Attentive Betweenness Centrality (ABC): Considering Options and Bandwidth when Measuring Criticality

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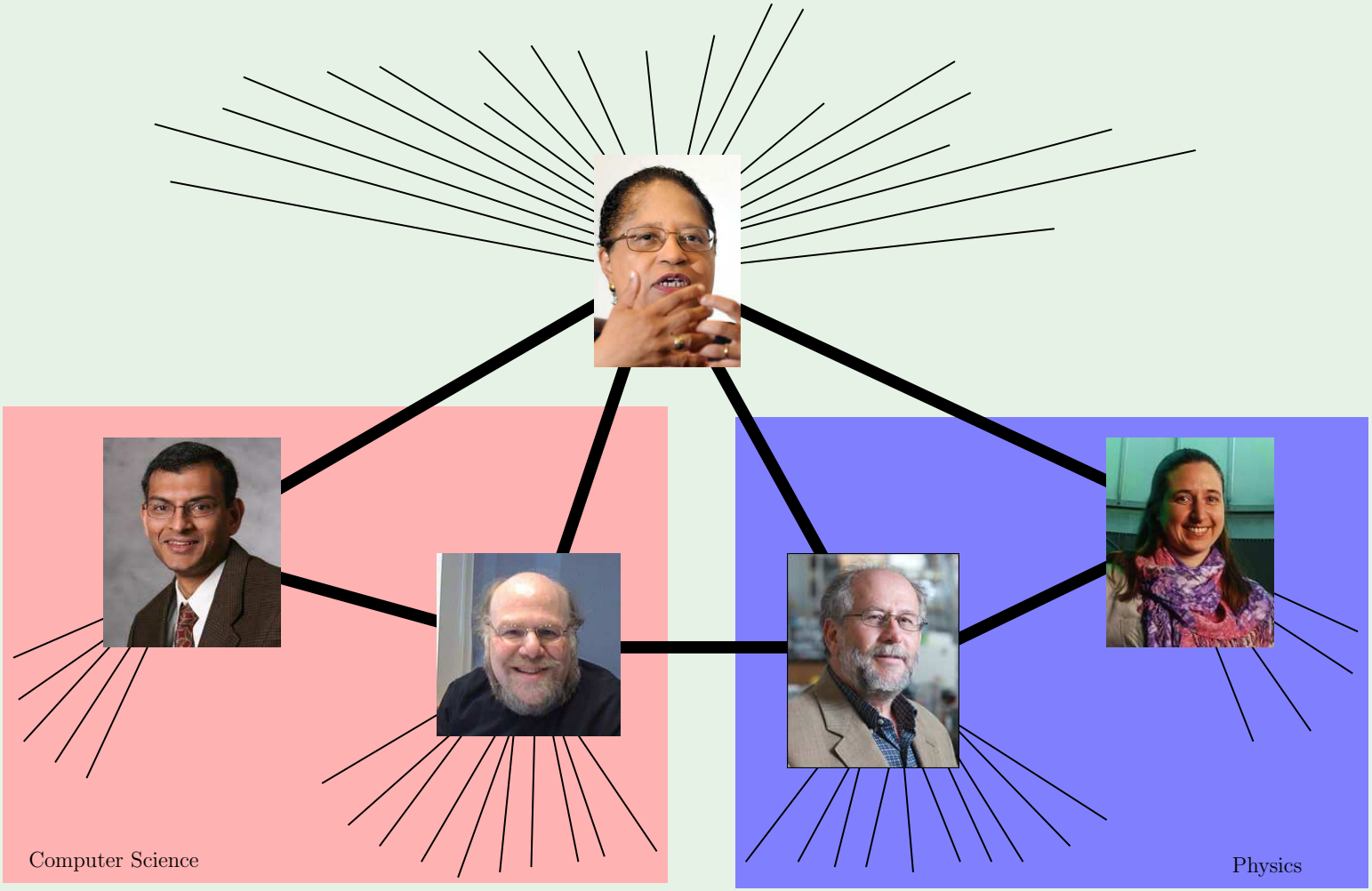
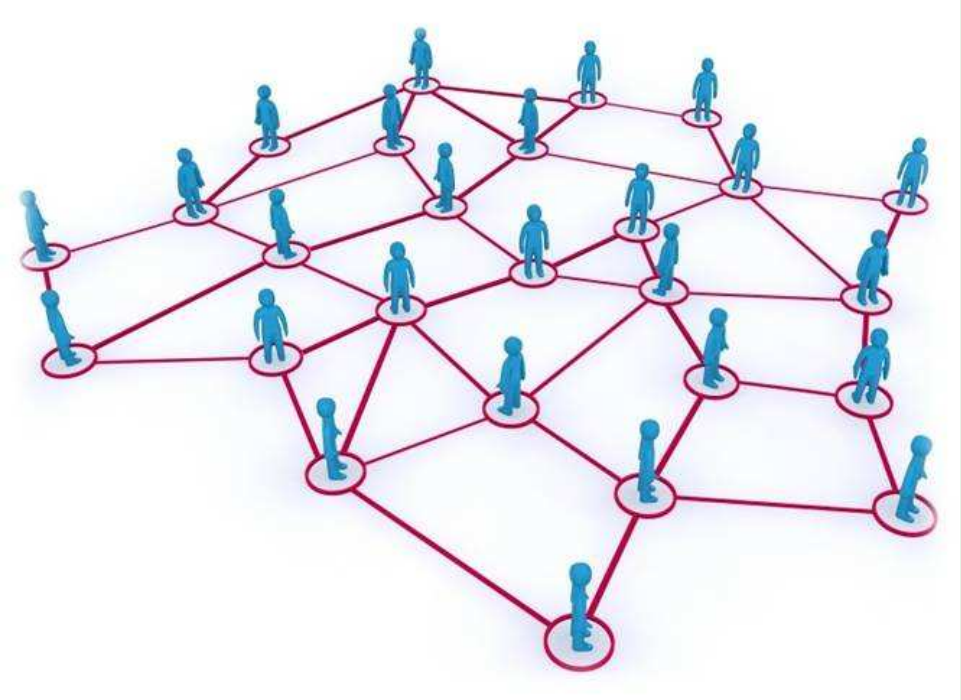
Who is the Most Critical?

Would you use the president of your university to reach a colleague?



How is it Done Today?

Network	# Nodes
Karate Club	34
RPI	8,000
DBLP	74,443
IMDB	33,557
Facebook	1 Billion
Twitter	1 Billion
World	7 Billion

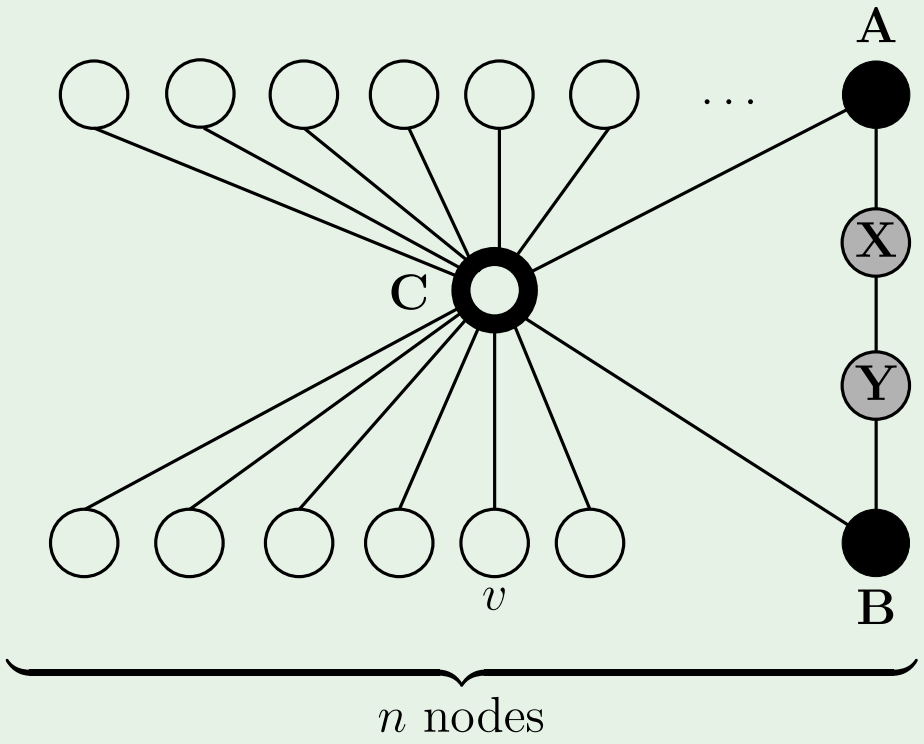


- degree-centrality
- pagerank
- closeness-centrality
- . . .
- betweenness-centrality

A node has high **betweenness centrality** if many shortest paths use the node.

Motivation: capture how critical a node is to the flow of information between other pairs of nodes.

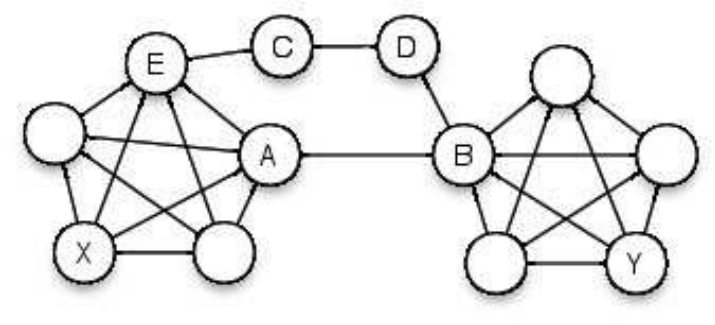
Betweenness



	actor		
	C	X	Y
Betweenness	1	0	0

What's Wrong?

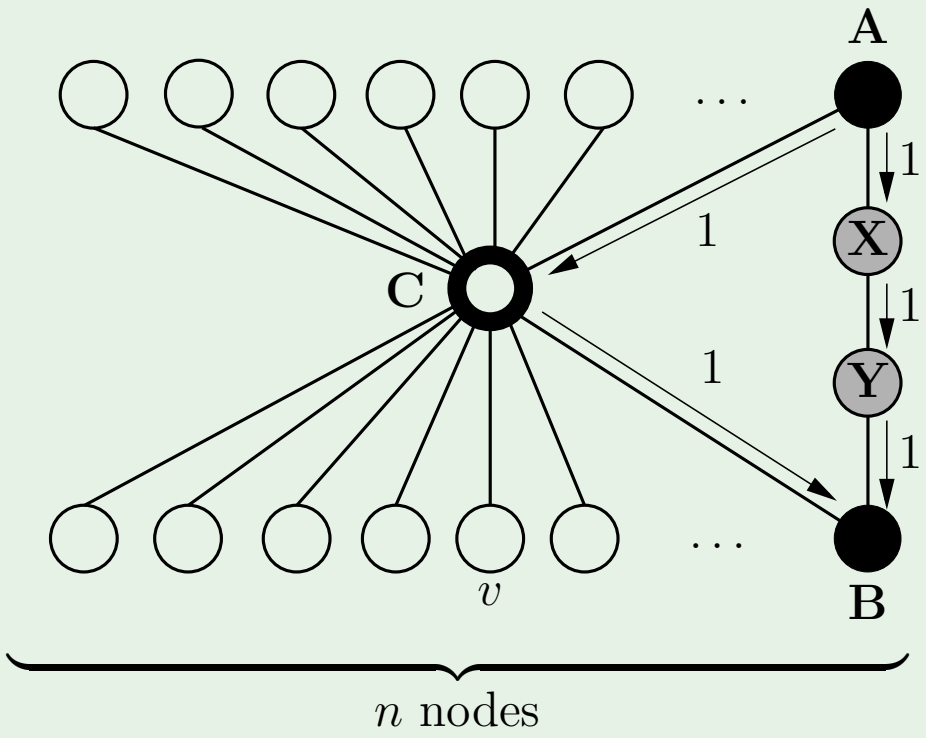
- Nodes on shortest paths are over emphasized (A, B); Nodes on almost shortest paths are marginalized (C).
- Information flow from X to Y uses just one path?
- How does X know the shortest path to Y?



Methods have evolved to address this:

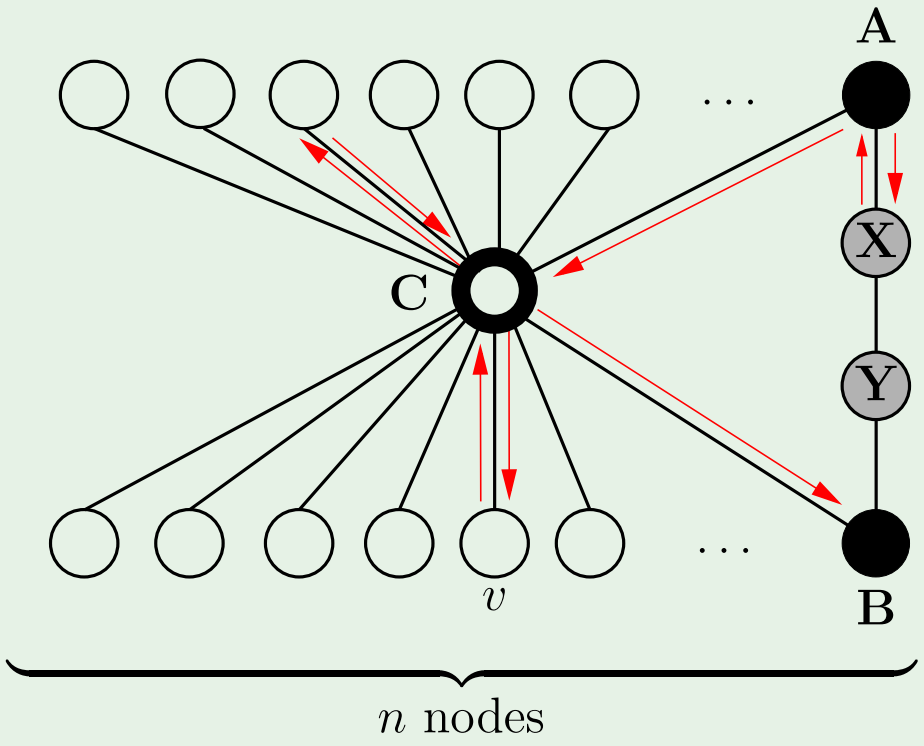
flow-betweenness; random walk betweenness

Flow-Betweenness



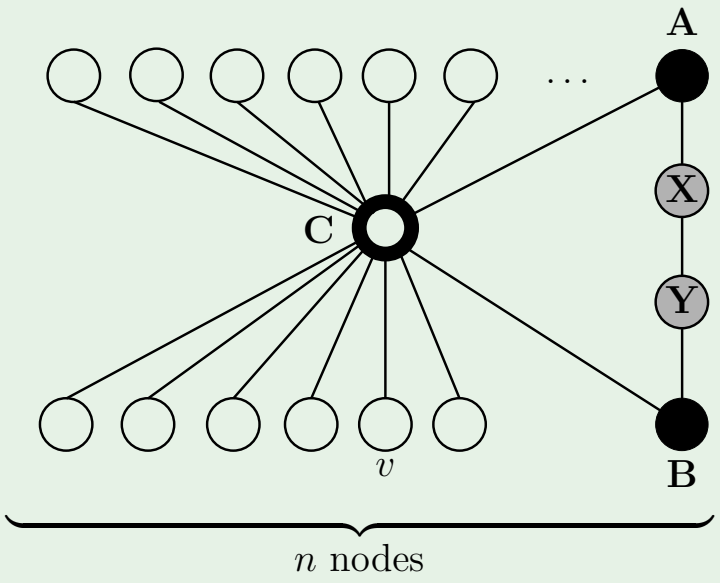
	actor		
	C	X	Y
Flow-Betweenness	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

Random Walk-Betweenness



	actor		
	C	X	Y
Random Walk-Betweenness	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{1}{2}$

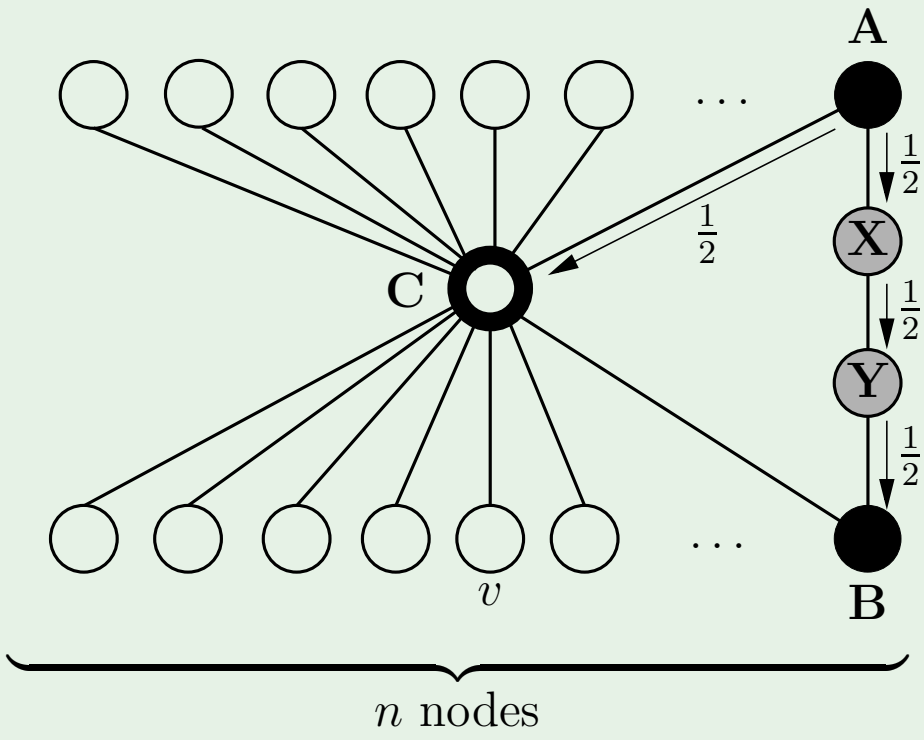
How it's Done Today



	actor		
	C	X	Y
Betweenness	**1**	0	0
Flow	**1**	1	1
Random Walk	**1**	$\frac{8}{9}$	$\frac{2}{3}$

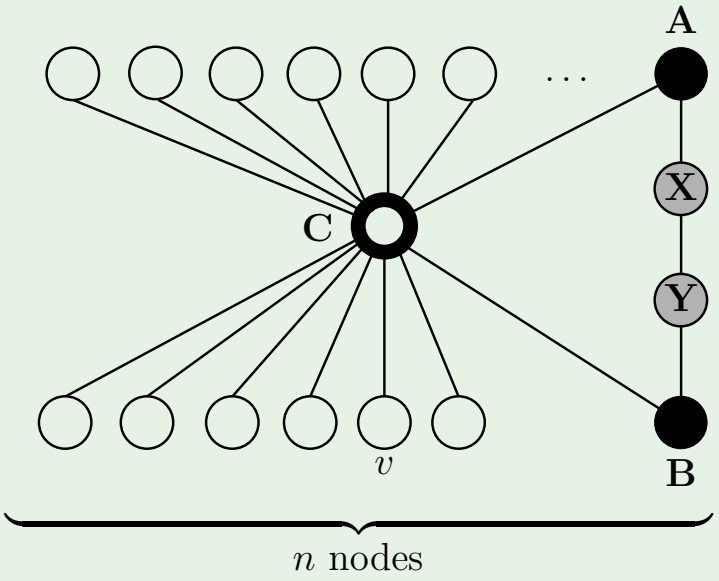
(scores renormalized so that max is 1)

Attentive Betweenness-Centrality (ABC)



- Imagine a unit of "information flow" starting at A
- *Only* $\frac{1}{2n-1}$ *of the flow to* C *makes it to* B.
- The total flow to B is therefore $\frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2n-1}$.
- The fraction of this total that flowed through C is $\frac{1}{2n}$

Attention Devalues High Degree Nodes



	actor		
	C	X	Y
Betweenness	1	0	0
Flow	1	1	1
Random Walk	1	$\frac{8}{9}$	$\frac{2}{3}$
ABC-Centrality	$\frac{1}{2n-1}$	1	1

(scores renormalized so that max is 1)

"In critical and baffling situations, it is always best to return to first principle and **simple action**"

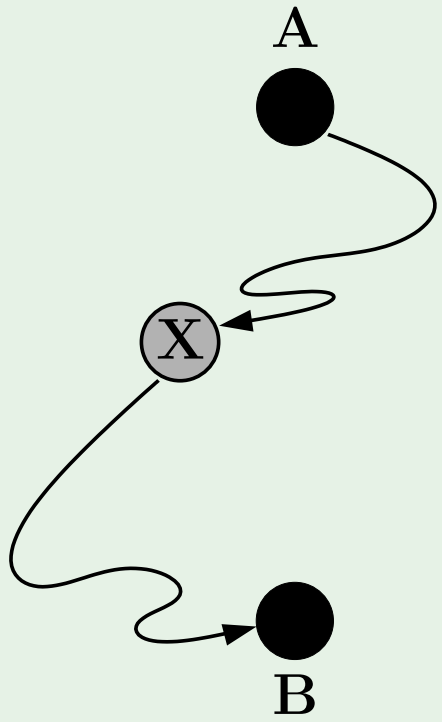
- Sir Winston Churchill

Information Flow is the Basis of Betweenness

- A sends information to B

- X is critical to some of this $A \to B$ flow.

