

## ASSIGNMENT 3

*Homeworks are due at the beginning of class on the due date. The point value for the 6000 level is indicated in small font.*

### 1 (80 (50) points) Trading Systems

Let  $r_i$ ,  $i = 0$  to  $N$  be a series for which we are interested in computing statistics.

- (a) **[30 (20)]** Give a linear time algorithm to compute the mean return, the Sharpe ratio and the Sterling ratio that makes *one* pass through the data to compute all three.
- (b) **[15 (10)]** From the website, you can download `ibm.dat`, `dell.dat` and `bond.dat`. The first column is the time (in minutes), and the second is the quote  $((bid + ask)/2)$ .

- (i) Consider the buy and hold strategy. Compute the the three statistics for the three intruments. You will encounter a problem with the Bond?

For this reason, one usually subtracts the risk-free return from each  $r_i$  at each time step, before computing the statistics. These are refered to as the *statistics adjusted for the risk-free rate*. Repeat the computations adjusting for the risk-free rate (assume that  $\frac{0}{0} = 0$ ). When you adjust for the risk free rate, which of the following will change:  $\mu, \sigma, MDD$ ?

These are buy and hold benchmarks. One could compare any trading strategy with these to determine if they are significantly better than these trivial strategies.

- (ii) Compute the statistics adjusted for the risk-free rate using a randomly generated strategy on bond and IBM. Assume that the trading costs for each instrument are the same,  $f_{bond} = f_{dell} = f_{ibm}$ . Use the trading costs:  $[0, 0.0001, 0.0002, 0.0005]$ .

Repeat this experiment 1000 times and give a table of the average values of the statistics for each trading cost.

This is another bench-mark for performance. If you have a trading system, you can see how much better it is than a random trading system and whether this difference is significant. What conclusion can you draw about the buy and hold strategies.

- (c) **[35 (20)]**

Implement the algorithm we discussed in class to determine the optimal trading strategy. Use this algorithm to determine the optimal trading strategy for (BOND, IBM), (BOND,DELL) and (DELL,IBM), using trading cost  $f_{bond} = f_{dell} = f_{ibm} = 0.02$

**NOTE:** Make sure that your algorithm is a linear time algorithm. This is not a completely trivial problem.

**[Hint:** The problem arises when you construct the optimal strategy to time  $t$  by appending a 1 or 0 to a previous optimal strategy. If you copy over the previously optimal strategy each time

you do this, the algorithm will be  $n^2$ . You might want to consider the relationship between back-tracking and dynamic programming. ]

Give the three statistics for each optimal strategy as well as a plot of the cumulative return with time for each.

Give an explanation for why (BOND,STOCK) is better than (STOCK,STOCK)?

## 2 (20 (50) points) Optimal Trade Entry/Exit

Consider the following exit scenario. You wish to sell 10 shares ( $K=10$ ) of a stock by time  $T = 10$ . Assume the stock price is decreasing linearly,  $P_t = 100 - \alpha t$ , where  $\alpha$  is a parameter we will play with. Assume that the impact functions are linear,  $f(x) = g(x) = \beta x$  where  $\beta$  is also a parameter to we will play with. You can only sell an integral number of shares at a time.

- (a) If you were to do brute force search for the optimal exit strategy, how many possible exit strategies are there?
- (b) Implement efficiently the dynamic programming algorithm to compute the optimal exit and determine the optimal exit strategy together with the maximum proceeds from the sale when  $\alpha = \{0, 1, 2\}$ , with  $\beta = 1$ . Repeat with  $\beta = 2$ .
- (c) Explain intuitively what is going on.