ASSIGNMENT 2

Homeworks are due at the begining of class on the due date. The point value for the 6000 level is indicated in small font.

1 (50 (20) points) Bond Portfolio Immunization

You have a future commitment of \$1000 in 0.75 years. There are three bonds availble on the market, with maturities 0.25, 0.5 and 1.0 years. Assume that the interest rate is 0.05. What is the optimaly immunized "portfolio" of these bonds that you will carry in order to meet your commitment in 0.75 years.

2 (40 (50) points) Exponential Moving Average

You are given a time series for the stock price, $S_1, S_2, \ldots, S_N$. Assume that $\tau = 1$ and that the exponential decay factor is λ. You may assume that $S_i = 0$ for $i < 1$.

(a) Give an algebraic expression (a summation formula) for $MV(t)$.

(b) Give a *linear time* algorithm (linear in N) to compute the time series $MV(t)$.
(Hint: First show that $MV(t+1) = e^{-\lambda}MV(t) + (1 - e^{-\lambda})S_{t+1}$.)

(c) From the website, you can download ibm.dat. The first column is the time (in minutes), and the second is the quote $((bid + ask)/2)$. Use your linear time algorithm to compute the moving average curves for $\lambda = \frac{1}{12}, \frac{1}{96}, \frac{1}{1920}$, and plot them together with the original.

3 (10 (30) points) Pattern Trading

Given a data base of patterns and a stock price time series, develop efficient algorithms to extract the set of paterns from the data base that are K-*active* at time t.

We define the K-active patterns as follows. View each pattern P as a string $P = p_1 p_2, \ldots, p_{|P|}$. Also view the stock price time series up to time t, $S(t) = S_0 S_1 \ldots S_t$ as a string. A pattern P is K-*active* at time t if the prefix of length K of the corresponding pattern string P matches the suffix of length K of the price string $S(t)$.

Define the database size D as the total length of all the patterns in the database. Let M_t be the total length of the patterns that are K-active at time t, and let $M = \sum_t M_t$ be the total length of the active patterns for this stock time series (M is the size of the output). Suppose that you are allowed to preprocess the database of patterns, and that the stock price time series $S_0, S_1, \ldots, S_N$ and the value of K are inputs to the algorithm. With $O(D)$ preprocessing, your algorithm should run in $O(NK + M)$.

[Hint: You may want to preprocess your database into a prefix tree.]

A typical application of such algorithms would be to define the stock price series as a string over a three letter alphabet: down (-1), no significant move (0), up (+1). The database of pattern strings are also similarily defined. One then takes all the K-active patterns at a time t. Those patterns whose matching prefix is a proper prefix give a prediction of the future. These patterns can be "voted together" somehow to give a prediction of the future prices of the stock. If, for example, the prediction is sufficiently positive, one might then consider a buy trade.