FINAL

Last Name: **Student ID#:**

Instructions

Fill in your Last Name and ID#. Answer **all** *questions in the space provided. Keep your answers BRIEF! You have 1 Hour and 30 min. You may consult a double sided sheet of notes during the exam.*

1	2	3	4	5	6	**TOTAL**
100	**50**	**100**	**50**	**100**	**100**	**500**

1. Let $B(T)$ be the zero coupon bond price with expiry T.

   (a) **[50]** What should the price of the following contract be:

   At $t_1 > 0$ I give you \$5 and at $t_2 > t_1$ you give me \$ 4.

   Your answer should be in terms of $B(t_1)$ and $B(t_2)$.

   (b) **[50]** Prove your answer using an arbitrage argument.

2. **[50]**Let $X_0, \ldots, X_n$ be an instruments price, and let $B_0, \ldots, B_n$ be the bond prices at times $t_0 = 0, t_1, \ldots, t_n$ (prices are the bid prices). Assume that the bid-ask spread in instrument is x and in bond is b, both constants. Assume that you have \$1 in bond at time 0. Give a linear time algorithm to compute the maximum possible wealth after time step n.

3. **[100]**Consider a stock with the dynamics $dS = S\sqrt{dt}\,\epsilon$. Consider a binomial model with time step $\Delta t = \frac{1}{4}$ for two periods $0, \Delta t, 2\Delta t$. Assume that $\lambda_{\pm} = 1 \pm \delta$ and that $e^{r\Delta t} = \frac{5}{4}$. Assume that the initial stock price $S(0) = 1$. Compute the price of the Asian call option with the strike as the arithmetic average over the three times $0, \Delta t, 2\Delta t$.

4. **[50]**Carefully define Type I and Type II arbitrage.

Carefully *state* the positive supporting price theorem and discuss how it relates to Monte-Carlo pricing methods?

5. **[100]**Consider a two period problem. Define the rate of return r by $r = \log \frac{W(T)}{W(0)}$, where W represents wealth. Two stocks S_1, S_2 have rates of return r_1, r_2 respectively. Consider a portfolio Π in which we spend a fraction f_1 of the initial wealth on S_1 and f_2 on S_2 $(f_1 + f_2 = 1)$. Give an expression for the rate of return r_Π of this portfolio and show that

$$\max\{r_1, r_2\} \geq r_\Pi \geq f_1 r_1 + f_2 r_2$$

When does the left inequality become equality; when does the right?

6. **[100]**From the previous problem, suppose that the rates of return r_1, r_2 have a Normal distribution with means $\mu_1 = 0.05, \mu_2 = 0.1$ and variances $\sigma_1^2 = \sigma_2^2 = 1$. Let the correlation between the rate of returns be $\rho = 0.5$. For the portfolio in which $f_1 = f_2 = 0.5$, Give an expression for the VaR with confidence threshold $\alpha = 0.95$ and initial wealth $W(0) = 100$.

   [Your final answer can be in terms of the CDF of the standard Normal, $\phi()$, or its inverse; if you need to, you may approximate the distriution of a sum of log-Normal random variables by a Normal distribution.]

SCRATCHWORK: