FINAL

Last Name: **Student ID#:**

Instructions

Fill in your Last Name and ID#. Answer **all** *questions in the space provided. Keep your answers BRIEF! This final is open book.*

1	2	3	4	5	6	**TOTAL**
50	**100**	**50**	**100**	**100**	**100**	**500**

1. Let $B(T)$ be the zero coupon bond price with maturity T.

   (a) **[25]** What should the price of the following contract be:

   At $t_1 > 0$ I give you \$5 and at $t_2 > t_1$ you give me \$ 4.

   Your answer should be in terms of $B(t_1)$ and $B(t_2)$.

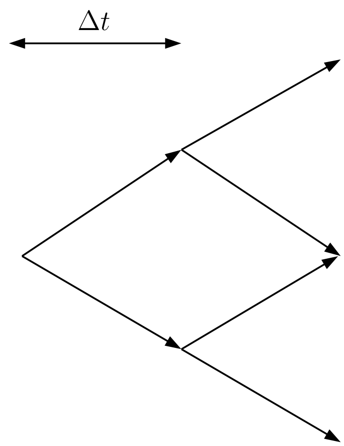
   (b) **[25]** Prove your answer using an arbitrage argument.

2. A stock has binomial dynamics (in the real world) given by $p = \frac{1}{2}$, $\lambda_+ = \frac{3}{2}$, $\lambda_- = \frac{1}{2}$ and suppose that the discount factor is $e^{r\Delta t} = \frac{5}{4}$. At time $t = 0$, the stock price is $S = 1$. Consider the 2-step economy at time steps $t = \Delta t, 2\Delta t$.

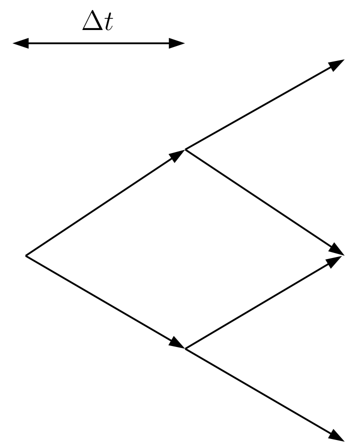
The derivative f is a European Call Option: maturity $T = 2\Delta t$ and strike $K = \frac{1}{2}$.

(a) **[50]**

Fill in the real world stock dynamics below, including probabilities



Show the cashflows of the derivative f for the various outcomes of the stock



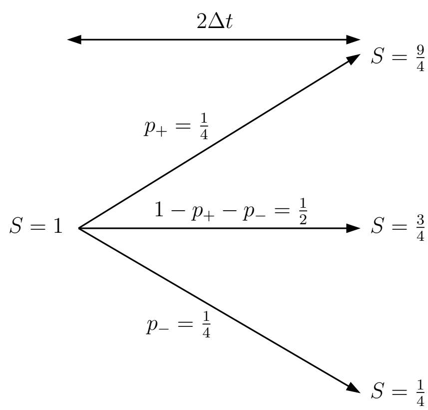
What is the risk neutral probability $\tilde{p}$?

(b) **[50]** Let F be the no-arbitrage price of the derivative f. **Compute** F.

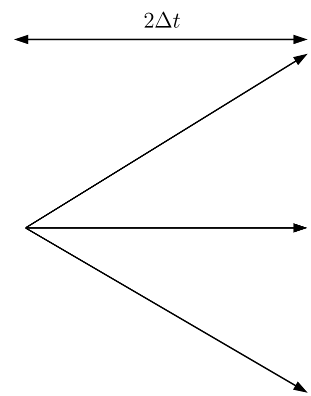
3. As in Question 2, set $e^{r\Delta t} = \frac{5}{4}$. But now the stock has trinomial dynamics over a *single* time-step $2\Delta t$. The derivative f is the same call option as in Question 2.

(a) **[25]**

The stock dynamics with probabilities are below, for one time step $T = 2\Delta t$.



Show the cashflows of the derivative f for the various outcomes of the stock



(b) **[25]** This is a two period economy with $T = 2\Delta t$. What is $\mathbf{S}$, the vector of *all* instrument prices at time 0. What is Z, the matrix of possible prices at time T in the various states of the world. (Use F to denote the (unknown) price of the derivative at time 0)

4. **[50] Use the positive supporting price theorem to obtain the best upper and lower bounds you can on the price F of the derivative f, for the setup in Question 3.**

[50] [Hard] In question 2 the no-arbitrage price F is unique. In Question 3 the no-arbitrage price is not defined uniquely even though the derivative has exactly the same cashflows with the same probabilities (in the real world). **Explain this discrepancy. Be precise.**

5. Two random variables I and G have correlation $\rho(I,G) = -\frac{1}{\sqrt{2}}$. Here is what you know:

$$E[G] = 0; \qquad var[I] = 2; \qquad var[G] = 3;$$

You generate 100 joint samples, $(I_1, G_1), (I_2, G_2), \cdots, (I_{100}, G_{100})$ and find that

$$\hat{I} = \frac{1}{100}\sum_{i=1}^{100} I_i = -\frac{1}{2}; \qquad \hat{G} = \frac{1}{100}\sum_{i=1}^{100} G_i = \frac{1}{20}$$

(a) **[50] You want to estimate $E[I]$ with Montecarlo. What is the MonteCarlo estimate of $E[I]$ and estimate the error bar?**

(b) **[50] Construct a better *unbiased* estimate than the simple Monte Carlo estimate and give its error bar (construct the best estimate you can).**

6. **[100]** Two stocks S_1 and S_2 follow a geometric Brownian motion,

$$S_i(T) = S_i(0)e^{\eta_i}$$

where $\eta_i \sim N(0.5, 1)$ and $\rho(\eta_1, \eta_2) = 0.5$ (correlation coefficient). A portfolio invests a *fraction* of wealth $w_i = \frac{1}{2}$ in each instrument. Your initial wealth $W(0) = \$100$.

Compute the 1%-**VaR** (the *loss* you may incurr with 1% chance)

[Your final answer can be in terms of the CDF of the standard Normal, $\phi()$, or its inverse; if you need to, you may approximate the distriution of a sum of log-Normal random variables by a Normal distribution.]

SCRATCHWORK: