QUIZ 1: 60 Minutes

Last Name: Solutions

First Name: _____

RIN: _____

Section: _____

Answer **ALL** questions.
NO COLLABORATION or electronic devices. Any violations result in an F.
NO questions allowed during the test. Interpret and do the best you can.

GOOD LUCK!

Circle at most one answer per question.
10 points for each correct answer.

You **MUST** show work to ensure getting full credit.

Total
150

1. $\sqrt{2}$ is what kind of number?

E

- [A] A natural number.
- [B] An integer.
- [C] A rational number.
- [D] A member of the set $\mathbb{Q}$.
- [E] None of the above.

$\sqrt{2}$ is irrational.
We proved it in class by contradiction

2. What is the set $\mathbb{Z} \cap \overline{\mathbb{N}} \cap S$, where S is the set of perfect square numbers. The universal set is $\mathbb{R}$.

B

- [A] $\varnothing$, the empty set.
- [B] $\{0\}$.
- [C] S.
- [D] The non-positive integers.
- [E] The set is not well defined.

only thing in common is 0.

$\mathbb{Z} = \{0, \pm 1, \ldots\}$
$\overline{\mathbb{N}} = \{$ numbers other than $1, 2, 3, \ldots\}$
eg. $1.2, -7$.
$S = \{0, 1, 4, 9, \ldots\}$

3. $A = \{2, 5\}$ and $B = \{3, 7\}$. What is the Cartesian Product $A \times B$?

C

- [A] $\{6, 14, 15, 35\}$.
- [B] $\{2, 3, 5, 7\}$.
- [C] $\{(2,3), (2,7), (5,3), (5,7)\}$.
- [D] $\{(2,3), (3,2), (2,7), (7,2)(5,3), (3,5), (5,7), (7,5)\}$.
- [E] None of the above.

$A \times B = \{(a,b) \mid a \in A, b \in B\}$
$= \{(2,3), (2,7), (5,3), (5,7)\}$

4. How many rows in the truth table of $(p \to q) \vee p$ are T?

E

- [A] 0.
- [B] 1.
- [C] 2.
- [D] 3.
- [E] 4.

T if p is T
if ? is F, T if $(p \to q)$ is T
p is F means $p \to q$ is T
∴ always T.

5. IF (you ace the final OR the quiz), THEN you get an A. You did get an A. *Did you ace the final*?

E

- [A] Yes, for sure.
- [B] No, for sure.
- [C] Yes, if and only if you did not ace the quiz.
- [D] Yes if you did not ace the quiz; otherwise we don't know.
- [E] None of the above.

p: ace final
q: ace quiz
r: get A
$(p \vee q) \to r$

p	q	r	$(p \vee q) \to r$
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	T

$(p \vee q \to r)$ and r are true
p, q can be anything

6. Which mathematical claims are T. Note, $(a,b,c) \in \mathbb{R}^3$ stands for triples of real numbers (a,b,c).

✓ (I) IF $\Big(\forall (a,b,c) \in \mathbb{R}^3 : ax^2+bx+c=0\Big)$, THEN $x=0$ → set $a,c=0$ $b=1$ → $x=0$.

X (II) $\forall (a,b,c) \in \mathbb{R}^3 : \Big(\text{IF } ax^2+bx+c=0, \text{ THEN x=0}\Big)$ → set $a=1$ $b=2$ $c=1$
$x^2+2x+1=0$ is T with $x=-1$
but $x=0$ is F
∴ this implication does not hold for all a,b,c

A

A I only.

B II only.

C Both I and II.

D Neither I nor II.

7. For a natural number n, consider the implication: IF $n \geq n+1$, THEN $n+1 \geq n+2$
Determine whether the *implication* is T or F?

A

A Always T no matter what n is.

B Always F no matter what n is.

C T only for positive n.

D T only for negative n.

E None of the above.

Suppose $n \geq n+1$
$n+1 \geq n+2$ ∴ implication is T.

8. What method of proof is used to prove that $\sqrt{2}$ is irrational?

C

A Direct proof.

B Contraposition proof.

C Proof by contradiction.

D Induction.

E Strong induction.

Assume $\sqrt{2} = \frac{a}{b}$ [lowest terms]
Contradiction: $2 = \frac{a^2}{b^2} \rightarrow a^2 = 2b^2$ ← even
$a = 2k$
$b^2 = 2k^2$ ← even
$b = 2l$
a and b are both even, which can't be.

9. Which gives a valid proof of the implication $(p \vee q) \rightarrow r$.

E

A Assume p is T and show that r must be T.

B Assume q is T and show that r must be T.

C Assume r is F and show that p must be F.

D Assume r is F and show that q must be F.

