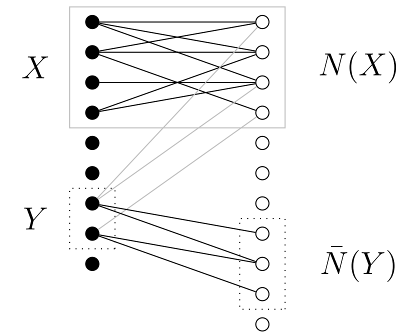
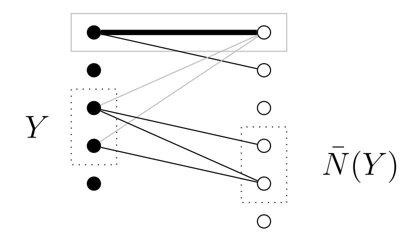



<u>**Case 2.**</u> For *every* proper left-subset $X$ (with $1 \leq |X| \leq n$), $|X| < |N(X)|$.

- Match the first left-vertex along any edge to a neighbor.



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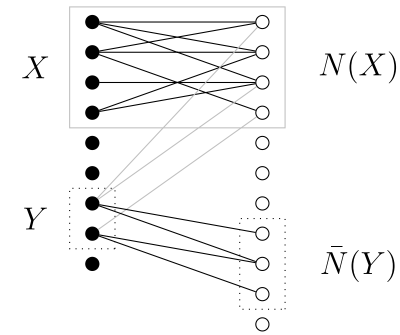


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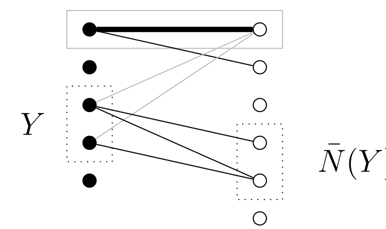


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$$|\bar{N}(Y)| \ge |N(Y)| - 1 \ge |Y| + 1 - 1 = |Y|$$

- The remaining left-vertices have a matching to the remaining right-vertices.

In both cases, there is a left-matching which covers the $n+1$ left-vertices. ■

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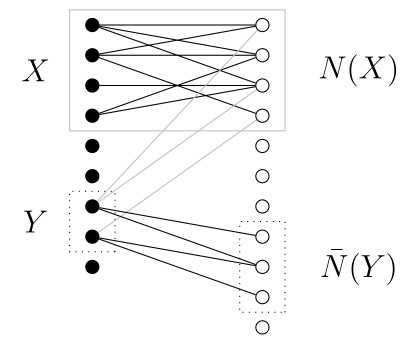


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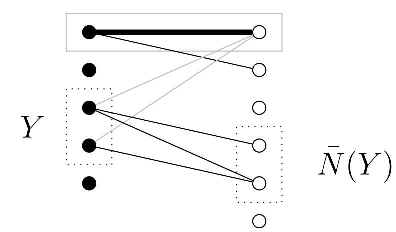


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**Exercise.** If (min left-degree) $\geq$ (max right-degree) then Hall's condition holds.

**Example 12.3.** Building Latin Squares.

# Stable Marriage: Mathematics of Dating

Matching with *preferences*: **A**lice, **B**arb and **C**arla want to date **X**avier, **Y**ariv and **Z**ach.

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|---|---|---|---|
| 1. | A | A | B |
| 2. | B | C | A |
| 3. | C | B | C |

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|---|---|---|---|
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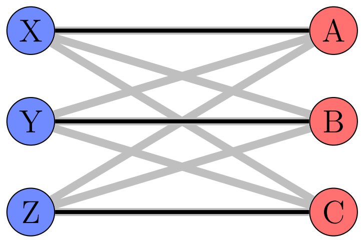

# Stable Marriage: Mathematics of Dating

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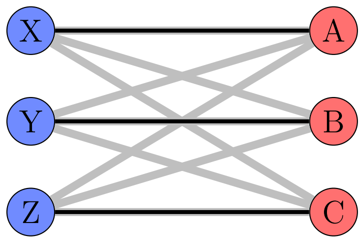
| | **X** | **Y** | **Z** |
|---|---|---|---|
| 1. | A | A | B |
| 2. | B | C | A |
| 3. | C | B | C |

| | **A** | **B** | **C** |
|---|---|---|---|
| 1. | Z | Y | Z |
| 2. | Y | X | X |
| 3. | X | Z | Y |




- Yariv prefers Alice to Barb (Barb is Yariv's current mate).

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| 1. | Z | Y | Z |
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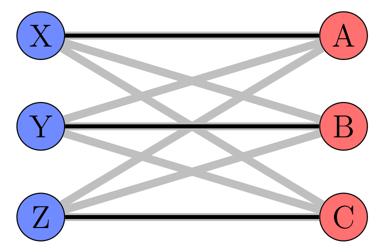


- Yariv prefers Alice to Barb (Barb is Yariv's current mate).
- *A*lice prefers Yariv to Xavier (Xavier is Alice's current mate).

# Stable Marriage: Mathematics of Dating

Matching with *preferences*: **A**lice, **B**arb and **C**arla want to date **X**avier, **Y**ariv and **Z**ach.

| | **X** | **Y** | **Z** |
|---|---|---|---|
| 1. | A | A | B |
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| 3. | C | B | C |

| | **A** | **B** | **C** |
|---|---|---|---|
| 1. | Z | Y | Z |
| 2. | Y | X | X |
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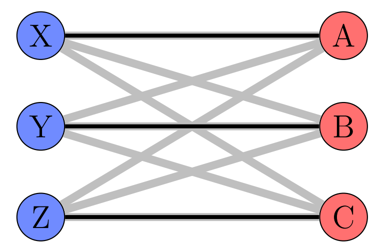


- Yariv prefers Alice to Barb (Barb is Yariv's current mate).
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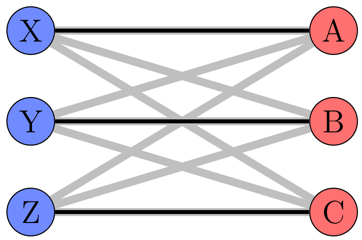
# Stable Marriage: Mathematics of Dating

Matching with *preferences*: **A**lice, **B**arb and **C**arla want to date **X**avier, **Y**ariv and **Z**ach.

| | **X** | **Y** | **Z** | | **A** | **B** | **C** |
|---|---|---|---|---|---|---|---|
| 1. | A | A | B | 1. | Z | Y | Z |
| 2. | B | C | A | 2. | Y | X | X |
| 3. | C | B | C | 3. | X | Z | Y |




- Yariv prefers Alice to Barb (Barb is Yariv's current mate).
- *A*lice prefers Yariv to Xavier (Xavier is Alice's current mate).
- Yariv and Alice both prefer each other to their current mates.
- That kind of *volatile* match-up leads to scandal.

# A Dating Ritual That Ends with No Volatile Pairs

**Day 1.** Ladies on balconies.
Each gent serenades their top choice.

| **X** | **Y** | **Z** |
|---|---|---|
| A | A | B |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| X, Y | Z | – |

| **A** | **B** | **C** |
|---|---|---|
| Z | Y | Z |
| Y | X | X |
| X | Z | Y |

# A Dating Ritual That Ends with No Volatile Pairs

**Day 1.** Ladies on balconies.
Each gent serenades their top choice.
Ladies ask only their favored suitor to come back (“dating”).

| **X** | **Y** | **Z** |
|---|---|---|
| A | A | B |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| ~~X~~, Y | Z | – |

| **A** | **B** | **C** |
|---|---|---|
| Z | Y | Z |
| Y | X | X |
| X | Z | Y |

# A Dating Ritual That Ends with No Volatile Pairs

**Day 1.** Ladies on balconies.
Each gent serenades their top choice.
Ladies ask only their favored suitor to come back (“dating”).
A rejected gent will never again woo that lady!

| X | Y | Z |
|---|---|---|
| ~~A~~ | A | B |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| ~~X~~, Y | Z | – |

| A | B | C |
|---|---|---|
| Z | Y | Z |
| Y | X | X |
| X | Z | Y |

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|---|---|---|
| ~~A~~ | A | B |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| ~~X~~, Y | Z | – |

| **A** | **B** | **C** |
|---|---|---|
| Z | Y | Z |
| Y | X | X |
| X | Z | Y |

**Day 2.** $Y$ and $Z$ return to $A$ and $B$ respectively. ("dating")

| **X** | **Y** | **Z** |
|---|---|---|
| ~~A~~ | A | B |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| Y | Z | – |

