

Foundations of Computer Science

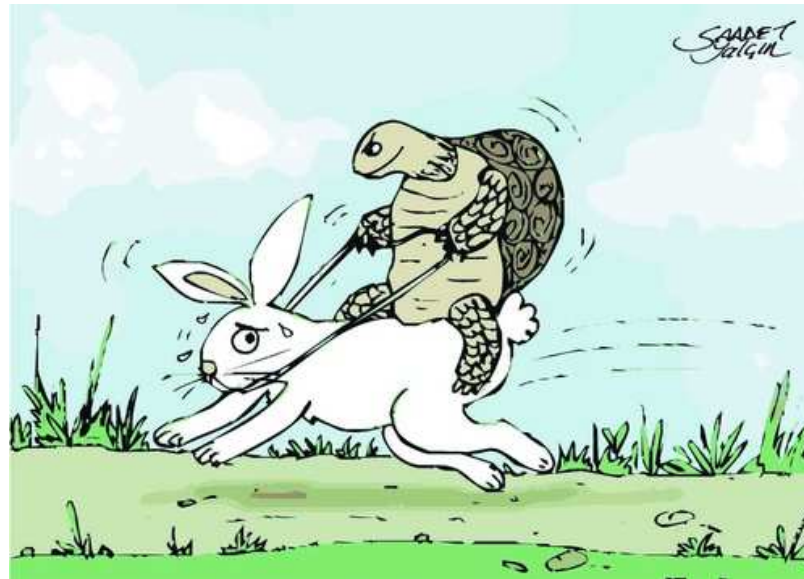
Lecture 28

Efficiency: The Class P, NP and NP-Completeness

Running Time

Efficiently Solvable Problems

Boundary Between Efficient and Inefficient



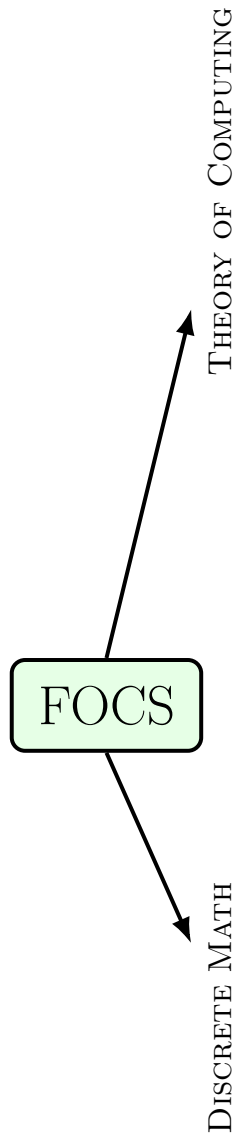
Last Time

- 1 Computer: Universal Turing Machine U_{TM}
- 2 Program and Input: $\langle M \rangle \# w$. U_{TM} simulates M on w .
- 3 No Program Verifier, no Ultimate-Debugger, no PCP-Solver. 😞
- 4 No means No.

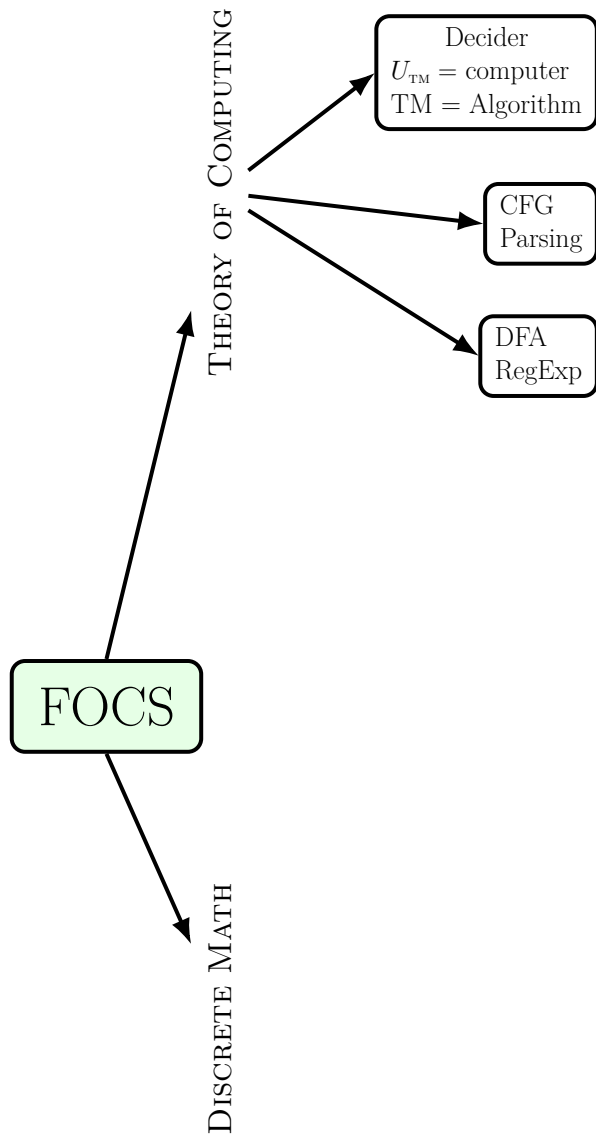
The Path Forward: Focus on Decidable Problems

