ASSIGNMENT 2

Homeworks are due at the beginning of class or in my mailbox by 2pm on the due date. The point value for the 6000 level is indicated in small font.

1 (25 (0) points) Bond Portfolio Immunization

You have a future commitment of $1000 in 0.75 years. There are three bonds available on the market, with maturities 0.25, 0.5 and 1.0 years. Assume that the interest rate is 0.05. What is the optimally immunized “portfolio” of these bonds that you will carry in order to meet your commitment in 0.75 years.

2 (25 (35) points) Exponential Moving Average

You are given a time series for the stock price, $S_1, S_2, \ldots, S_N$. Assume that $\tau = 1$ and that the exponential decay factor is $\lambda$. You may assume that $S_i = 0$ for $i < 1$.

(a) Give an algebraic expression (a summation formula) for $MV(t)$.

(b) Give a linear time algorithm (linear in $N$) to compute the time series $MV(t)$ for all $t = 1, 2, \ldots, N$.

(Hint: First show that $MV(t + 1) = e^{-\lambda}MV(t) + (1 - e^{-\lambda})S_{t+1}$.)

(c) From the website, you can download ibm.dat. The first column is the time (in minutes), and the second is the quote $((\text{bid} + \text{ask})/2)$. Use your linear time algorithm to compute the moving average curves for $\lambda = \frac{1}{12}, \frac{1}{96}, \frac{1}{1920}$, and plot them together with the original.
3 (0 (15) points) Pattern Trading

You have a data base of patterns and a stock price time series of length $N$. In both cases they are discretized into $+$ (up) and $-$ (down). Here is an example with 3 patterns and the price time series:

<table>
<thead>
<tr>
<th>time</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
<td>$+$</td>
<td>$+$</td>
<td>$-$</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

Patterns

$P_1$  $++-+$

$P_2$  $+-+-+-$

$P_3$  $+++-+$

A pattern is $K$-active if its prefix of size $K$ matches the suffix of size $K$ of the stock price time series. For example: at time step 10, $P_1$ is 3-active, $P_2$ is 2-active and $P_3$ is not active; at time step 9, $P_1$ is 2-active, $P_2$ is 1-active and $P_3$ is 3-active. In general, define a $K$-active patterns as follows. A pattern $P$ as a string $P = p_1p_2,...,p_{|P|}$. At time $t$, the stock price time series is also a string (up to time $t$), $S(t) = S_1...S_t$. A pattern $P$ is $K$-active at time $t$ if the prefix of length $K$ of the corresponding pattern string $P$ matches the suffix of length $K$ of the price string $S(t)$.

(a) Develop the most efficient algorithm you can to extract the set of patterns from the data base that are $K$-active at time $t$ for $t = 1, 2, 3, ... N$. Give the running time of your algorithm in terms of $D, M, K, N$ (see below).

Define the database size $D$ as the total length of all the patterns in the database. Let $M_t$ be the total length of the patterns that are $K$-active at time $t$, and let $M = \sum_t M_t$ be the total length of the active patterns for this stock time series ($M$ is the size of the output). Suppose that you are allowed to preprocess the database of patterns, and that the stock price time series $S_1, ..., S_N$ and the value of $K$ are inputs to the algorithm. With $O(D)$ preprocessing, there is an algorithm which runs in $O(NK + M)$ time. (You preprocess the database of patterns into a prefix tree).

(b) Suggest ways in which you could use your algorithm to do stock trading?