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|---|---|---|
| ~~A~~ | A | B |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| ~~X~~, Y | Z | – |

| **A** | **B** | **C** |
|---|---|---|
| Z | Y | Z |
| Y | X | X |
| X | Z | Y |

**Day 2.** $Y$ and $Z$ return to $A$ and $B$ respectively. ("dating")
$X$ goes to $B$'s balcony.

| **X** | **Y** | **Z** |
|---|---|---|
| ~~A~~ | A | B |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| Y | X, Z | – |

# A Dating Ritual That Ends with No Volatile Pairs

**Day 1.** Ladies on balconies.
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| **X** | **Y** | **Z** |
|---|---|---|
| ~~A~~ | A | B |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| ~~X~~, Y | Z | – |

| **A** | **B** | **C** |
|---|---|---|
| Z | Y | Z |
| Y | X | X |
| X | Z | Y |

**Day 2.** $Y$ and $Z$ return to $A$ and $B$ respectively. ("dating")
$X$ goes to $B$'s balcony.
$B$ rejects $Z$. $Z$ erases $B$; $X$ and $Y$ will return.

| **X** | **Y** | **Z** |
|---|---|---|
| ~~A~~ | A | ~~B~~ |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| Y | X, ~~Z~~ | – |

# A Dating Ritual That Ends with No Volatile Pairs

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| **X** | **Y** | **Z** |
|---|---|---|
| ~~A~~ | A | B |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| ~~X~~, Y | Z | – |

| **A** | **B** | **C** |
|---|---|---|
| Z | Y | Z |
| Y | X | X |
| X | Z | Y |

**Day 2.** $Y$ and $Z$ return to $A$ and $B$ respectively. ("dating")
$X$ goes to $B$'s balcony.
$B$ rejects $Z$. $Z$ erases $B$; $X$ and $Y$ will return.

| **X** | **Y** | **Z** |
|---|---|---|
| ~~A~~ | A | ~~B~~ |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| Y | X, ~~Z~~ | – |

**Day 3.** $Y$ and $X$ return to $A$ and $B$ respectively. ("dating")
$Z$ goes to $A$'s balcony.

| **X** | **Y** | **Z** |
|---|---|---|
| ~~A~~ | A | ~~B~~ |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| Y, Z | X | – |

# A Dating Ritual That Ends with No Volatile Pairs

**Day 1.** Ladies on balconies.
Each gent serenades their top choice.
Ladies ask only their favored suitor to come back ("dating").
A rejected gent will never again woo that lady!

| **X** | **Y** | **Z** |
|---|---|---|
| ~~A~~ | A | B |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| ~~X~~, Y | Z | – |

| **A** | **B** | **C** |
|---|---|---|
| Z | Y | Z |
| Y | X | X |
| X | Z | Y |

**Day 2.** $Y$ and $Z$ return to $A$ and $B$ respectively. ("dating")
$X$ goes to $B$'s balcony.
$B$ rejects $Z$. $Z$ erases $B$; $X$ and $Y$ will return.

| **X** | **Y** | **Z** |
|---|---|---|
| ~~A~~ | A | ~~B~~ |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| Y | X, ~~Z~~ | – |

**Day 3.** $Y$ and $X$ return to $A$ and $B$ respectively. ("dating")
$Z$ goes to $A$'s balcony.
$A$'s patience is rewarded. $A$ rejects $Y$ for her top-choice $Z$.

| **X** | **Y** | **Z** |
|---|---|---|
| ~~A~~ | ~~A~~ | ~~B~~ |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| ~~Y~~, Z | X | – |

# A Dating Ritual That Ends with No Volatile Pairs

**Day 1.** Ladies on balconies.
Each gent serenades their top choice.
Ladies ask only their favored suitor to come back ("dating").
A rejected gent will never again woo that lady!

| **X** | **Y** | **Z** |
|---|---|---|
| ~~A~~ | A | B |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| ~~X~~, Y | Z | – |

| **A** | **B** | **C** |
|---|---|---|
| Z | Y | Z |
| Y | X | X |
| X | Z | Y |

**Day 2.** $Y$ and $Z$ return to $A$ and $B$ respectively. ("dating")
$X$ goes to $B$'s balcony.
$B$ rejects $Z$. $Z$ erases $B$; $X$ and $Y$ will return.

| **X** | **Y** | **Z** |
|---|---|---|
| ~~A~~ | A | ~~B~~ |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| Y | X, ~~Z~~ | – |

**Day 3.** $Y$ and $X$ return to $A$ and $B$ respectively. ("dating")
$Z$ goes to $A$'s balcony.
$A$'s patience is rewarded. $A$ rejects $Y$ for her top-choice $Z$.

| **X** | **Y** | **Z** |
|---|---|---|
| ~~A~~ | ~~A~~ | ~~B~~ |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| ~~Y~~, Z | X | – |

**Day 4.** Each girl has found her boy.

| **X** | **Y** | **Z** |
|---|---|---|
| ~~A~~ | ~~A~~ | ~~B~~ |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| Z | X | Y |

# A Dating Ritual That Ends with No Volatile Pairs

**Day 1.** Ladies on balconies.
Each gent serenades their top choice.
Ladies ask only their favored suitor to come back ("dating").
A rejected gent will never again woo that lady!

| **X** | **Y** | **Z** |
|---|---|---|
| ~~A~~ | A | B |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| ~~X~~, Y | Z | – |

| **A** | **B** | **C** |
|---|---|---|
| Z | Y | Z |
| Y | X | X |
| X | Z | Y |

**Day 2.** $Y$ and $Z$ return to $A$ and $B$ respectively. ("dating")
$X$ goes to $B$'s balcony.
$B$ rejects $Z$. $Z$ erases $B$; $X$ and $Y$ will return.

| **X** | **Y** | **Z** |
|---|---|---|
| ~~A~~ | A | ~~B~~ |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| Y | X, ~~Z~~ | – |

**Day 3.** $Y$ and $X$ return to $A$ and $B$ respectively. ("dating")
$Z$ goes to $A$'s balcony.
$A$'s patience is rewarded. $A$ rejects $Y$ for her top-choice $Z$.

| **X** | **Y** | **Z** |
|---|---|---|
| ~~A~~ | ~~A~~ | ~~B~~ |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| ~~Y~~, Z | X | – |

**Day 4.** Each girl has found her boy.
The dating ritual ends with non-volatile marriages

$A$—$Z$ $B$—$X$ $C$—$Y$

| **X** | **Y** | **Z** |
|---|---|---|
| ~~A~~ | ~~A~~ | ~~B~~ |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| Z | X | Y |

# A Dating Ritual That Ends with No Volatile Pairs

**Day 1.** Ladies on balconies.
Each gent serenades their top choice.
Ladies ask only their favored suitor to come back ("dating").
A rejected gent will never again woo that lady!

| **X** | **Y** | **Z** |
|---|---|---|
| ~~A~~ | A | B |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| ~~X~~, Y | Z | – |

| **A** | **B** | **C** |
|---|---|---|
| Z | Y | Z |
| Y | X | X |
| X | Z | Y |

**Day 2.** $Y$ and $Z$ return to $A$ and $B$ respectively. ("dating")
$X$ goes to $B$'s balcony.
$B$ rejects $Z$. $Z$ erases $B$; $X$ and $Y$ will return.

| **X** | **Y** | **Z** |
|---|---|---|
| ~~A~~ | A | ~~B~~ |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| Y | X, ~~Z~~ | – |

**Day 3.** $Y$ and $X$ return to $A$ and $B$ respectively. ("dating")
$Z$ goes to $A$'s balcony.
$A$'s patience is rewarded. $A$ rejects $Y$ for her top-choice $Z$.

| **X** | **Y** | **Z** |
|---|---|---|
| ~~A~~ | ~~A~~ | ~~B~~ |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| ~~Y~~, Z | X | – |

**Day 4.** Each girl has found her boy.
The dating ritual ends with non-volatile marriages

$A$—$Z$ $B$—$X$ $C$—$Y$

| **X** | **Y** | **Z** |
|---|---|---|
| ~~A~~ | ~~A~~ | ~~B~~ |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| Z | X | Y |

