Quadratic Convergence to the Optimum

Suppose that we wish to minimize $E(w) = g(w)^2$. Suppose that $g(w)$ has a single zero at $w = a$, i.e., $g(a) = 0$. Suppose that $g(w)$ is a continuously differentiable function of all orders. We assume that $g'(a) \neq 0$ and that $g''(a) \neq 0$. Suppose that w_n is close to a. The Newton method attempts to find the solution a by constructing a series of estimates as follows.

$$w_{t+1} = w_t - \frac{g(w_t)}{g'(w_t)}$$

1. If ϵ_t is sufficiently close to zero so that the quadratic taylor approximation is accurate, show that

   $$\epsilon_{t+1} = -\frac{1}{2} g''(a) \left[1 - \frac{2}{g'(a)} \right] \epsilon_t^2$$

   In other words, the convergence is quadratic! Thus quadratic convergence is not too good to be true!

2. Show that if $g'(a) = 0$, one loses the quadratic convergence.