$$Bet_{A\to B}(X) = \frac{A \to B \text{ information flow through } X}{A \to B \text{ information flow}}$$



- The betweenness of X is the average.

$$Bet(X) = \text{average}_{A,B}\Big[Bet_{A\to B}(X)\Big]$$

Four "Axioms" of Information Flow

I. **Forward Propagation**

An actor will not send information back along edges from where the information came.

II. **Locality**

An actor cannot process global information and perform global algorithms in determining how to forward.

III. **Attention**

Actors have a finite attention they can give a piece of information – cannot service all neighbors all the time.

IV. **Multipath**

Information may flow along multiple paths; longer paths are less valuable than shorter ones.

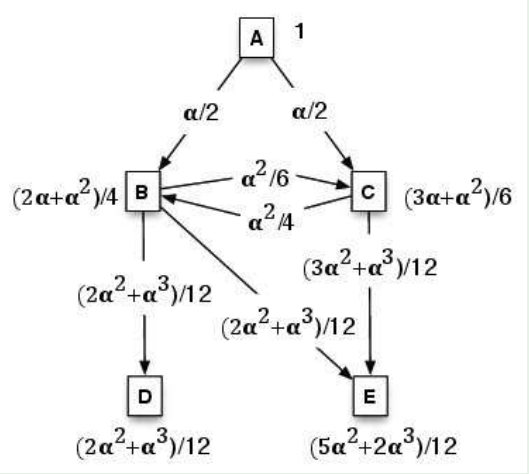
ABC-Centrality Satisfies the Four Axioms

	Betweenness measure			
	Bet.	ABC	Flow	Rand Walk
Forw. Prop.	✓	✓	✓	✘
Locality	✘	✓	✘	✓
Attention	✘	✓	✘	✘
Non-shortest	✘	✓	✓	✓
Complexity	$O(mn)$	$O(mn)$	$O(m^2n)$	$O(mn^2)$

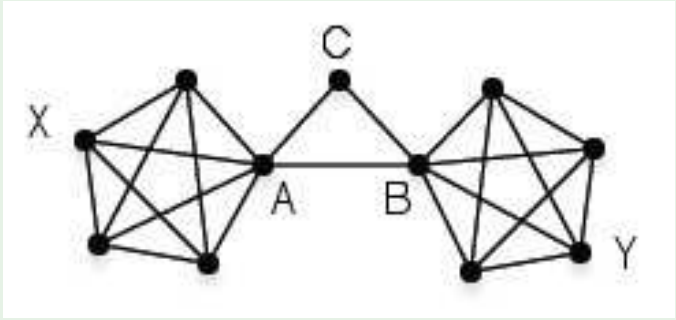
- Efficient BFS-like algorithm.
- Applies to directed and weighted graphs.

Attenuation parameter $\boldsymbol{\alpha}$ determines how important longer paths are.

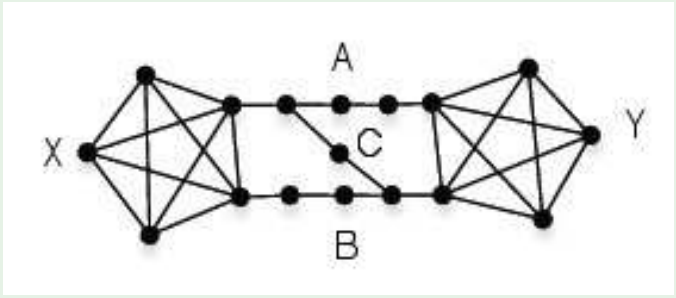
$\alpha \to 0$: ABC is similar to betweenness with *attention*.