**Theorem.** [Gale-Shapely, 1962]

1. For $n$ men and women, the dating ritual ends after at most $n^2$ days of dating.

# A Dating Ritual That Ends with No Volatile Pairs

**Day 1.** Ladies on balconies.
Each gent serenades their top choice.
Ladies ask only their favored suitor to come back ("dating").
A rejected gent will never again woo that lady!

| **X** | **Y** | **Z** |
|---|---|---|
| ~~A~~ | A | B |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| ~~X~~, Y | Z | – |

| **A** | **B** | **C** |
|---|---|---|
| Z | Y | Z |
| Y | X | X |
| X | Z | Y |

**Day 2.** $Y$ and $Z$ return to $A$ and $B$ respectively. ("dating")
$X$ goes to $B$'s balcony.
$B$ rejects $Z$. $Z$ erases $B$; $X$ and $Y$ will return.

| **X** | **Y** | **Z** |
|---|---|---|
| ~~A~~ | A | ~~B~~ |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| Y | X, ~~Z~~ | – |

**Day 3.** $Y$ and $X$ return to $A$ and $B$ respectively. ("dating")
$Z$ goes to $A$'s balcony.
$A$'s patience is rewarded. $A$ rejects $Y$ for her top-choice $Z$.

| **X** | **Y** | **Z** |
|---|---|---|
| ~~A~~ | ~~A~~ | ~~B~~ |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| ~~Y~~, Z | X | – |

**Day 4.** Each girl has found her boy.
The dating ritual ends with non-volatile marriages

$A$—$Z$ $B$—$X$ $C$—$Y$

| **X** | **Y** | **Z** |
|---|---|---|
| ~~A~~ | ~~A~~ | ~~B~~ |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| Z | X | Y |

**Theorem.** [Gale-Shapely, 1962]

1. For $n$ men and women, the dating ritual ends after at most $n^2$ days of dating.
2. Every man and woman will be matched at the end.

# A Dating Ritual That Ends with No Volatile Pairs

**Day 1.** Ladies on balconies.
Each gent serenades their top choice.
Ladies ask only their favored suitor to come back ("dating").
A rejected gent will never again woo that lady!

| X | Y | Z |
|---|---|---|
| ~~A~~ | A | B |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| ~~X~~, Y | Z | – |

| A | B | C |
|---|---|---|
| Z | Y | Z |
| Y | X | X |
| X | Z | Y |

**Day 2.** $Y$ and $Z$ return to $A$ and $B$ respectively. ("dating")
$X$ goes to $B$'s balcony.
$B$ rejects $Z$. $Z$ erases $B$; $X$ and $Y$ will return.

| X | Y | Z |
|---|---|---|
| ~~A~~ | A | ~~B~~ |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| Y | X, ~~Z~~ | – |

**Day 3.** $Y$ and $X$ return to $A$ and $B$ respectively. ("dating")
$Z$ goes to $A$'s balcony.
$A$'s patience is rewarded. $A$ rejects $Y$ for her top-choice $Z$.

| X | Y | Z |
|---|---|---|
| ~~A~~ | ~~A~~ | ~~B~~ |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| ~~X~~, Z | X | – |

**Day 4.** Each girl has found her boy.
The dating ritual ends with non-volatile marriages
$A$—$Z$ $B$—$X$ $C$—$Y$

| X | Y | Z |
|---|---|---|
| ~~A~~ | ~~A~~ | ~~B~~ |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| Z | X | Y |

**Theorem.** [Gale-Shapely, 1962]

1. For $n$ men and women, the dating ritual ends after at most $n^2$ days of dating.
2. Every man and woman will be matched at the end.
3. The resulting set of marriages is stable (no volatile pairs).

# A Dating Ritual That Ends with No Volatile Pairs

**Day 1.** Ladies on balconies.
Each gent serenades their top choice.
Ladies ask only their favored suitor to come back ("dating").
A rejected gent will never again woo that lady!

| **X** | **Y** | **Z** |
|---|---|---|
| ~~A~~ | A | B |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| ~~X~~, Y | Z | – |

| **A** | **B** | **C** |
|---|---|---|
| Z | Y | Z |
| Y | X | X |
| X | Z | Y |

**Day 2.** $Y$ and $Z$ return to $A$ and $B$ respectively. ("dating")
$X$ goes to $B$'s balcony.
$B$ rejects $Z$. $Z$ erases $B$; $X$ and $Y$ will return.

| **X** | **Y** | **Z** |
|---|---|---|
| ~~A~~ | A | ~~B~~ |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| Y | X, ~~Z~~ | – |

**Day 3.** $Y$ and $X$ return to $A$ and $B$ respectively. ("dating")
$Z$ goes to $A$'s balcony.
$A$'s patience is rewarded. $A$ rejects $Y$ for her top-choice $Z$.

| **X** | **Y** | **Z** |
|---|---|---|
| ~~A~~ | ~~A~~ | ~~B~~ |
| B | C | A |
| C | B | C |

| A | B | C |
|---|---|---|
| ~~Y~~, Z | X | – |

**Day 4.** Each girl has found her boy.
The dating ritual ends with non-volatile marriages

$A$—$Z$ $B$—$X$ $C$—$Y$

| **X** | **Y** | **Z** |
|---|---|---|
| ~~A~~ | ~~A~~ | ~~B~~ |
| B | C | A |
| C | B | C |

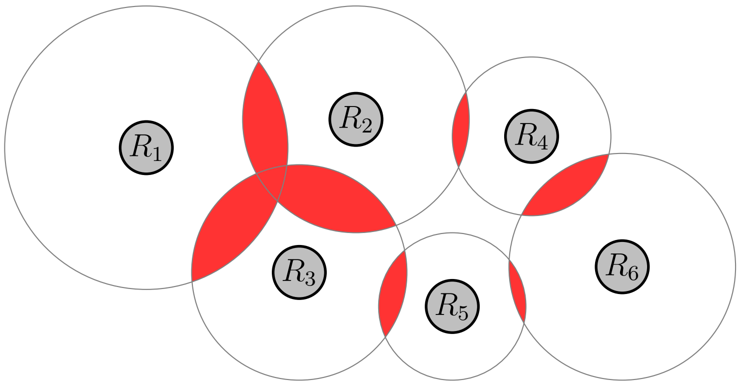
| A | B | C |
|---|---|---|
| Z | X | Y |

**Theorem.** [Gale-Shapely, 1962]

1. For $n$ men and women, the dating ritual ends after at most $n^2$ days of dating.
2. Every man and woman will be matched at the end.
3. The resulting set of marriages is stable (no volatile pairs).
4. The girls are in control but they lose. ☹

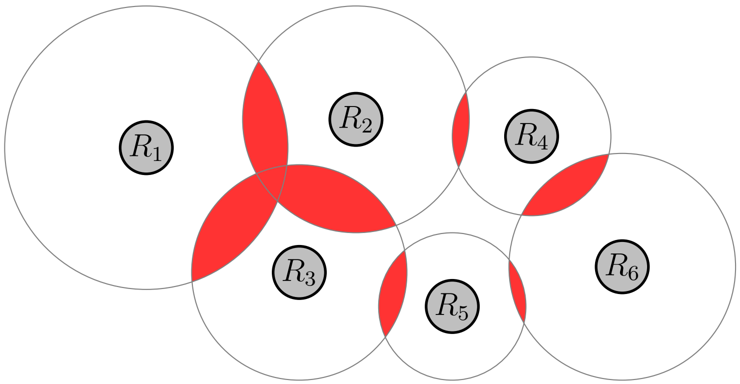
# Conflict Graphs and Coloring

**Task 1:** Assigning Radio Frequencies

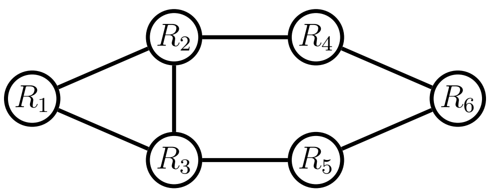


# Conflict Graphs and Coloring

**Task 1:** Assigning Radio Frequencies

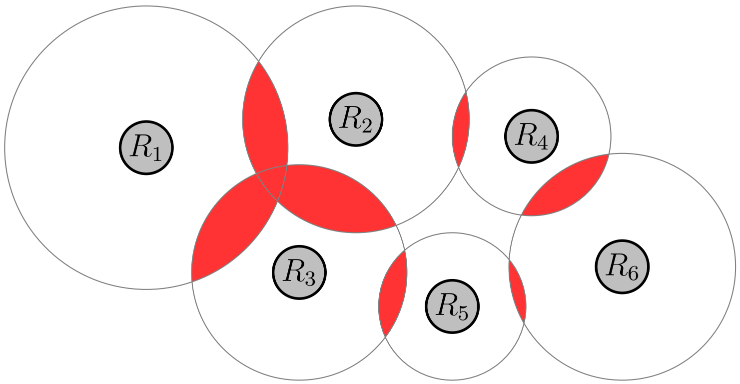



Stations broadcasting to the same listener (red areas) need different frequencies (conflict).