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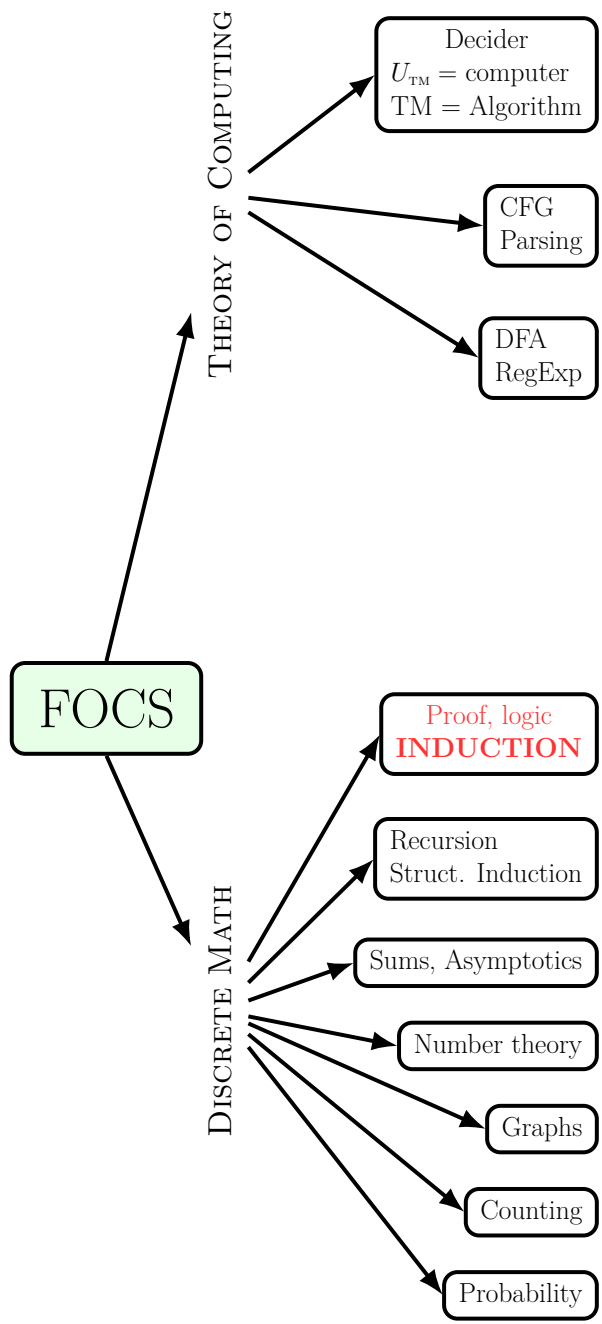
The Path Forward: Focus on Decidable Problems



