QUIZ 2: 60 Minutes

Last Name: Solutions

First Name: _____

RIN: _____

Section: _____

Answer **ALL** questions.
NO COLLABORATION or electronic devices. Any violations result in an F.
NO questions allowed during the test. Interpret and do the best you can.

GOOD LUCK!

Circle at most one answer per question.
10 points for each correct answer

You **MUST** show **CORRECT** work to get full credit.

When in doubt, TINKER.

Total
200

1. How many subsets of $\{a, b, c, d, e\}$ contain the letter a?

B

A 8.

B 16.

C 32.

D 64.

E None of the above.

a + subset of $\{b, c, d, e\}$.

2^4 subsets

$= 16$

2. How many subsets of $\{a, b, c, d, e\}$ have exactly 3 letters?

E

A 8.

B 16.

C 32.

D 64.

E None of the above.

Choose 3 from 5

$\therefore \binom{5}{3} = \frac{5!}{3!\,2!} = \frac{5\times 4}{2} = 10$

3. All women are either young or gray or both young and gray. In a set of 50 women, 40 are young and 40 are gray. How many are both young and gray?

C

A 10.

B 20.

C 30.

D 40.

E None of the above.

$|\text{Gray} \cup \text{young}| = 50$

$|\text{Gray}| = 40$

$|\text{young}| = 40$

$|\text{Gray} \cap \text{young}| = |\text{Gray}| + |\text{young}| - |\text{Gray} \cup \text{young}|$

$= 40 + 40 - 50 = 30.$

4. You flip a 2-sided coin and roll a 6-sided die. How many outcomes are in the probability space?

D

A 6.

B 8.

C 10.

D 12.

E None of the above.

2 for Coin

6 for die

$2 \times 6 = 12$ ← Product Rule

5. An experiment has 3 possible outcomes A, B, C. $P(A) = 1/3$, $P(B) = 1/6$. What is $P(C)$?

C

A 0

B 1/4.

C 1/2.

D 1.

E None of the above.

$P(A) + P(B) + P(C) = 1$

$\rightarrow P(C) = 1 - \frac{1}{3} - \frac{1}{6} = \frac{1}{2}.$

6. Make 3 flips of a biased coin, with probability of heads 2/3. What are the chances of more heads than tails?

[A] 1/2.
[B] 8/9.
[C] 16/27.
[D] 20/27.
[E] None of the above

D

2 heads $\binom{3}{2}\left(\frac{2}{3}\right)^2\frac{1}{3} = 3\cdot\frac{4}{27} = \frac{12}{27}$

3 heads $\binom{3}{3}\cdot\left(\frac{2}{3}\right)^3 = \frac{8}{27}$

Prob $= \frac{12}{27} + \frac{8}{27} = \frac{20}{27}$

7. In Problem 6, what are the chances of more heads than tails if all three flips match?

[A] 1/2.
[B] 8/9.
[C] 16/27.
[D] 20/27.
[E] None of the above

B
No Work
No Credit

Match = TTT or HHH

$P[\text{match}] = \frac{1}{27} + \frac{8}{27} = \frac{9}{27}$

$P[TTT] = \binom{3}{0}\left(\frac{2}{3}\right)^0\left(\frac{1}{3}\right)^3 = \frac{1}{27}$

$P[HHH] = \binom{3}{3}\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^0 = \frac{8}{27}$

$P[\text{more H} \mid \text{Match}] = \frac{P[\text{more H} \wedge \text{Match}]}{P[\text{Match}]} = \frac{P[HHH]}{P[\text{Match}]} = \frac{8/27}{9/27} = \frac{8}{9}$

8. 1-in-20 men are color blind and 1-in-400 women are color blind. There are an equal number of men and women. What are the chances a random person is color blind?

[A] 1/20.
[B] 20/800.
[C] 21/800.
[D] 22/800.
[E] None of the above.

C
No work
No Credit

$P[CB] = P[CB \mid \text{Man}]\,P[\text{Man}] + P[CB \mid \text{Woman}]\,P[\text{Woman}]$

$= \frac{1}{20}\cdot\frac{1}{2} + \frac{1}{400}\cdot\frac{1}{2}$

$= \frac{1}{40} + \frac{1}{800} = \frac{21}{800}$

9. In Problem 8, what are the chances a random color blind person is a man?

[A] 1/20.
[B] 20/21.
[C] 20/800.
[D] 21/800.
[E] None of the above

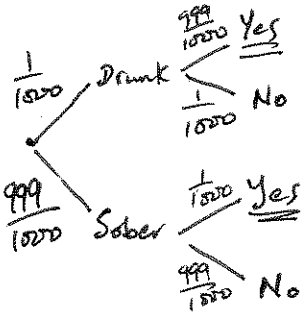
B
No Work
No Credit

$P[\text{Man} \mid CB] = \frac{P[\text{Man} \wedge CB]}{P[CB]} = \frac{\frac{1}{20}\cdot\frac{1}{2}}{\frac{21}{800}} = \frac{20/800}{21/800} = \frac{20}{21}$

10. 1-in-1000 drivers are drunk. A breathalyzer test is correct at saying if you are drunk or not 99.9% of the time. The breathalyzer says you are drunk. What are the chances you are drunk?

[A] 1/2.
[B] 99/100.
[C] 999/100.
[D] 999/1000.
[E] None of the above.