# Conflict Graphs and Coloring

**Task 1:** Assigning Radio Frequencies

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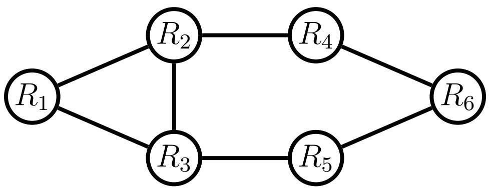


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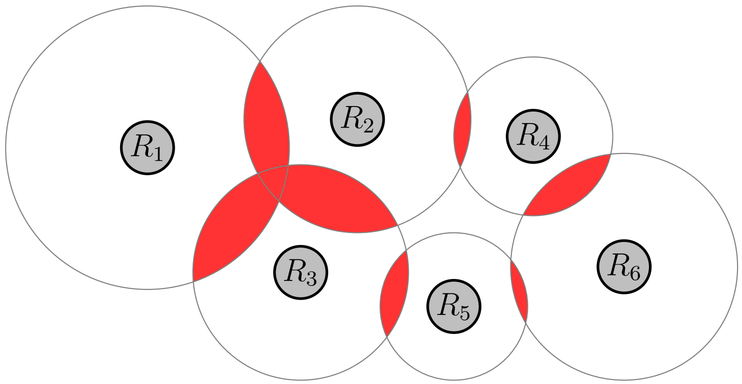


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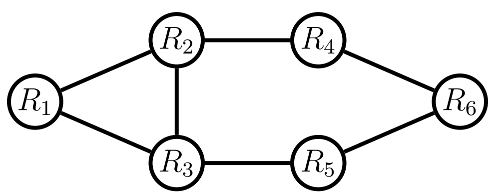


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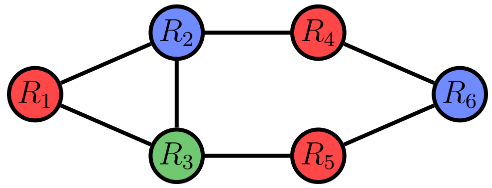

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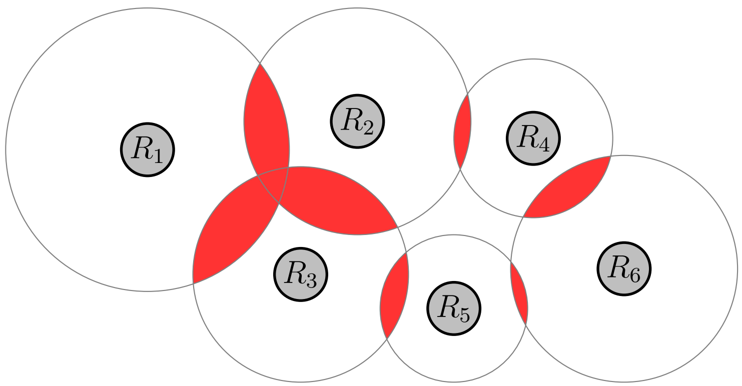


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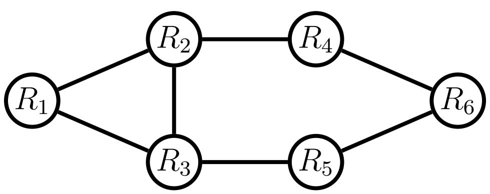


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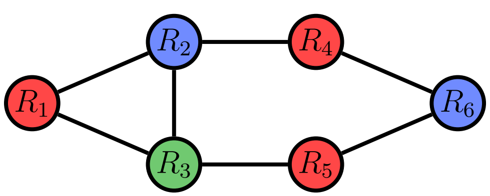


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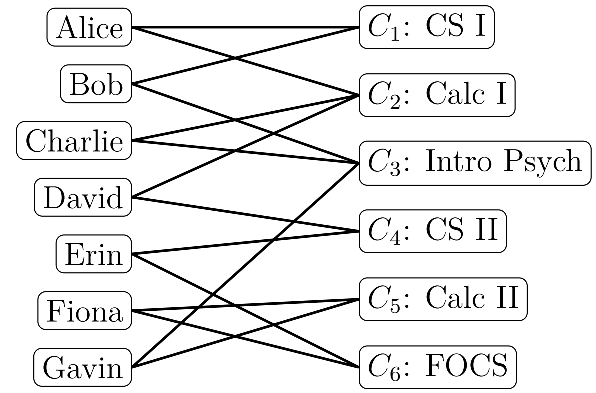

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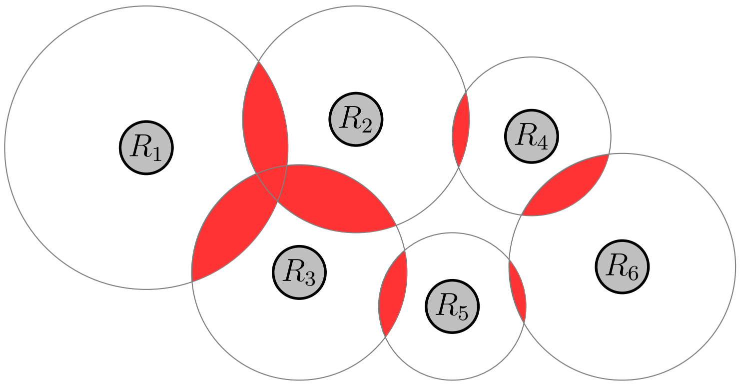


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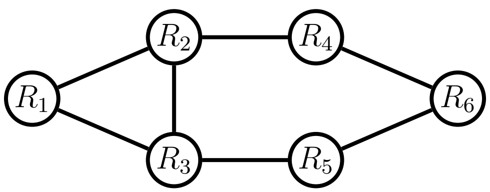


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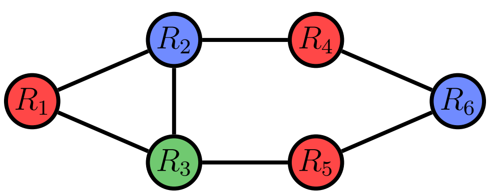


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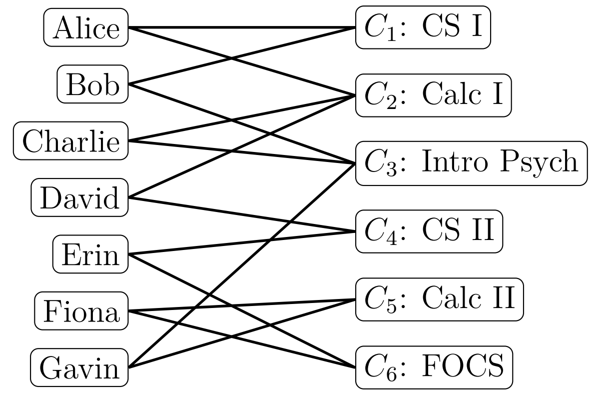


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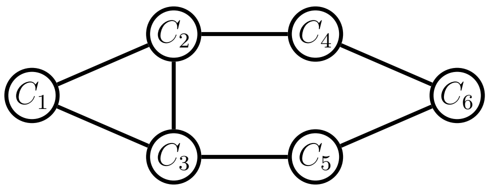

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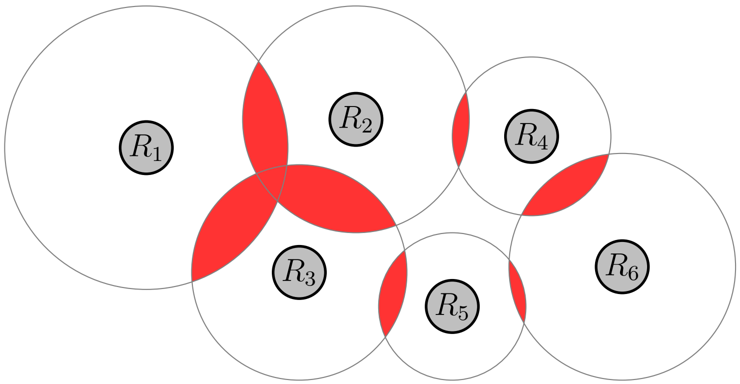


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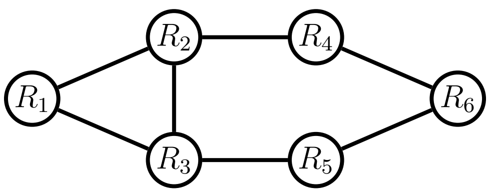


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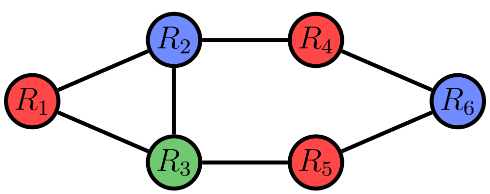


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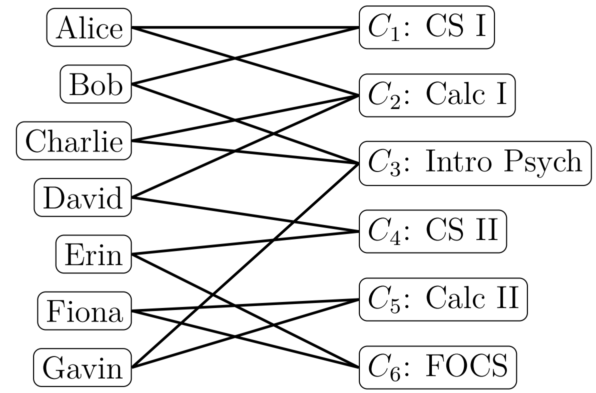


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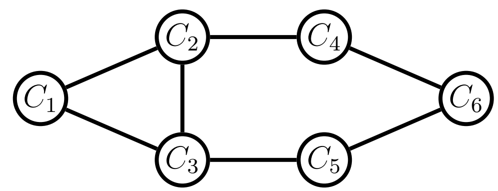


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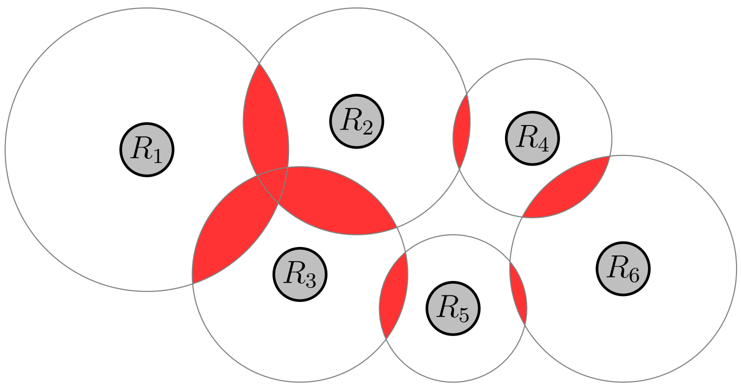


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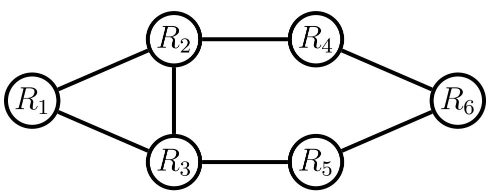


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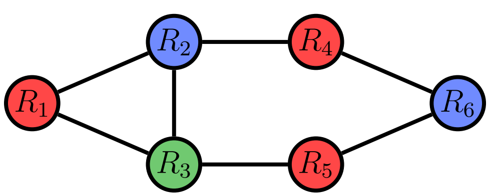


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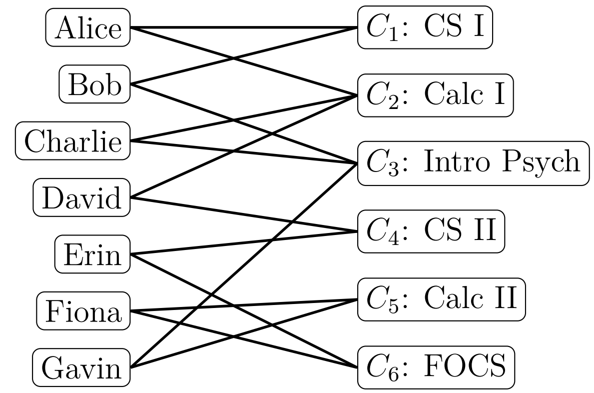


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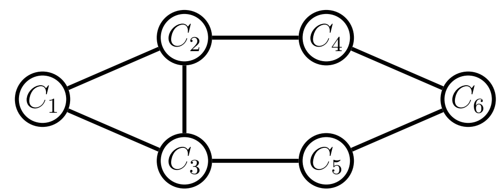


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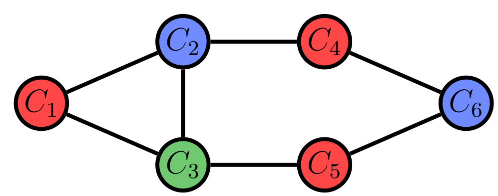

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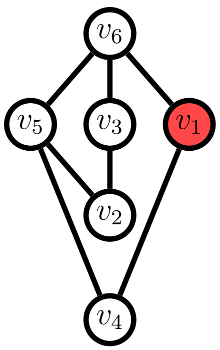
For visual effect, pick the colors $\{1, 2, 3, 4\}$ as $\{, , , \}$.



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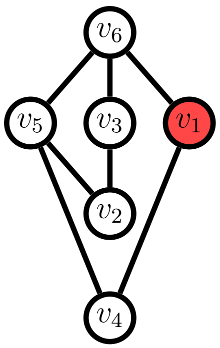

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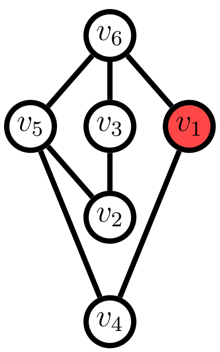

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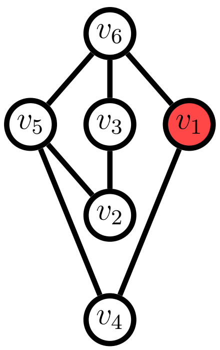


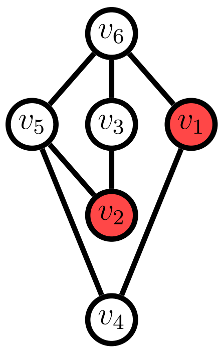

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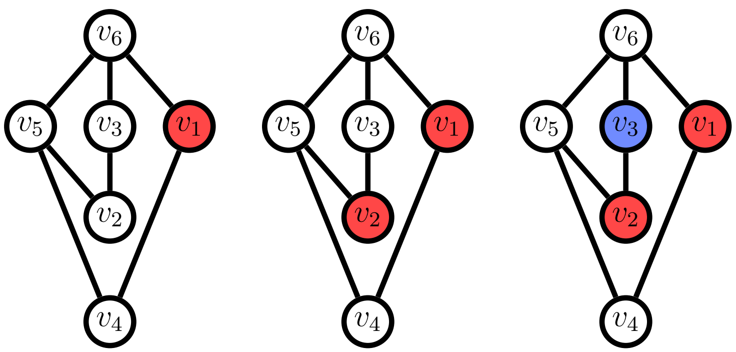

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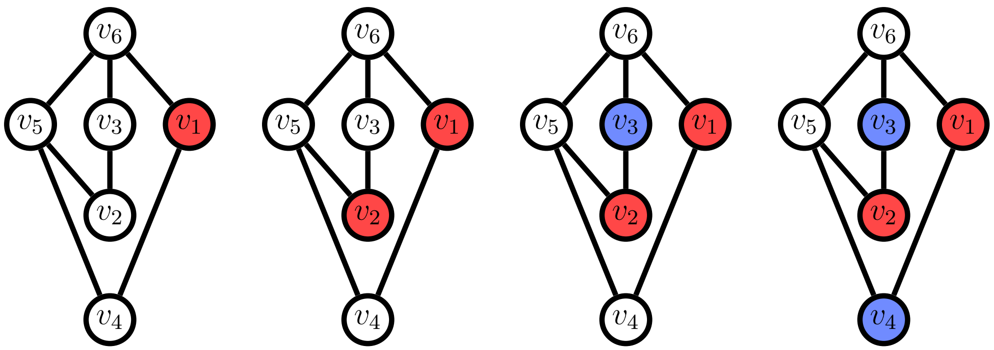

