

# Foundations of Computer Science

## Lecture 25

### P versus NP

Non-deterministic Turing Machines.

Polynomial Time (P) and Non-deterministic Polynomial Time (NP).

Reducibility and NP-completeness: SAT.

Is  $NP \subseteq P$ ?



"I can't find an efficient algorithm, but neither can all these famous people."

# Last Time

1. The infinite loop.
2. Recognizable languages (allowing the infinite loop for “reject”).
3. Decidable languages (no-infinite loops allowed).
4. Programs versus computing machines.
  - The string (text/mathematical) description of a TM.
  - The Universal Turing Machine (UTM).
5. Feeding the string  $\langle TM \rangle$  as input to TM (TM is a Turing machine).
6. The language corresponding to “program verification”
  - A simpler language: TMs that reject “themselves”.
7. The *simpler* language is **undecidable**.
8. We **cannot** automate program verification!
9. Challenge problem #2, though it looks harmless, is **UNSOLVABLE!**

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# Today: Decidable and Undecidable Problems

1. Non-deterministic Turing Machines.
2. Polynomial: a definition of efficiency that stood the test of time.
3. Non-deterministic Polynomial (**NP**) versus Deterministic Polynomial (**P**).  
NP does not mean Non-Polynomial. 😊
4. The **NP**-complete problem **SAT** and the complexity hierarchy.
5. Does  $P = NP$ ? That is **THE QUESTION**.