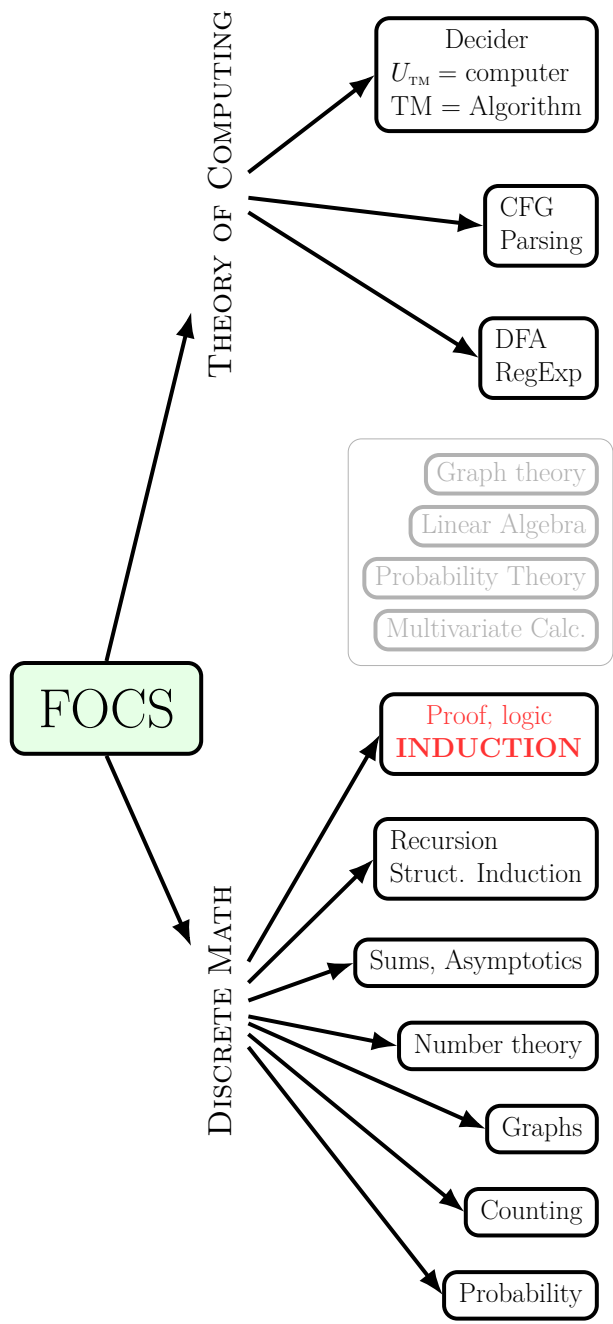
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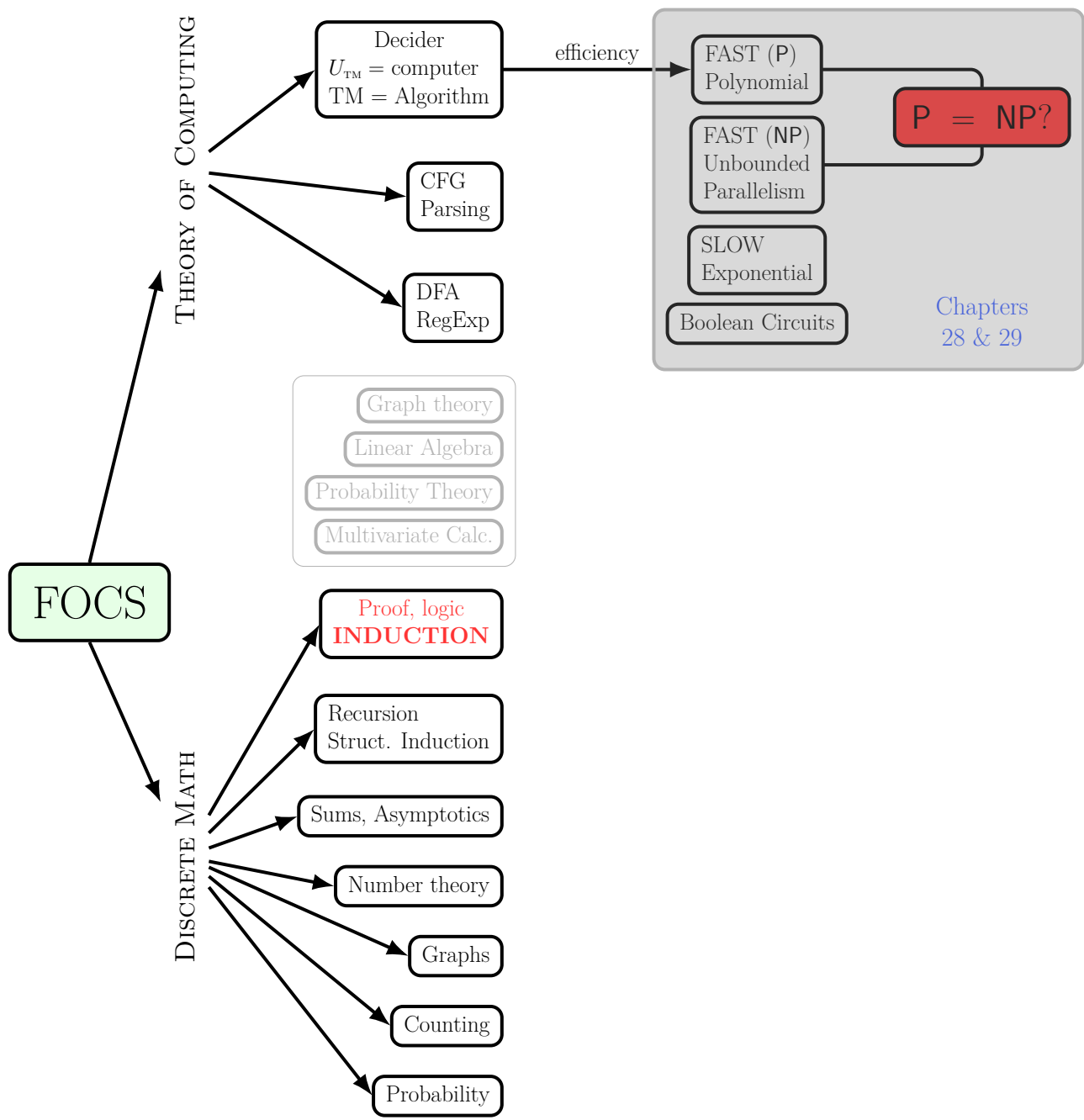