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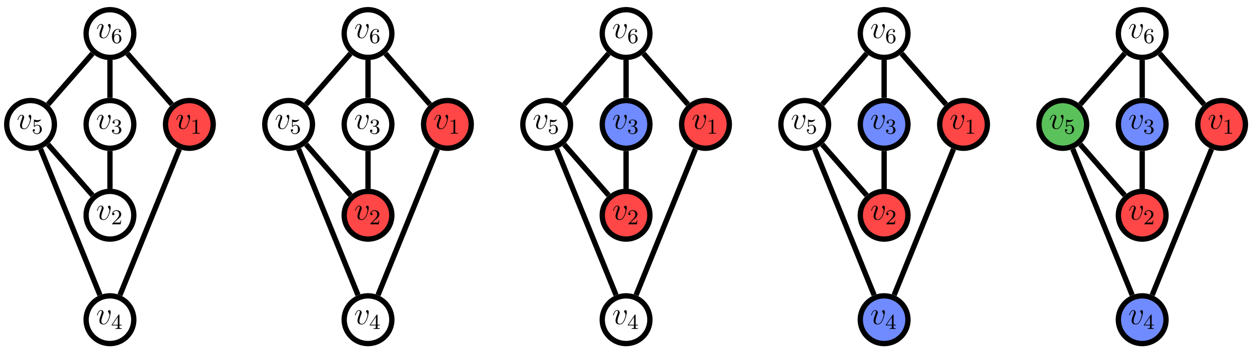

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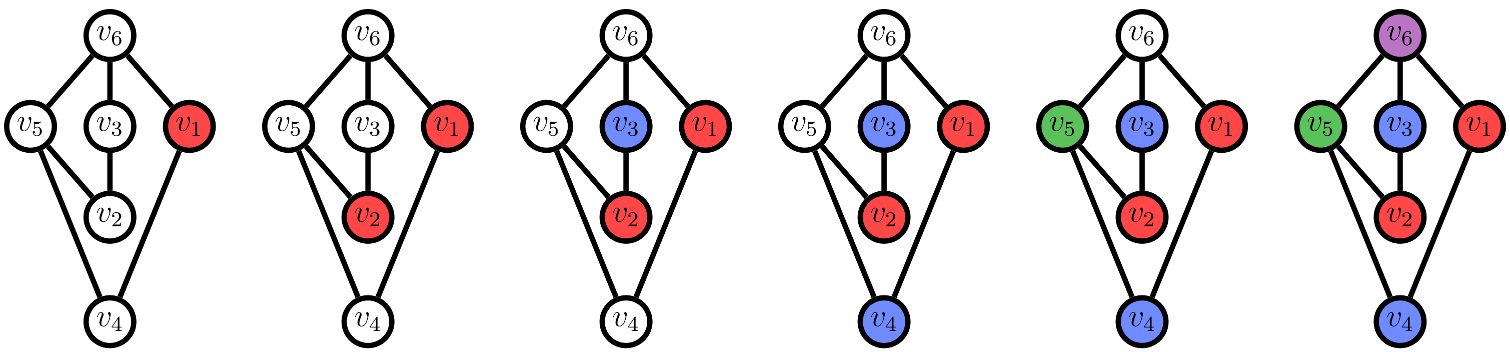

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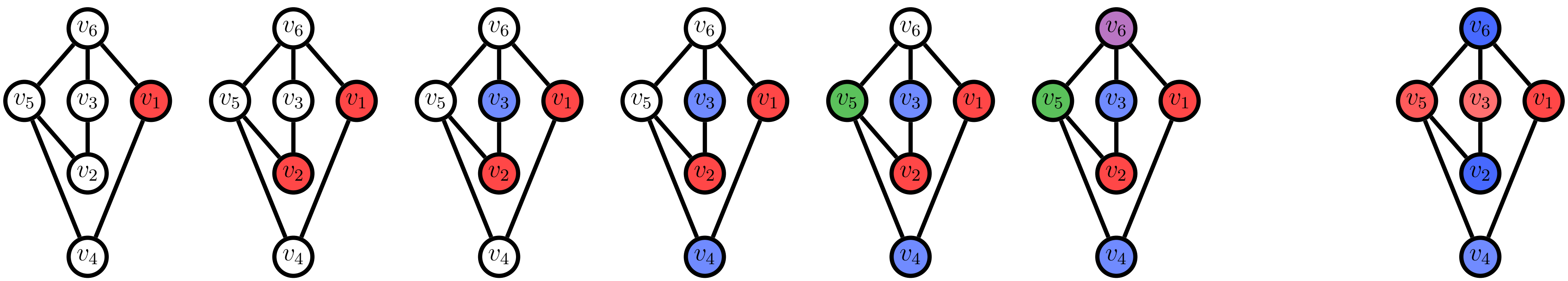


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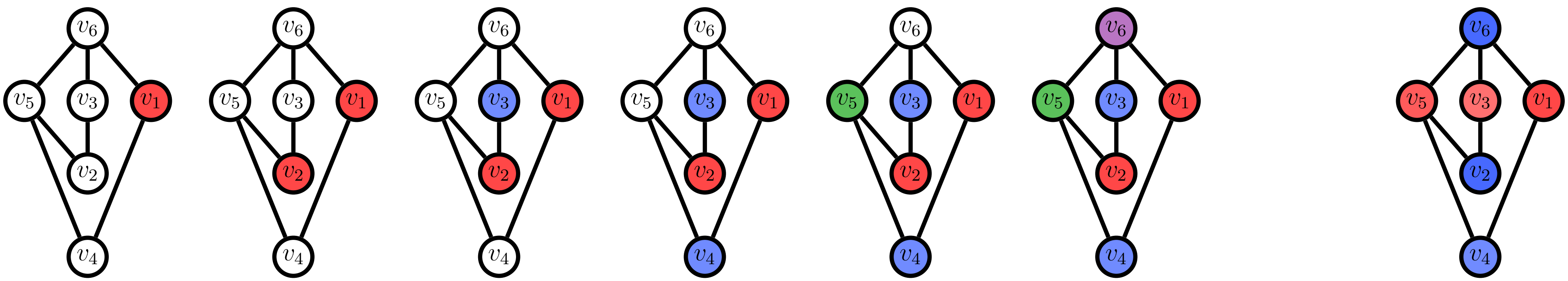


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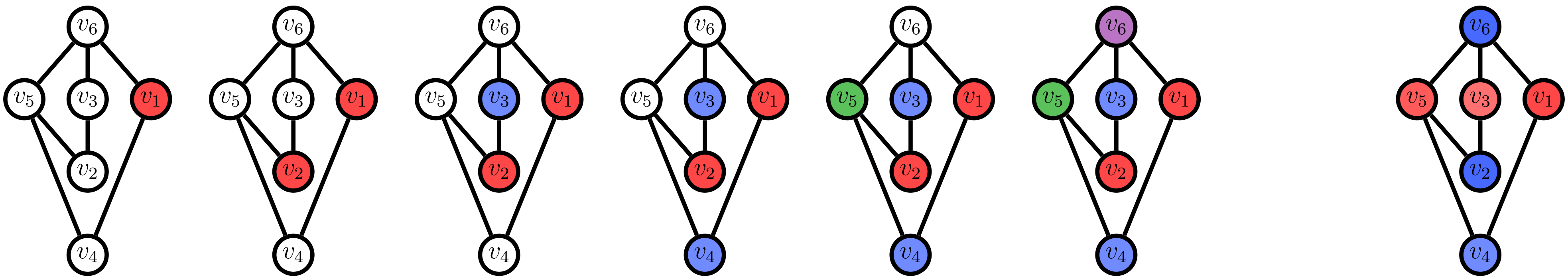


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Theorem. Chromatic number is bounded by maximum degree.
$\chi(G) \leq \Delta(G) + 1$, where $\Delta(G)$ is the maximum degree in $G$, $\Delta(G) = \max_i \delta_i$.

