Intro to Comp. Finance RPI
Magdon-Ismail 2022

ASSIGNMENT 5

Homeworks are due at the begining of class on the due date. The point value for the 6000 level is indicated in small font.

Consider the following options on a stock S (in all cases S_0 is the initial stock price):

1. $C(S_0, K, T)$: the *European Call* option with strike K and exercise time T.
2. $B(S_0, K, B, T)$: the *Barrier option* with payoff K, barrier B and horizon T. If and when the price hits the barrier B the holder may buy at B and sell at K.
3. $A(S_0, T)$: the *average strike Asian call option* with expiry T – a European call option with strike at the average value of the stock price over $[0, T]$.
4. $M(S_0, T)$: the *minimum strike call option* – a European call option with strike at the minimum price over $[0, T]$.

1 (50 (50) points) Simulating Stock Paths

Assume the initial stock price is S_0 and it follows real and risk neutral dynamics given by

$$\Delta S = \mu S \Delta t + \sigma S \Delta W \qquad \Delta\tilde{S} = r\tilde{S}\Delta t + \sigma\tilde{S}\Delta\tilde{W}.$$

Write a program that takes as input $\mu,\ r,\ \sigma,\ S_0,\ T, \Delta t$ and simulates the stock price from time 0 to T in time steps of Δt for the risk neutral world, using each of the following modes:

(a) Binomial mode I: compute $\lambda_\pm$ from μ, σ assuming that $p = \frac{1}{2}$, and then computing $\tilde{p}$.

(b) Binomial mode II: compute $\lambda_\pm$ from μ, σ assuming that $p = \frac{2}{3}$, and then computing $\tilde{p}$.

(c) Continuous mode: using the continuous risk neutral dynamics r, σ generate at time step Δt as if the discrete model were taken to the limit $dt \to 0$.

For each of the three methods, give plots of representative price paths for $S_0 = 1,\ \mu = 0.07,\ r = 0.03,\ \sigma = 0.2,\ T = 2$ using $\Delta t = 0.1, 0.01, 0.0001$.

2 (50 (50) points) Pricing Options Using Monte Carlo

Use Monte Carlo simulation to price the 4 options. Assume that $S_0 = 1$, $\mu = 0.07$, $r = 0.03$, $\sigma = 0.2$, $T = 2$. For each case, use each of the three modes above, and compute the price using each of the three time discretizations, $\Delta t = 0.1, 0.01, 0.0001$.

In all cases, make some intelligent choice for the number of Monte Carlo samples that you need to take to get an accurate price. [Hint: first take a few samples to get an estimate of the variance of the Monte Carlo sample values.]

(a) Compute $C(1, 1, 2)$ as efficiently as you can and compare with the analytic formula.

(b) Compute $B(1, 1, 0.95, 2)$

(c) Compute $A(1, 2)$, for three possible definitions of "average": the harmonic, arithmetic, and geometric means. Explain the relative ordering of these prices.

(d) Compute $M(1, 2)$.

3 Bonus - Unspecified Number of Points

The barrier and the minimum option values can be computed analytically. Can you compute them, and compare with your numerical estimates?