E None of the above.

Assume $(p \vee q)$ is T show r is T
Assume r is T show $(p \vee q)$ is F
both p, q are F.

10. $P(n)$ = "n is even" and $Q(n)$ = "n is a sum of two primes". Translate "$\forall n \in \mathbb{N} : P(n) \rightarrow Q(n)$."

D.

A If n is a natural number then n is a sum of two primes.

B Every prime number is a natural number.

C There is a natural number which is a prime number.

D Every positive even number is a sum of two primes.

E Some positive even number is a sum of two primes.

For every n, if n is even then n is a sum of two primes

Every even n is a sum of two primes

11. $P(n)$ is a predicate (n is an integer). $P(1)$ is true; and, $P(n) \to P(2n-1) \wedge P(2n)$ is true for $n \geq 1$. Which set captures *all* n for which we can be **sure** $P(n)$ is T?

A

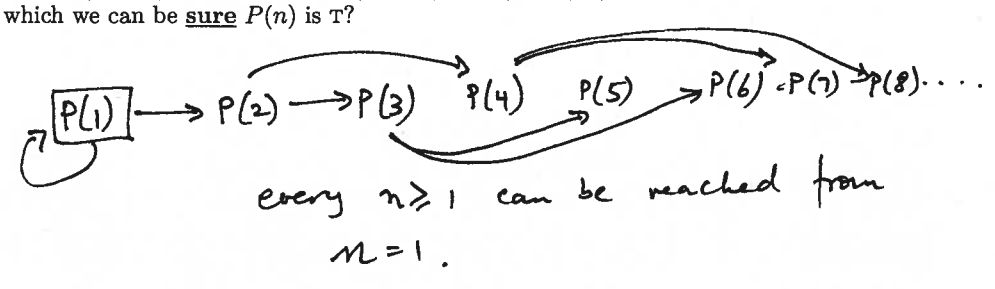
[A] All $n \geq 1$.

[B] All $n \geq 2$.

[C] All even $n \geq 1$.

[D] All even $n \geq 2$.

[E] None of the above.



every $n \geq 1$ can be reached from $n = 1$.

12. Which of the following, if any, is a valid way to prove $P(n) \to P(n+1)$ in an induction proof.

(I) Let's see what happens if $P(n)$ is T.
⋮ (valid derivations)
Look! $P(n+1)$ is T. ✓
Direct

(II) Let's see what happens if $P(n+1)$ is F.
⋮ (valid derivations)
Look! $P(n)$ is F. ✓
Contraposition

D

[A] None. [B] I only. [C] II only. [D] Both I and II

13. We wish to break a group of n students into project-teams of 4 or 7 students.

[A] IF $n \geq 7$, THEN it can be done.

[B] IF $n \geq 11$, THEN it can be done.

[C] IF $n \geq 14$, THEN it can be done.

[D] IF $n \geq 19$, THEN it can be done.

[E] None of the above are T.

D

4	5	6	7	8	9	10	11	12	13	14	15	16	17
4	×	×	7	4,4	×	×	4,7	4,4,4	×	7,7	4,4,7	4,4,4,4	×

18	19	20	21
4,7,7	4,4,4,7	4,4,4,4,4	7,7,7

$P(n) \to P(n+4)$

$\therefore$ $n \geq 18$ can do.

14. $A = \{x \mid x = 12m + 21n, \text{ for } m, n \in \mathbb{Z}\}$. T or F: $A = \mathbb{Z}$?

[A] T.

[B] F.

[C] Depends on m.

[D] Depends on n.

[E] None of the above.

B

Consider $2 \in \mathbb{Z}$

suppose $2 \in A \to 12m + 21n = 2$.

LHS div by 3 RHS not div by 3

FISHY

$\therefore 2 \notin A \therefore A \neq \mathbb{Z}$.

15. What is the function defined recursively on the right for integer $n \geq 0$.

[A] $f(n) = n!$.

[B] $f(n) = 2^n$.

[C] $f(n) = 2^n \times n^n$.

[D] $f(n) = 2^n \times n!$.

[E] None of the above.

D

$$f(n) = \begin{cases} 1 & n = 0; \\ 2n f(n-1) & n \geq 1. \end{cases}$$

n	0	1	2	3
$f(n)$	1	2	8	48
$2^n n!$	1	2	8	48

Looks like $2^n n!$.

Proof by induction

$f(0) = 2^0 0!$ ✓

Prove: $f(n) \to f(n+1)$: Assume $f(n) = 2^n n!$

$f(n+1) = 2(n+1) f(n)$

$= 2(n+1) 2^n n!$

$= 2^{n+1} (n+1)!$ ✓