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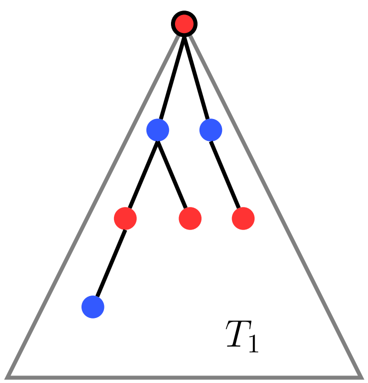

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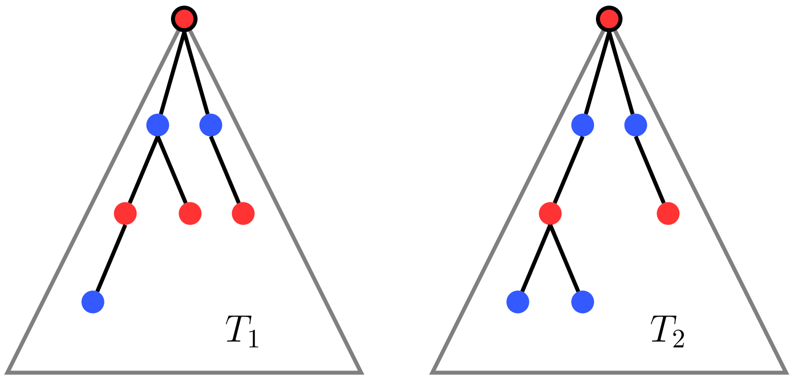

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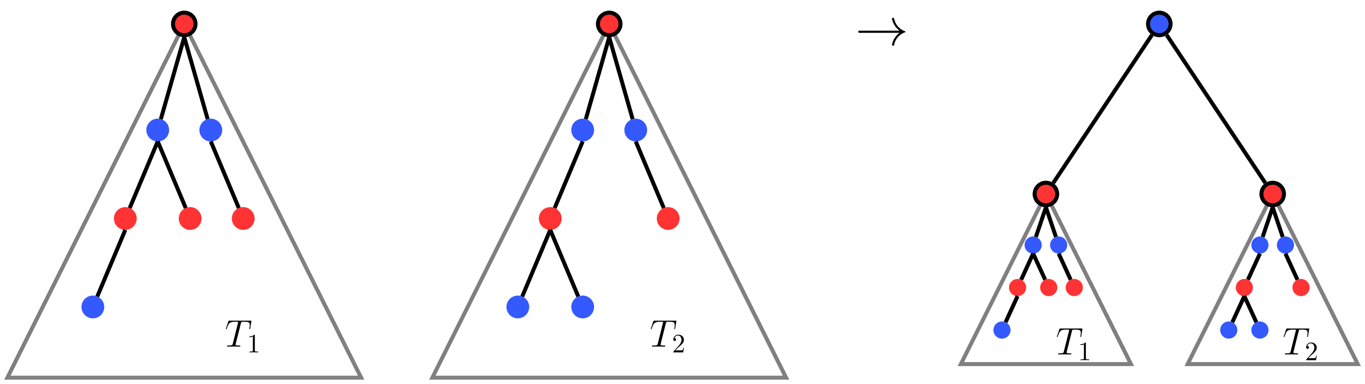

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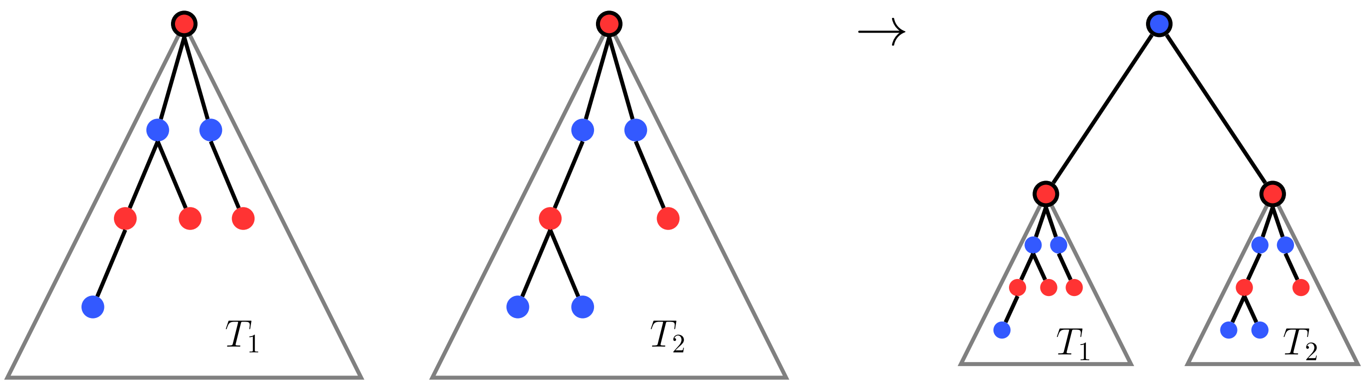


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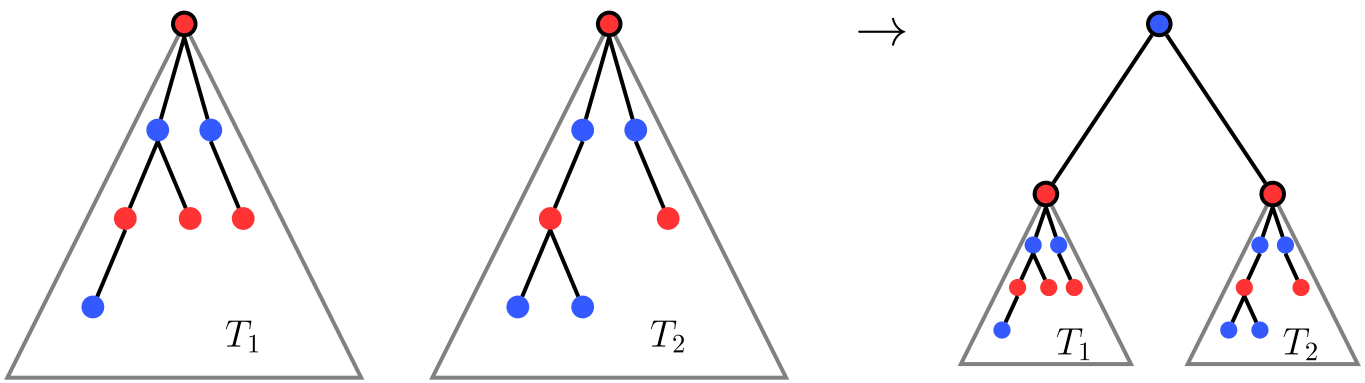


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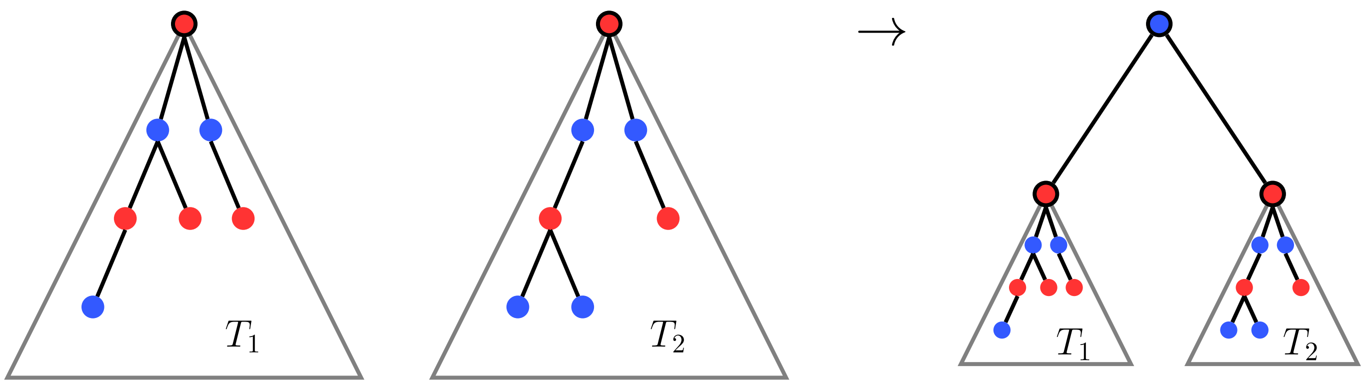


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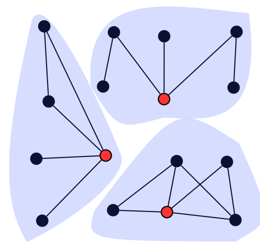
A graph is bipartite if and only if its chromatic number is 2. Trees are bipartite.

