

ASSIGNMENT 1

Homeworks are due at the beginning of class on the due date. The point value for the 6000 level is indicated in small font.

1 (90 (50) points) Bonds and Arbitrage

- (a) [30 (20)] A coupon paying bond gives coupon payments of X_1, X_2, \dots, X_n at times t_1, t_2, \dots, t_n and a final principal payment of F at time T . Use an arbitrage argument to show that the price of this bond today should be

$$P = \sum_{i=1}^n X_i B(0, t_i) + F \cdot B(0, T),$$

where $B(0, t)$ is the usual zero coupon bond.

Rewrite this formula in terms of the interest rate, r , which you can assume is a constant.

- (b) [40 (20)] On the website for this problem set, you can download zero coupon bond data in the file `zero_coupon.dat`. This data was obtained from www.bondtrac.com on 09/02/05.

This file contains five columns, the first three are the month, day and year of expiry of the bond. The fourth is the time to expiry in years, T , and the fifth is the price of the bond, $B(0, T)$. The value of the payment (face value) at time T is \$100, not \$1 as discussed in class. Note that some dates are repeated because typically data is collected from multiple sources. You need to deal with this issue.

- (i) Give a plot of $B(0, T)$ versus T and explain the general observed shape.
 - (ii) Using the prices and times to expiry, obtain the implied interest rate to that time. Plot the implied interest rate as a function of T .
Is this curve a constant?
If not, suggest explanations for why not.
- (c) [20 (10)] Using the zero coupon bond data, price the following contract.

I will pay you \$1 on 02/15/2006 in return for \$2 on 08/15/2009

2 (10 (50) points) Bond Portfolio Immunization

Exercise 2.10 in the notes for instruments. (Exercises 2.9, 2.8, 2.9 may be useful.)