A
No Work
No Credit



$P[\text{Yes}] = \frac{999}{10^6} + \frac{999}{10^6}$

$P[\text{Drunk} \mid \text{Yes}] = \frac{P[\text{Drunk} \wedge \text{Yes}]}{P[\text{Yes}]} = \frac{\frac{999}{10^6}}{\frac{999}{10^6} + \frac{999}{10^6}}$

$= \frac{1}{2}$

11. A box has 9 fair coins and 1 two-headed coin. Pick a random coin and flip. What is are the chances of H?

A 9/20.

B 10/20.

C 11/20.

D 12/20.

E None of the above

C
No work
No credit

$$P[H] = \underbrace{P[H \mid \text{fair}]}_{\frac{1}{2}} \underbrace{P[\text{fair}]}_{\frac{9}{10}} + \underbrace{P[H \mid \text{biased}]}_{1} \underbrace{P[\text{Biased}]}_{\frac{1}{10}}$$

$$= \frac{9}{20} + \frac{1}{10} = \frac{11}{20}$$

12. In problem 11, your flip was H. What are the chances you picked a fair coin?

A 9/10.

B 9/11.

C 9/20.

D 11/20.

E None of the above

B

$$P[\text{fair} \mid H] = \frac{P[\text{fair} \text{ and } H]}{P[H]} = \frac{\frac{9}{10} \cdot \frac{1}{2}}{\frac{11}{20}} = \frac{9}{11}.$$

13. $\mathbf{X}$ is a uniform random variable taking a value in the set $\{1, 2, 3, 4\}$. What is $\mathbb{P}[\mathbf{X} = 2]$?

A 0.1.

B 0.2.

C 0.3.

D 0.4.

E None of the above.

E

All outcome probabilities the same.

4 outcomes → each probability = $\frac{1}{4}$

$$P[X = 2] = 0.25$$

14. In Problem 13, what is $\mathbb{P}[\mathbf{X} \geq 2]$?

A 0.

B 0.25.

C 0.5.

D 0.75.

E None of the above.

D

$$P[X \geq 2] = P[\{2, 3, 4\}] = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$$

$$= 0.75$$

15. In Problem 13, what is $\mathbb{E}[\mathbf{X}]$?

A 2.

B 2.5.

C 3.

D 3.5.

E None of the above.

B

$$E[X] = \sum_{x=1}^{4} x \, P(x)$$

$$= \sum_{x=1}^{4} x \cdot \frac{1}{4} = \frac{1}{4} \underbrace{\sum_{x=1}^{4} x}_{10}$$

$$= \frac{10}{4}$$

$$= 2.5$$

16. Roll a fair 6-sided die 4 times. What are the chances to roll exactly 2 fours?

A $140/6^4$.

B $150/6^4$.

C $160/6^4$.

D $170/6^4$.

E None of the above.

Binomial $n=4$ $p=\frac{1}{6}$ $P[2] = \binom{4}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2 = \frac{6\cdot 5^2}{6^4} = \frac{150}{6^4}$

Uniform Prob Space: # outcomes $6\times6\times6\times6 = 6^4$

outcomes with 2 fours: $\binom{4}{2}$ choices for where to place 4's.

Product Rule: $\binom{4}{2}\times 5\times 5 = 6\cdot 25 = 150$ (5×5 choices for other 2 rolls)

$\therefore$ Prob $= \frac{150}{6^4}$

B — No Work No Credit

17. A biased coin has probability of heads 3/4. What is the expected number of heads in 20 flips?

A 5.

B 10.

C 15.

D 20.

E None of the above.

$n=20$

$p=\frac{3}{4}$

$E[\#\text{ successes}] = np = 20\cdot\frac{3}{4} = 15$

C

18. A box has 5 fair and 5 two-headed coins. Pick a random coin and make 10 flips. What is $\mathbb{E}$[number of heads]?

A 6

B 6.5

C 7

D 7.5

E None of the above.

$E[\#H] = \underbrace{E[\#H \mid \text{fair}]}_{10\cdot\frac{1}{2}}\underbrace{P[\text{fair}]}_{\frac{5}{10}} + \underbrace{E[\#H \mid 2\text{-heads}]}_{10}\underbrace{P[2\text{-heads}]}_{\frac{5}{10}}$

$= 5\cdot\frac{1}{2} + 10\cdot\frac{1}{2}$

$= 2.5 + 5 = 7.5$

D — No work No credit.

19. Boys are twice as likely as girls. What is the expected number of kids till you have a boy?

A 1.

B 2.

C 3.

D 4.

E None of the above.

$P[B] = 2P[G]$

$P[B] + P[G] = 1 \rightarrow 3P[G] = 1 \rightarrow P[G] = \frac{1}{3} \rightarrow P[B] = \frac{2}{3}$

Expected wait to Boy $= \frac{1}{P[B]} = \frac{3}{2} = 1.5$

E

20. Ana and Amy have kids till a boy. For Ana, boys and girls are equally likely. For Amy, boys are twice as likely as girls. What is the expected number of kids Ana and Amy will have in total?

A 3.

B 3.5.

C 4.

D 4.5.

E None of the above.

X_1 = # Kids for Ana $\quad E[X_1] = \frac{1}{1/2} = 2.$

X_2 = # Kids for Amy $\quad E[X_2] = \frac{1}{P[B]} = \frac{1}{2/3} = 1.5$

total $= X_1 + X_2$

$E[X_1 + X_2] = E[X_1] + E[X_2] = 2 + 1.5 = 3.5$

Linearity of Expectation

B — No Work No Credit