# Other Graph Problems

**Connected Components.** For "viral" marketing, pick one vertex in each *connected component* (e.g. target the "central (red)" vertices). [EASY]



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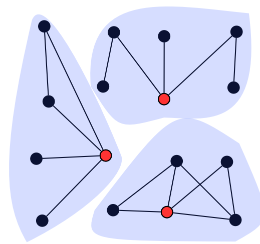
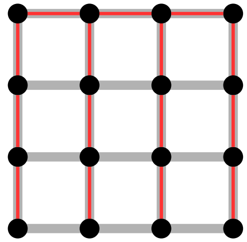


**Spanning Tree.** In a road grid (gray), to maintain a *minimal* "highway system" that offers high-speed travel we can use a *spanning tree* (red). [EASY]



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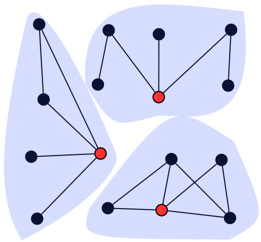

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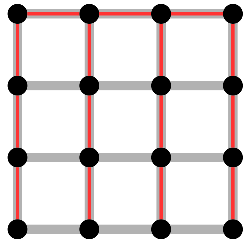

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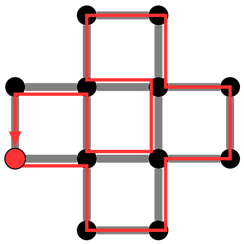

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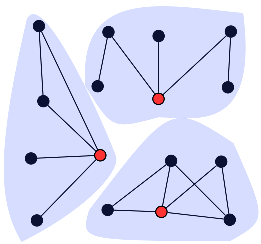

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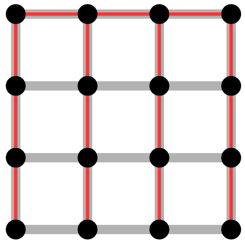

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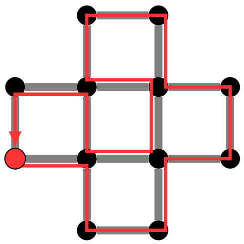

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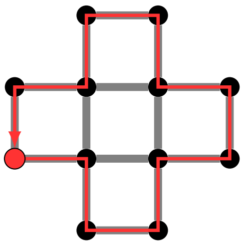

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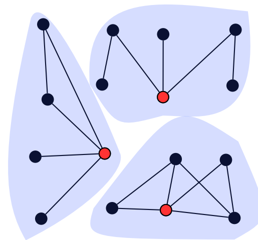

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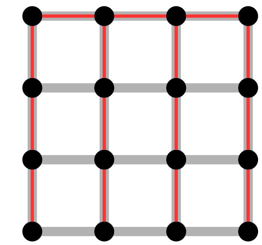

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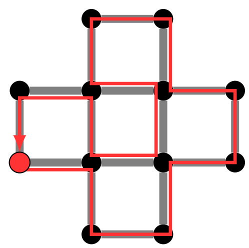

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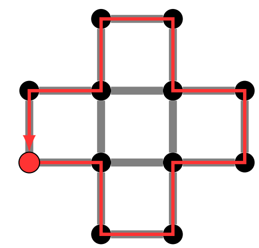

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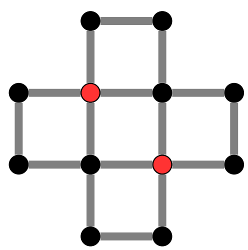

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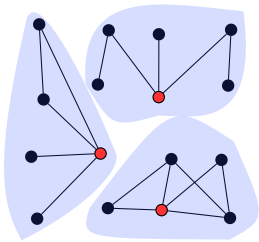

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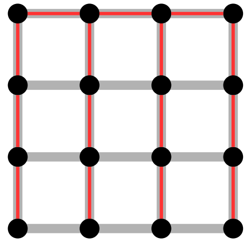

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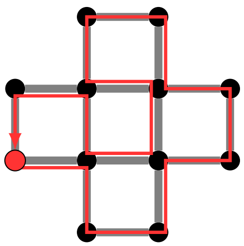

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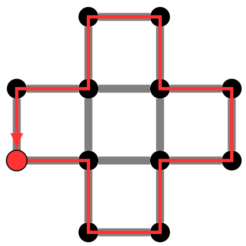

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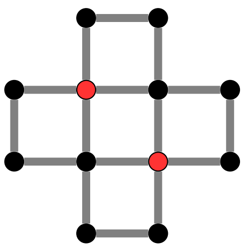

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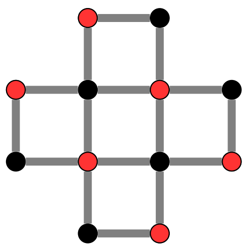

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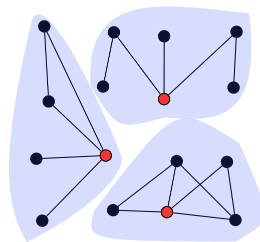

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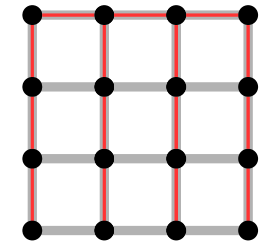

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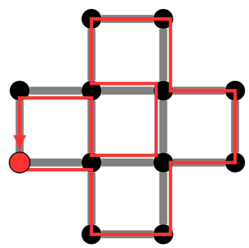

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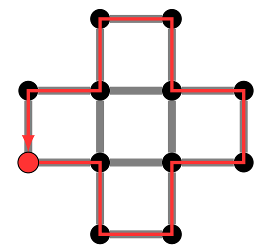

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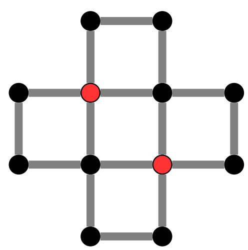

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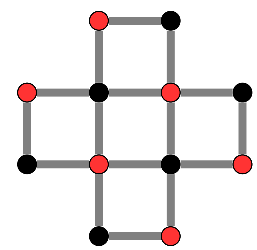

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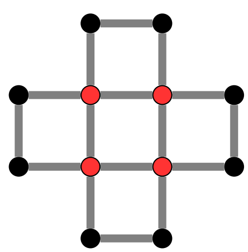

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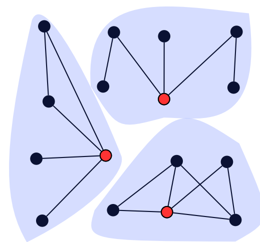

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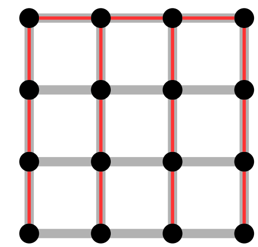

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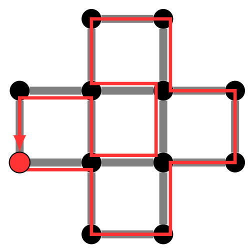

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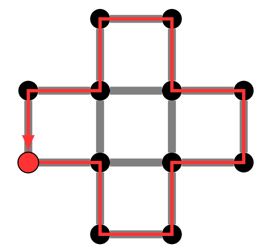

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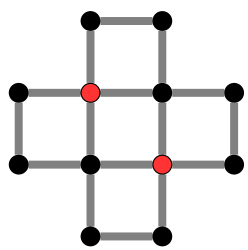

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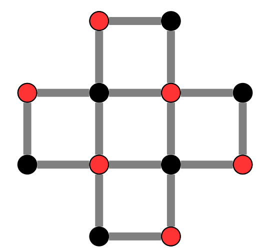

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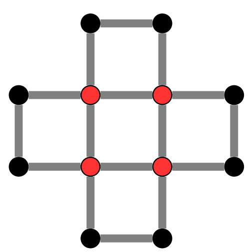

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