ASSIGNMENT 4

Homeworks are due at the begining of class on the due date. The point value for the 6000 level is indicated in small font.

1 (70 (50) points) Arbitrage - Two Period

Consider the following two period economies. Determine (to within reasonable precision) which ones have arbitrage opportunities. If you think that an economy has an arbitrage opportunity, give the portfolio that results in the arbitrage opportunity. If you believe that there is no arbitrage opportunity, then give the risk neutral probabilities (or the Martingale measure). Note, sometimes the risk neutral probabilities may not be unique. In all cases, justify your answer.

We use **S** to denote the current instrument prices, and **Z** to denote the future instrument price matrix.

(a)

$$(i) \qquad \mathbf{S}_1 = \begin{bmatrix} 5 \\ 14 \\ 20 \\ 33.5 \end{bmatrix} \qquad \mathbf{Z}_1 = \begin{bmatrix} 14 & 5 & 5 \\ 10 & 12 & 17 \\ 10 & 17 & 25 \\ 10 & 9 & 8 \end{bmatrix}$$

$$(ii) \qquad \mathbf{S}_2 = \begin{bmatrix} 5.47 \\ 14 \\ 20 \\ 8.825 \end{bmatrix} \qquad \mathbf{Z}_2 = \begin{bmatrix} 14 & 5 & 5 \\ 10 & 12 & 17 \\ 10 & 17 & 25 \\ 10 & 9 & 8 \end{bmatrix}$$

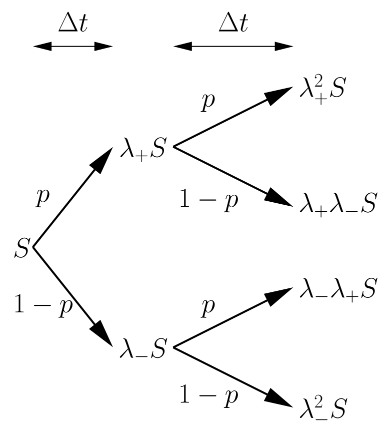
(b)

$$(i) \qquad \mathbf{S}_1 = \begin{bmatrix} 15 \\ 10 \end{bmatrix} \qquad \mathbf{Z}_1 = \begin{bmatrix} 14 & 5 & 5 \\ 10 & 12 & 17 \end{bmatrix}$$

$$(ii) \qquad \mathbf{S}_2 = \begin{bmatrix} 8 \\ 13 \end{bmatrix} \qquad \mathbf{Z}_2 = \begin{bmatrix} 14 & 5 & 5 \\ 10 & 12 & 17 \end{bmatrix}$$

2 (30 (50) points) Arbitrage - Multi-Period

Consider the following stock dynamics according to our *binomial* model with probability p.



A bond has values B, $Be^{r\Delta t}$ and $Be^{2r\Delta t}$ at the three times, where $e^{r\Delta t} = \frac{5}{4}$. There is a variant of the call option, instrument I, with current price C. The strike is $K = \frac{S}{4}$ and if the stock initially goes down, you *must* exercise, getting a cashflow $\lambda_- S - K$ at time Δt. If the stock initially goes up, then you *must* wait and exercise at the end of the next period.

The stock dynamics are given by $p = \frac{1}{2}$, $\lambda_+ = \frac{3}{2}$, $\lambda_- = \frac{1}{2}$. There are two possible prices (P, $\tilde{P}$) that one could select for instrument I. P is the expected value of the discounted future cashflows according to the real dynamics, p. $\tilde{P}$ is the expected value of the discounted future cashflows according to the risk neutral (fictitious pricing) dynamics, $\tilde{p} = \frac{e^{r\Delta t} - \lambda_-}{\lambda_+ - \lambda_-}$.

(a) Compute P, $\tilde{P}$.

(b) If the price is actually P, then then show that there is an arbitrage opportunity – i.e., construct the arbitrage opportunity.

(c) What happens to your arbitrage opportunity if the price were $\tilde{P}$.

(d) Show that if the price is not $\tilde{P}$, then there is an arbitrage opportunity.

[Hint: You may want to consider what the price of I should be at time Δt if the stock went up. Use this knowledge to construct an arbitrage *strategy*.

Note: that in multi-period economies, we have to generalize the concept of an arbitrage protfolio to an arbitrage strategy, where one is allowed to change the portfolio according to the stock path. As long as at any time, there is no nett investment using the strategy, and the strategy always yields, at time $2\Delta T$, non-negative and sometimes positive return, then there is arbitrage.]