QUIZ 2: 60 Minutes

Last Name: Solutions

First Name: _____

RIN: _____

Section: _____

Answer **ALL** questions.
NO COLLABORATION or electronic devices. Any violations result in an F.
NO questions allowed during the test. Interpret and do the best you can.

GOOD LUCK!

Circle at most one answer per question.
10 points for each correct answer

You **MUST** show work to get full credit.

When in doubt, TINKER.

Total
200

1. A drawer has 5 red and 5 blue socks. It's dark and you can't see. What is the *minimum* number of socks must you pull out to *guarantee* getting at least one red sock and at least one blue sock?

Worst case
5 blue + 1 red = 6.

[A] 3.
[B] 4.
[C] 5.
[D] 6
[E] None of the above.

2. What is the *minimum* number of children needed to *guarantee* two are born on the same day of the week?

pigeon hole 8
Worst case MTWRFSS

[A] 5.
[B] 6.
[C] 7.
[D] 8.
[E] None of the above.

3. What is the *minimum* number of children needed to *guarantee* two are born on a Monday?

All could be born on Tuesday → Can't be done

[A] 6.
[B] 7.
[C] 8.
[D] 367.
[E] None of the above.

4. In how many ways can you pick a debate team of 3 students from 6 students?

$\binom{6}{3} = \frac{6\times5\times4}{3!} = 20.$

[A] 20.
[B] 120.
[C] 6^3.
[D] 3^6.
[E] None of the above.

5. Which number could be a probability of some event?

[A] 3/2 > 1
[B] $\sqrt{2}$. > 1
[C] $\sqrt{2}-1$. ✓
[D] $\sqrt{2}-2$. < 0
[E] π. > 1

6. You randomly flip two independent fair coins. What is the probability of at least one flip being heads?

[A] 0.
[B] 1/4.
[C] 1/4.
[D] 3/4.
[E] 1.

Both T = 1/4
∴ At least 1 H = $1 - \frac{1}{4} = \frac{3}{4}$

7. You randomly roll a pair of fair 6-sided dice. What is the most likely sum of the dice?

[A] 5.
[B] 6.
[C] 7
[D] 8.
[E] 9.

2 3 4 5 6 7 8 9 10 11 12
$\frac{1}{36}$ $\frac{2}{36}$ $\frac{3}{36}$ $\frac{4}{36}$ $\frac{5}{36}$ $\frac{6}{36}$ $\frac{5}{36}$ $\frac{4}{36}$ $\frac{3}{36}$ $\frac{2}{36}$ $\frac{1}{36}$

8. Random variable **X** has a uniform distribution on the ten values $\{1, 2, \ldots, 10\}$. What is $\mathbb{P}[\mathbf{X} \text{ is prime}]$?

[A] 0.1.
[B] 0.2.
[C] 0.3.
[D] 0.4.
[E] None of the above.

prime: 2, 3, 5, 7 → $\frac{4}{10}$.

9. Random variable **X** has values $\{1, 2, \ldots, 10\}$ with probabilities $\{x, 2x, \ldots, 10x\}$. What is $\mathbb{P}[\mathbf{X} \text{ is prime}]$?

[A] 4/55.
[B] 17/55.
[C] 19/55.
[D] 21/55.
[E] It cannot be determined without knowing the value of x.

NO WORK NO CREDIT

$x + 2x + \cdots + 10x = \frac{10 \cdot 11}{2} x = 55x = 1$
→ $x = \frac{1}{55}$
prime: $\frac{2+3+5+7}{55} = \frac{17}{55}$

10. Randomly pick a 5-bit sequence (independent bits and each bit is 1 with probability $\frac{1}{2}$). What is the probability that the sequence starts and ends with the same bit?

[A] 1/4.
[B] 1/2.
[C] 3/4.
[D] 2/32.
[E] None of the above.

0 _ _ _ 0 or 1 _ _ _ 1
$\frac{1}{4}$ + $\frac{1}{4}$
$\frac{1}{2}$

11. Randomly pick a 5-bit sequence (independent bits and each bit is 1 with probability $\frac{1}{2}$). What is the probability that the sequence has at least one 1?

A 1/32.

B 9/32.

C 27/32.

D 31/32.

E None of the above.

All 0 has prob $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$.

At least one 1 = $1 - \text{Prob}[\text{All } 0] = 1 - \frac{1}{32} = \frac{31}{32}$

12. Which inequality for the AND of two events A and B is always correct? Assume $\mathbb{P}[A] > 0$ and $\mathbb{P}[B] > 0$.

A $\mathbb{P}[A \cap B] \leq \mathbb{P}[A] \times \mathbb{P}[B]$. × can equal $P[A]$

B $\mathbb{P}[A \cap B] \geq \mathbb{P}[A] \times \mathbb{P}[B]$. × can be 0

C $\mathbb{P}[A \cap B] \leq \min(\mathbb{P}[A], \mathbb{P}[B])$.

D $\mathbb{P}[A \cap B] \geq \min(\mathbb{P}[A], \mathbb{P}[B])$. × can be 0.

E None of the above.

$A \cap B$ is a subset of A and B

($\min(\cdot,\cdot)$ takes the minimum.)