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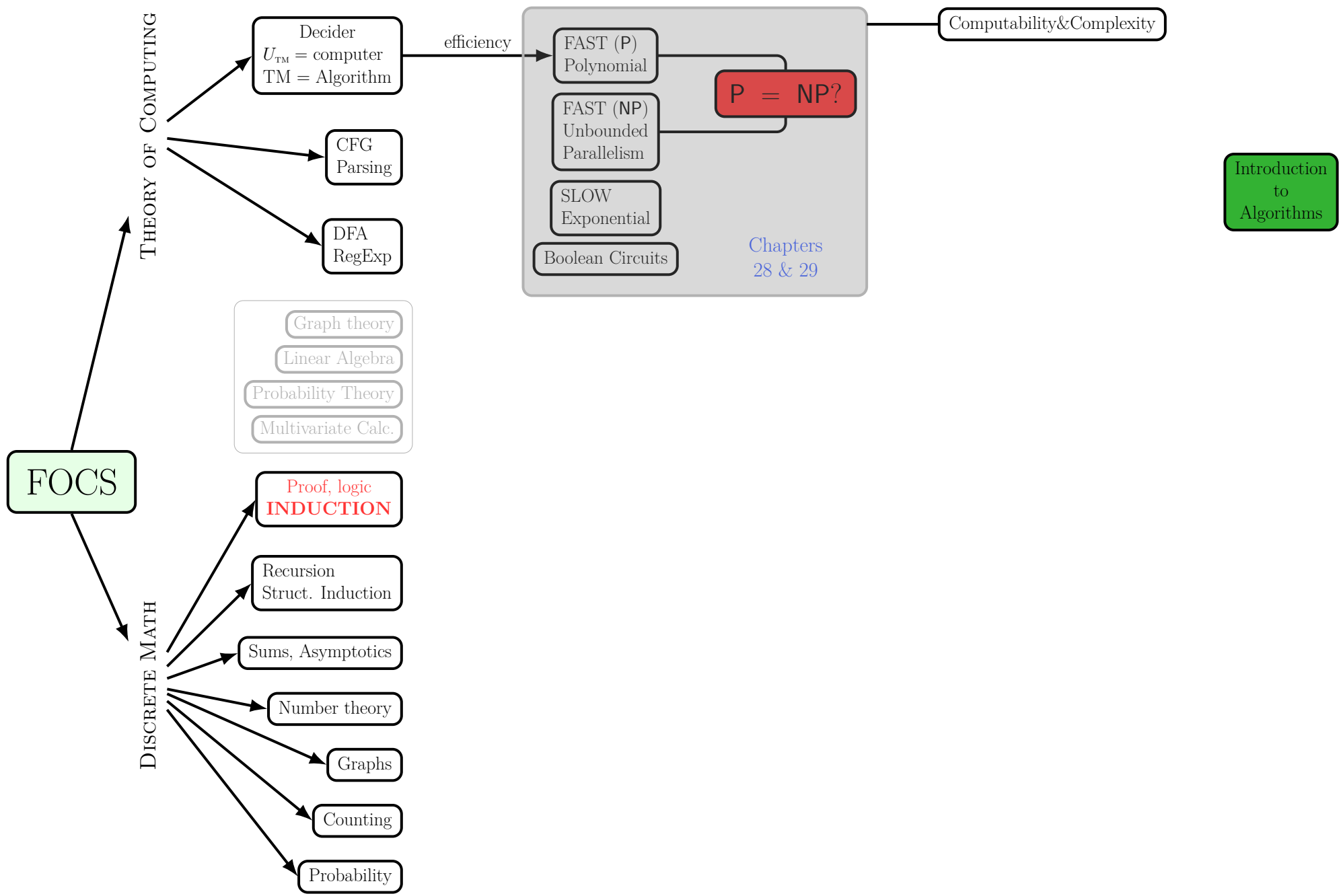