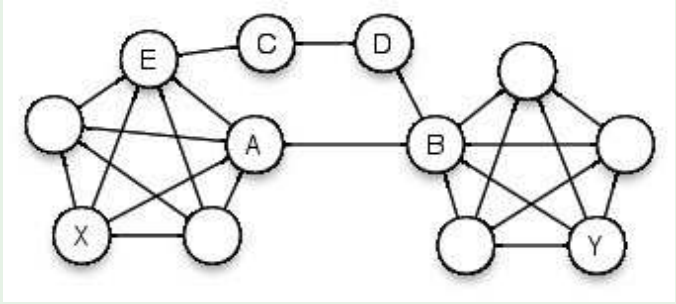
ABC Works Well in Stylized Networks



Measure	A, B	C	X, Y
BET	1.00	0.00	0.00
FLOW	1.00	0.45	0.10
Random Walk	1.00	0.49	0.40
PageRank	1.00	0.39	0.69
ABC	1.00	0.33	0.12

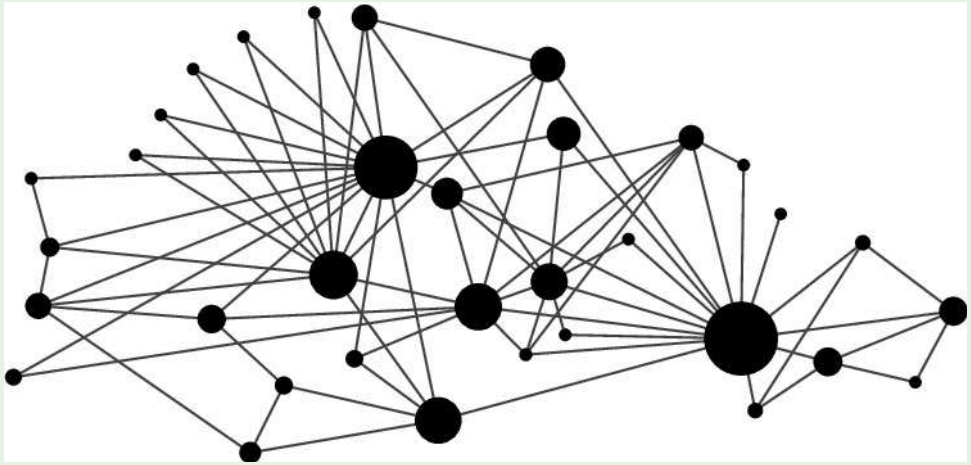


Measure	A, B	C	X, Y
BET	1.00	0.81	0.00
FLOW	1.00	0^+	0.06
Random Walk	1.00	0.84	0.59
PageRank	0.68	0.68	1.00
ABC	1.00	0.75	0.19

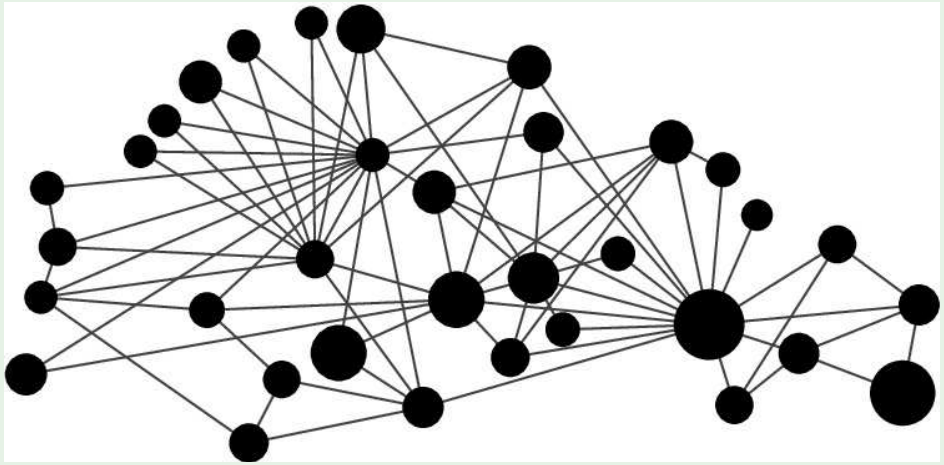


Measure	A	B	C	D	E	X	Y
BET	0.71	1.00	0.09	0.16	0.18	0.00	0.00
FLOW	0.62	1.00	0.46	0.46	0.62	0.08	0.08
Random Walk	0.75	1.00	0.48	0.50	0.60	0.50	0.38
PageRank	0.83	1.00	0.40	0.40	0.83	0.66	0.66
ABC	0.63	1.00	0.24	0.27	0.29	0.10	0.10