$\therefore P[A \cap B] \leq \min[P(A), P(B)]$

13. Which formula for the AND of two events A and B is always correct? Assume $\mathbb{P}[A] > 0$ and $\mathbb{P}[B] > 0$.

A $\mathbb{P}[A \cap B] = \mathbb{P}[A] \times \mathbb{P}[B]$. ×

B $\mathbb{P}[A \cap B] = \mathbb{P}[A] + \mathbb{P}[B]$. ×

C $\mathbb{P}[A \cap B] = \mathbb{P}[A \mid B] + \mathbb{P}[B \mid A]$. ×

D $\mathbb{P}[A \cap B] = \mathbb{P}[A \mid B] \times \mathbb{P}[B]$.

E $\mathbb{P}[A \cap B] = \mathbb{P}[A \mid B] \times \mathbb{P}[B \mid A]$. ×

$P[A|B] = \frac{P[A \cap B]}{P[B]}$

14. A box has 6 fair coins and 4 two-headed coins You pick a random coin and flip. What is $\mathbb{P}[H]$?

A 4/10

B 5/10

C 6/10

D 7/10

E None of the above.

NO WORK NO CREDIT

$P[H] = P[H|\text{fair}]P[\text{fair}] + P[H|\text{two H}]P[\text{two H}]$.

$= \frac{1}{2}\cdot\frac{6}{10} + 1\cdot\frac{4}{10} = \frac{3}{10} + \frac{4}{10} = \frac{7}{10}$.

Alternatively there are 20 sides of which $6 + 4\times 2 = 14$ are H. You pick a random side so H has prob $\frac{14}{20}$

15. A box has two coins, one is fair and one is two-headed. You picked a coin randomly, flipped it twice and got HH. What are the chances you have the fair coin?

A 1/2.

B 1/3.

C 1/4.

D 1/5.

E None of the above.

NO WORK NO CREDIT

$P[\text{fair}|HH] = \frac{P[\text{fair} \wedge HH]}{P[HH]} = \frac{\frac{1}{2}\cdot\frac{1}{4}}{P[HH|\text{fair}]P[\text{fair}] + P[HH|\text{two H}]P[\text{two H}]}$

($\frac{1}{4}$, $\frac{1}{2}$, 1, $\frac{1}{2}$)

$= \frac{\frac{1}{2}\cdot\frac{1}{4}}{\frac{1}{2}\cdot\frac{1}{4} + 1\cdot\frac{1}{2}} = \frac{\frac{1}{8}}{\frac{1}{8} + \frac{1}{2}} = \frac{1}{1+4} = \frac{1}{5}$

16. Which random variable **X** has a binomial distribution?

[A] Flip a fair coin until the second head appears. **X** is the number of flips made. waiting time

[B] Draw 10 cards from a randomly shuffled deck. **X** is the number of aces drawn. dependent

[C] Hats of 100 men randomly land on the 100 heads. **X** is the number of men who get their hat back. dependent

[D] Randomly answer 20 multiple-choice questions, each with 5 answers. **X** is the number correct. 20 trials, succ prob = $\frac{1}{5}$, Count succ independent.

[E] None of them have a binomial distribution.

17. Flip 5 fair coins independently. What is the probability to get exactly 3 heads.

[A] 3/16.

[B] 4/16.

[C] 5/16.

[D] 6/16.

[E] None of the above.

NO WORK NO CREDIT

$$\binom{5}{3}\left(\frac{1}{2}\right)^3\left(1-\frac{1}{2}\right)^{5-3} = 10\cdot\frac{1}{8}\cdot\frac{1}{4}$$

$$= \frac{10}{32} = \frac{5}{16}.$$

18. You flip a fair coin 3 times. What is the expected number of heads?

[A] 0.

[B] 1.

[C] 2.

[D] 3.

[E] None of the above.

$p = \frac{1}{2}$

$E[H] = np = \frac{3}{2} = 1.5$

19. A box has two fair coins and one two-headed coin. You randomly pick a coin and flip the coin you picked 3 times. What is the expected number of heads?

[A] 0.

[B] 1.

[C] 2.

[D] 3.

[E] None of the above.

NO WORK NO CREDIT

$E[H] = E[H|\text{fair}]P[\text{fair}] + E[H|\text{twoH}]P[\text{twoH}]$

$= \frac{3}{2}\cdot\frac{2}{3} + 3\cdot\frac{1}{3}$

$= 1 + 1$

$= 2$

20. Each sex is equally likely. A couple has kids until they have at least one boy and at least one girl. What is the expected number of kids the couple will have?

[A] 2.

[B] 3.

[C] 4.

[D] 5.

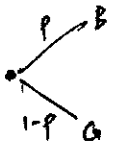
[E] None of the above.

NO WORK NO CREDIT

First is B or G, then wait for other with succ. prob $\frac{1}{2}$

→ expected wait after first = 2 ∴ 1+2=3

or



$E[X] = E[X|B]P[B] + E[X|G]P[G] = p + \frac{p}{1-p} + 1-p+\frac{1-p}{p}$

$\left(1+\frac{1}{1-p}\right)p \qquad \left(1+\frac{1}{p}\right)(1-p)$

$= 1 + \frac{p}{1-p} + \frac{1-p}{p} \qquad p=\frac{1}{2} \rightarrow 1+1+1=3$