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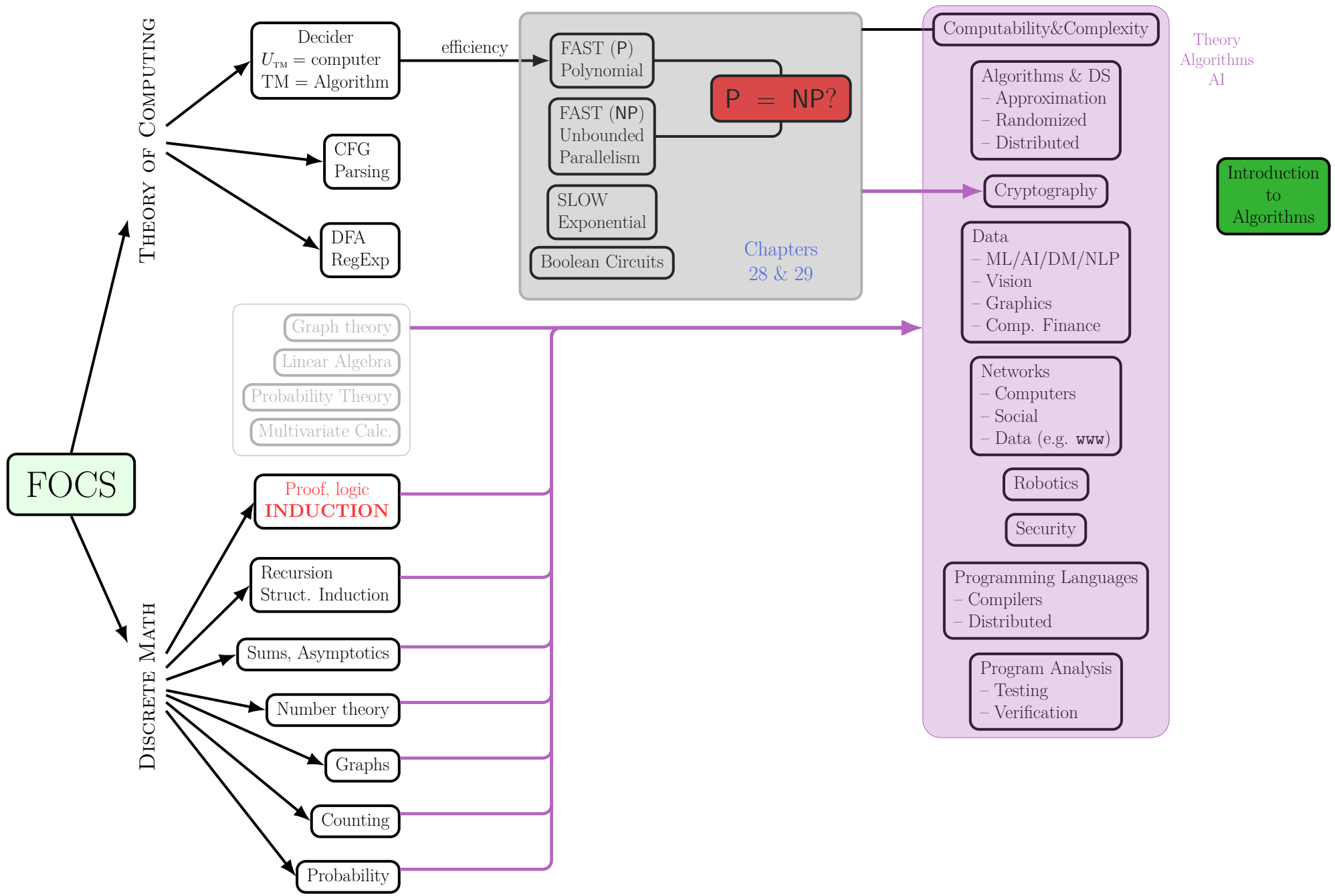
The Path Forward: Focus on Decidable Problems



