ASSIGNMENT 7

Homeworks are due at the begining of class on the due date. The point value for the 6000 level is indicated in small font.

1 (80 (60) points) Price the American Put Option

Implement the following three methods for pricing the American Put option.

$B(S_0, K, T, r, \sigma, N)$: Use the Binomial model with N discretization steps - use the risk neutral probability.

$LSM(S_0, K, T, r, \sigma, N, M, L)$: Use the LSM algorithm with N discretization steps and M paths to obtain the exercise thresholds, doing regression with second order polynomials, and then use another L paths to price the option.

$OPT(S_0, K, T, r, \sigma, N, M, L)$: Use N discretization steps and M paths to obtain the exercise thresholds. To get the thresholds, do not use regression, but use the algorithm to obtain the optimal threshold as discussed in class. Then use another L paths to price the option.

Test your algorithm: I computed $B(10, 9, 1, 0.04, 0.5, N)$ for $N \in \{10^2, 10^3, 10^4, 10^5\}$ and obtained $\{1.26369, 1.26590, 1.26558, 1.26554\}$. In the rest of this homework, set $S_0 = 10, K = 10, T = 2, r = 0.05; \sigma = 0.2$, and for the Monte Carlo approaches, use the continuous model for generating the paths.

(a) For $N \in \{100, 1000\}$, obtain $B(S_0, K, T, r, \sigma, N)$.

(b) For $(N, M, L) \in \{(50, 10000, 10000), (100, 5000, 5000), (50, 5000, 50000)\}$, obtain $LSM(S_0, K, T, r, \sigma, N, M, L)$. Compare the runtimes and accuracy.

(c) For (N, M, L) as in the previous part, obtain $OPT(S_0, K, T, r, \sigma, N, M, L)$. Compare the runtimes and accuracy.

(d) Suppose that you did not use *new* paths to compute the price, but the same paths on which you got the exercise thresholds. Compute $OPT(S_0, K, T, r, \sigma, 50)$ using 50 paths in this way. Repeat this many times, and take the average.

Now compute $OPT(S_0, K, T, r, \sigma, 50, 50, 50)$ many times and take the average. Explain your results.

2 (20 (40) points) Monte Carlo for the American Put Option under Max Exercise

In this problem, consider two very simple Monte Carlo approaches to pricing the American put.

$MAX_1(S_0, K, T, r, \sigma, N, M)$**:** For a Monte Carlo path, a natural way to exercise the option is exercise at the minimum stock price (if it is below the strike K). Thus, we can construct the cash flows for any path to be $K - \min_t S_{path}(t)$. Since this has to be discounted to time 0, what will the discount factor be?

Let $MAX_1(S_0, K, T, r, \sigma, N, M)$ denote the price obtained this way, using N time discretization steps, and M paths.

$MAX_2(S_0, K, T, r, \sigma, N, M)$**:** We immediately notice that the above prescription may be sub-optimal. Why not exercise when the discounted cash-flow is maximum. Thus, we can construct the *discounted* cash flow for a path to be $\max_t\{e^{-rt}(K - S_{path}(t))^+\}$.

Let $MAX_2(S_0, K, T, r, \sigma, N, M)$ denote the price obtained this way, using N time discretization steps, and M paths.

Use the same values of S_0, K, T, r, σ as in the previous problem.

(a) Compute $MAX_1(S_0, K, T, r, \sigma, 50, 50000)$.

(b) Compute $MAX_2(S_0, K, T, r, \sigma, 50, 50000)$.

(c) Compare your answers above to your answers for the previous problem and explain your findings.