Random Walk has Trouble Differentiating (Karate Club)



ABC-Centrality



Random Walk Centrality

Centrality score correlation matrix

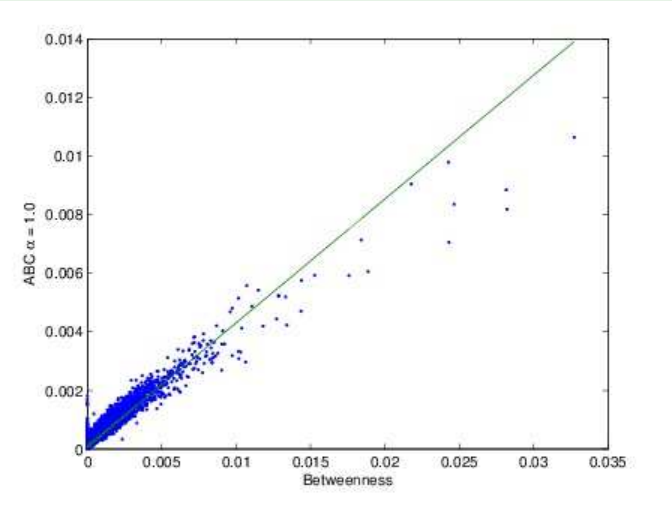
	BET	ABC^1	ABC^{0^+}	RW	DEG	CL	FLOW	PG
BET	1	**.98**	$\mathbf{1^-}$	.51	.92	.72	.95	.92
ABC^1	.98	1	.98	.51	.96	.77	.96	.97
ABC^{0^+}	1^-	.98	1	.51	.92	.73	.96	.93
RW	.51	.51	.51	1	.41	.32	.53	.42
DEG	.92	.96	.92	.41	1	.77	.91	1^-
CL	.72	.77	.73	.32	.77	1	.59	.74
FLOW	.95	.96	.96	.53	.91	.59	1	.93
PG	.92	.97	.93	.42	1^-	.74	.93	1

BET=betweenness; RW=Random Walk; DEG=degree; CL=closeness; PG=PageRank

ABC Works in Real Networks (IMDB)

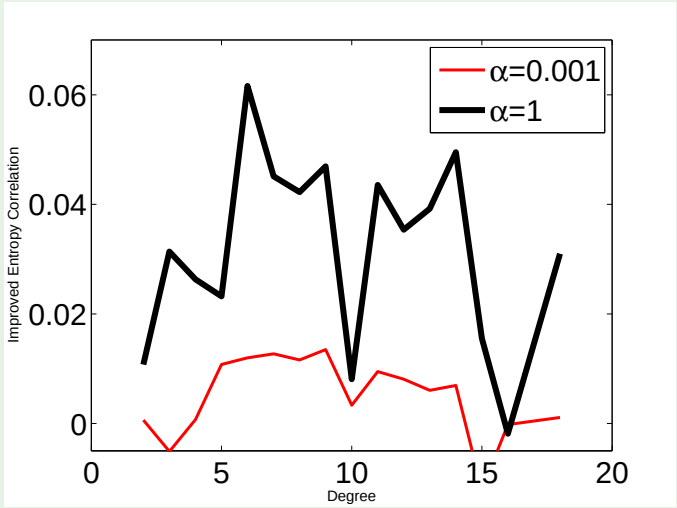
Betweenness and ABC are correlated

- Very high betweenness gets dampened;
- Low betweenness nodes can improve.



Most "critical" actors are diverse

Rank	Actor
1	Michael Masden
2	David Carradine
3	James Russo
4	Joe Estevez
5	Eric Roberts
⋮	⋮



ABC scores correlate better with diversity

Wrapping Up

Critical nodes are Caring Conduits

- Information flow is at the hear of betweenness.

	Betweenness measure			
	Bet.	ABC	Flow	Rand Walk
Forw. Prop.	✓	✓	✓	✘
Locality	✘	✓	✘	✓
Attention	✘	✓	✘	✘
Non-shortest	✘	✓	✓	✓
Complexity	$O(mn)$	$O(mn)$	$O(m^2n)$	$O(mn^2)$

Our principles of information flow are very general.

- ABC-centrality satisfies the basic principles, yet captures the essence of betweenness.
- Validated on stylized and real networks.
- Software: http://www.cs.rpi.edu/lfdlab

Thank You

Questions?