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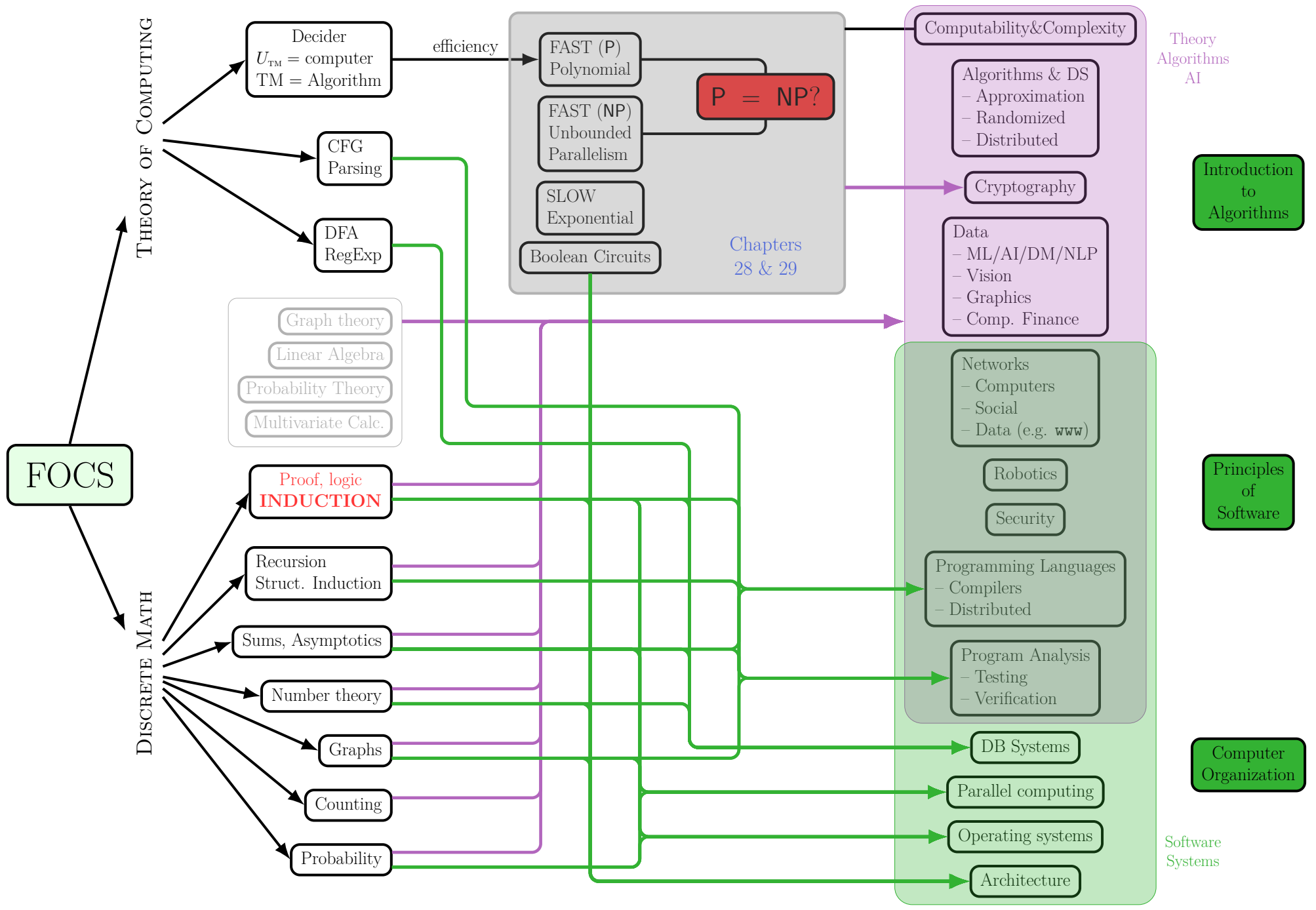
The Path Forward: Focus on Decidable Problems



Theory
Algorithms
AI

Introduction
to
Algorithms

The Path Forward: Focus on Decidable Problems



Today: Efficiency: The Class P, NP and NP-Completeness

- 1 Time complexity: Asymptotic worst-case analysis.
- 2 The class P: Efficiently solvable problems.
- 3 Polynomial on one architecture means polynomial on pretty much any architecture.

Running Time

Time Complexity

Two Tapes vs. One Tape

Decidable But Non -Polynomial

Boundary Between Efficient and Inefficient

...the high technology so celebrated today is essentially a mathematical technology.

“To err is human, but to really foul things up you need a computer.” – Paul Ehrlich

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- **Mariner rocket explodes (1962)**. Formula into code bug resulted in no smoothing of deviations.
- **WWIII (1983)?** Soviet EWS detects 5 US-missiles (bug detected sunlight reflections).
 - ▶ **Luckily Stanislav “funny feeling in my gut” Petrov** thought: “surely they’d use more missiles?”
- **Therac 25 (1985)**. Concurrent programming bug killed patients through massive 100× radiation overdose.
- **AT&T Lines Go Dead (1990)**. 75 million calls dropped (one line of buggy code in software upgrade).
- **Patriot missile defense fails (1991)**. **28 soldiers dead, 100 injured** (rounding error in scud-detection).
- **Pentium floating point long-division bug (1993)**. Cost: \$475 million – flawed division table.
- **Ariane rocket explosion (1996)**. Cost: \$500 million – overflow in 64-bit to 16-bit conversion.
- **Y2K (1999)**. Cost: \$500 billion spent because year was stored as 2 digits to save space.
- **Mars Climate Orbiter Crash (1998)**. Cost: \$125 million lost due to metric to imperial units bug.
- **Tesla Self-Driving Car (2016)**. **1 dead**. Auto-pilot didn’t “see” tractor-trailer.
- **Financial Disasters:** London Stock Exchange down due to single server bug (**2009**; billions of pounds of trading); Knight Capital computer glitch triggers stock sale (**2012**; 500 million lost and Knight’s value drops by 75%).
- **Airline Disasters:**
 - ▶ AirFrance 447 2009, **228 dead**: pitot-tube failure feeds inconsistent data to programs which then panic pilot.
 - ▶ Spanair 5022, 2008, **154 dead**: malware virus.
 - ▶ AdamAir 574, 2007, **102 dead**: navigation system errors (and pilot errors).
 - ▶ KoreanAir 801, 1997, **228 dead**: ground proximity warning system bug.
 - ▶ AeroPerú 603, 1996, **70 dead**: altimeter failures.
 - ▶ Scottish RAF Chinook, 1994, **29 dead**: faulty test program
 - ▶ AirFrance 296, 1988, **3 dead**: altimeter bug.
 - ▶ IranAir 655, 1988, **290 dead**: shot down by US Aegis combat system (misidentified as attacking military plane).
 - ▶ KoreanAir 007, 1983, **269 dead**: autopilot took plane into Soviet airspace where it got shot down.
 - ▶ Boeing 737 Max, 2018,2019, **346 dead**: attack sensor + algorithm errors.



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- **Software errors cost the U.S. \$60 billion annually in rework, lost productivity and actual damages.**

Put effort to make *sure* your program works **fully** correctly **all** the time.

